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15:1 Resonance effects on the orbital motion of artificial satellites

***Abstract**: The motion of an artificial satellite is studied considering geopotential perturbations and resonances between the frequencies of the mean orbital motion and the Earth rotational motion. The behavior of the satellite motion is analyzed in the neighborhood of the resonances 15:1. A suitable sequence of canonical transformations reduces the system of differential equations describing the orbital motion to an integrable kernel. The phase space of the resulting system is studied taking into account that one resonant angle is fixed. Simulations are presented showing the variations of the semi-major axis of artificial satellites due to the resonance effects.*
***Keywords**: Resonance, Artificial satellites, Celestial mechanics.*

INTRODUCTION

The problem of resonance effects on orbital motion of satellites falls under a more categorical problem in astrodynamics, which is known as the one of zero divisors. The influence of resonances on the orbital and translational motion of artificial satellites has been extensively discussed in the literature under several aspects. In fact, for instance, it has been considered the (Formiga, 2005): resonance of the rotation motion of a planet with the translational motion of a satellite (Lima Jr., 1998; Formiga, 2005); sun-synchronous resonance (Hughes, 1980); spin-orbit resonance (Beleskii, 1975; Vilhena De Moraes e Silva, 1990); resonances between the frequencies of the satellite rotational motion (Hamill and Blitizer, 1974); and resonance including solar radiation pressure perturbation (Ferraz Mello, 1979). Generally, the problem involves many degrees of freedom because there can be several zero divisors, but although the problem is still analytically unsolved, it has received considerable attention in the literature from an analytical standpoint. It justifies the great attention that has been given in literature to the study of resonant orbits characterizing the dynamics of these satellites, as can be seen in recent published papers (Deleflie, *et al*., 2011; Anselmo and Pardini, 2009; Chao and Gick, 2004; Rossi, 2008).

In this paper, the type of resonance considered is the commensurability between the frequencies of the satellite mean orbital motion and the Earth rotational one. Such case of resonance occurs frequently in real cases. In fact, in a survey from a sample of 1818 artificial satellites, chosen in a random choice from the NORAD 2-line elements (Celestrak, 2004), about 85% of them are orbiting near some resonance's region. In our study, it satellites in the neighbourhood of the 15:1 resonance (Fig. 1), or satellites with an orbital period of about 1.6 hours will be considered. In our choice, the characteristic of the 356 satellites under this condition can be seen in Table 1 (Formiga, 2005).

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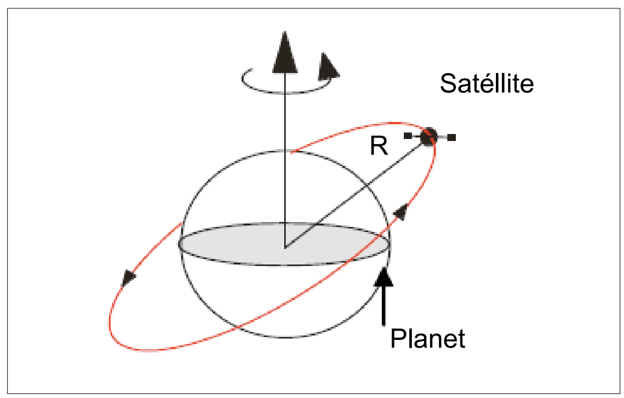

Figure 1. Orbital resonance.

Table 1. Orbital characteristics for 15:1 resonance.

Orbital characteristic for 15:1 resonance	Number of satellites
e≤0,1 e i≤5°	1
e≤0,1 e i≥70°	290
e≤0,1 e 55°<i<70°	41
e≤0,1 e 5°<i<55°	26

The system of differential equations describing the orbital motion of an artificial satellite under the influence of perturbations due to the geopotential, is described here in a canonical form. In order to study the effects of resonances, a suitable sequence of canonical transformations was performed reducing the system of differential equations to an integrable kernel (Lima Jr., 1998). This system is integrated numerically, and simulations can show the behaviour of motion in the neighbourhood of the exact resonance. Some results are presented for the 15:1 resonances, where graphics representing the phase space and the time variation of some Keplerian elements are exhibited.

THE CONSIDERED POTENTIAL

Using Hansen's coefficients, the geopotential can be written as in Eq. 1 (Osório, 1973):

$$U = \frac{\mu}{2a} + \sum_{\ell=2}^{\infty}\sum_{m=0}^{\ell}\sum_{p=0}^{\ell}\sum_{q=-\infty}^{\infty}\frac{\mu}{a}\left(\frac{a_e}{a}\right)^{\ell} J_{\ell m}F_{\ell mp}(i) H_q^{-(\ell+1),(\ell-2p)}(e)\cos\varphi_{\ell mpq}(M,\omega,\Omega,\Theta) \quad (1)$$

where:

a, e, I, M, Ω, ω are the Keplerian elements;

$\Theta = \omega_E t$ is the sidereal time;

ω_E is the Earth's angular speed;

$J_{\ell m}$ are coefficients depending on the Earth's mass distribution;

$F_{\ell mp}(i)$ represents the inclination functions; and

$H_q^{(\ell+1),(\ell-2p)}$(e) are the Hansen's coefficients.

The argument is

$$\varphi_{\ell mpq}(M,\omega,\Omega,\Theta) = qM + (\ell-2p)\omega + m(\Omega-\Theta-\lambda_{\ell m}) + (\ell-m)\frac{\pi}{2} \quad (2)$$

where,

$\lambda_{\ell m}$ is the corresponding reference longitude of semi-major axis of symmetry for the harmonic (ℓ,m).

Considering perturbations due to geopotential and the classical Delaunay variables, the Lagrange equations describing the motion can be expressed in canonical forms and a Hamiltonian formalism can be used. The Delaunay canonical variables are given by Eq. 3:

$$L = \sqrt{\mu a},\ G = \sqrt{\mu a(1-e^2)},\ H = \sqrt{\mu a(1-e^2)}\cos i,\ \ell' = M,\ g = \omega,\ h = \Omega \quad (3)$$

where,

ℓ', g, h are coordinates; and
L, G, H are the conjugated moment.

Extending the phase space, where a new variable Θ, conjugated to Θ(t), is introduced to eliminate the explicit time dependence, the Hamiltonian F=F (L, G, H, Θ, ℓ', g, h, Θ) of the corresponding dynamical system is

$$F = \frac{\mu^2}{2L^2} + R_{\ell mpq} \quad (4)$$

with

$$R_{\ell mpq} = \sum_{\ell=2}^{\infty}\sum_{m=0}^{\ell}\sum_{p=0}^{\ell}\sum_{q=-\infty}^{\infty}\frac{\mu^2}{L^2}\left(\frac{\mu\, a_e}{L^2}\right)^{\ell} J_{\ell m}x\, xF_{\ell mp}(L,G,H)H_q^{-(\ell+1),((\ell-2p)}(L,G)x \cos\varphi_{\ell mpq}(\ell',g,h,\Theta) \quad (5)$$

where,

$$\varphi_{\ell mpq}(\ell', g,h,\Theta) = q\ell' + (\ell-2p)g + m(h-\Theta-\lambda_{\ell m}) + (\ell'-m)\frac{\pi}{2} \quad (6)$$

METHODOLOGY

The following procedure, including a sequence of canonical transformations, enables us to analyse the influence of the resonance upon the orbital elements (Lima Jr., 1998):

a) canonical variables (X, Y, Z, Θ, x, y, z, Θ) related with the Delaunay variables are introduced by the canonical transformation described as follows:

$$\begin{array}{lll} X = L & Y = G-L & Z = H-G \\ x = \ell' + g + h & y = g+h & z = h \end{array} \quad (7)$$

Thus, the new Hamiltonian is

$$H(X,Y,Z,\Theta,x,y,z,\theta) = \frac{\mu^2}{2X^2} + \omega_e\theta + R'_{\ell mpq} \quad (8)$$

where,

$$R'_{\ell mpq} = \sum_{\ell=2}^{\infty}\sum_{m=0}^{\ell}\sum_{p=0}^{\ell}\sum_{q=-\infty}^{\infty}\frac{\mu^{\ell+2}}{X^{2\ell+2}}a_e^{\ell}J_{\ell m}F_{\ell mp}(X,Y,Z) H_q^{-(\ell+1),(\ell-2p)}(X,Y)\cos\varphi_{\ell mpq}(x,y,z,\Theta) \quad (9)$$

b) a reduced Hamiltonian, containing only secular and periodic terms containing a commensurability between the frequencies of the motion, is constructed.

c) A new Hamiltonian considering a given resonance is constructed. Thus, if n stands for the orbital mean motion, the resonance condition can be expressed by

$$qn - m\omega_E = 0 \quad (10)$$

where, q and m are integers. We will denote by α=q/m the commensurability of the resonance.

The Hamilton is simplified as

$$H_r = \frac{\mu^2}{2X^2} + \omega_e\theta + \sum_{j=1}^{\infty} B_{2j,0,j,0}(X,Y,Z) + \sum_{\ell=2}^{\infty}\sum_{m=2}^{\ell}\sum_{p=0}^{\ell} B_{\ell mp(\alpha m)}(X,Y,Z)\cos\varphi_{\ell mp(\alpha m)} \tag{11}$$

with

$$\varphi_{\ell mp(\alpha m)} = m(\alpha x - \Theta) + (\ell - 2p + m\alpha)y + (m - \ell - 2p)z - m\lambda_{\ell m} + (\ell - m)\frac{\pi}{2}, \tag{12}$$

$$B_{\ell mp(\alpha m)}(X,Y,Z) = \frac{\mu^{\ell+2}}{X^{2\ell+2}} a_e^{\ell} J_{\ell m} F_{\ell mp}(X,Y,Z)\, H_{\alpha m}^{-(\ell+1),\,(\ell-2p)}(X,Y) \tag{13}$$

where

$$B_{2j,0,j,0}(X,Y,Z) = \frac{\mu^{2j+2}}{X^{4j+2}} a_e^{2j} J_{2j,0} F_{2j,0,j} H_0^{-(2j+1),2j} \tag{14}$$

are secular terms obtained from the conditions $q=m=0$ e $\ell=2p$. Equation 14 shows all the resonant terms with tesseral harmonics concerned in the resonance α.

A first integral given by

$$(1-\frac{1}{\alpha})X + Y + Z = C_1$$

can be obtained for the new Hamiltonian system with the Hamiltonian given by H_r

d) Using the first integral C1, the following Mathieu transformation (second canonical transformation) is introduced reducing the order of the system.

$$X_1 = X \qquad x_1 = x = (1-\frac{1}{\alpha})z$$
$$Y_1 = Y \qquad y_1 = y - z$$
$$Z_1 = (1-\frac{1}{\alpha})X + Y + Z \qquad z_1 = z$$
$$\Theta_1 = \Theta \qquad \theta_1 = \theta$$

Fixing a value for one resonant angle, we can consider as short period terms all periodic terms different from the fixed one.

A frequency α is selected and we consider a new Hamiltonian system containing just secular and resonant terms

$$H_c = \frac{\mu^2}{2X_1^2} + \omega_E\theta_1 + \sum_{j=1}^{\infty} B_{2,2j,0,j,0}(X_1,Y_1,C) + \sum_{\ell=2}^{\infty}\sum_{p=0}^{\ell} B_{lmp,(\alpha m)}\cos\varphi_{\ell mp,(\alpha m)}(x_1,y_1,\Theta_1)$$

A first integral C2 given by

$$(l-2p-m\alpha)X_1 - m\alpha Y_1 = C_2$$

can be found for this new Hamiltonian system.

e) A third canonical transformation given by

$$X_2 = X_1 \qquad x_2 = x_1 + (\frac{k-m\alpha)}{m\alpha}$$
$$Y_2 = (k-m\alpha)X_1 - m\alpha Y_1 \qquad y_2 = -\frac{1}{m\alpha}y_1$$
$$\Theta_2 = \Theta_1 \qquad \theta_2 = \theta_1$$

is performed and A new Hamiltonian is defined as critical Hamiltonian (H_c), with secular terms in the X_1 variable and the resonant terms with critical frequencies.

Introducing the coefficients k=ℓ-2p, with $\ell^3 \geq 2$, $s \leq p \leq \infty$, where s is value minimum by p to $s^3 \geq 0$ and k depends on the frequency chosen, we can determine a new dynamical system as function of $H_c = H_c(X_1, \Theta_1, x_1, \theta_1)$, where H_c is

$$H_c = \frac{\mu^2}{2X_1^2} + \omega_e\theta_1 + \sum_{j=1}^{\infty} B_{2j,0,j,0}(X_1,C_1,C_2) + \sum_{p=s}^{\infty} B_{(2p+k)mp(\alpha m)}(X_1,C_1,C_2)\cos\varphi_{(2p+k)mp(\alpha m)}(x_1,\Theta_1) \tag{15}$$

ONE RESONANT ANGLE

The influence of the resonance on the orbital motion can be analyzed integrating a system of differential equations, where the reduced Hamiltonian is obtained from the one with secular and resonant terms:

$$\frac{dX_1}{dt} = -\sum_{p=s}^{\infty} B_{(2p+k)mp(\alpha m)}(X_1,C_1,C_2)\sin\varphi^*_{(2p+k)mp(\alpha m)}(x_1,\Theta_1)$$

and

$$\frac{d\varphi^*_{(2p+k)mp(\alpha m)}}{dt} = m\alpha\frac{\mu^2}{X_1^3} - m\omega_e - m\alpha\sum_{j=1}^{\infty}\frac{\partial B_{2j,0,j,0}(X_1,C_1,C_2)}{\partial X_1} - m\alpha\sum_{p=s}^{\infty}\frac{\partial B_{(2p+k)mp(\alpha m)}(X_1,C_1,C_2)}{\partial X_1}\cos\varphi^*_{(2p+k)mp(\alpha m)}(x_1,\Theta_1) \tag{16}$$

The analysis of the effects of the 15:1 resonance will be done here considering the tesseral $J_{15,15,}$ and the zonal harmonic J_2 with k=13. The dynamical systems (Eq.16) is:

$$\frac{dX_{1,15,15,1,1}}{dt} = -B_{15,15,1,1}(X_1,C_1,C_2)$$
$$\sin\varphi^*_{15,\,15,11}(x_1,\Theta_1)$$

and

$$\frac{d\varphi^*_{15,15,1,1}}{dt} = \frac{\mu^2}{X_1^3} - 15\omega_e -$$
$$\frac{\partial B_{2,0,1,0}(X_1,C_1,C_2)}{\partial X_1} \tag{17}$$
$$-\frac{\partial B_{15,15,1,1}(X_1,C_1,C_2)}{\partial X_1}$$
$$\cos\varphi^*_{15,15,1,1}(x_1,\Theta_1)$$

The secular and resonance terms are, respectively, Eqs. 18 and 19:

$$B_{15,15,1,1}(X_1,C_1,C_2) = \frac{\mu^{17}}{X_1^{32}} a_e^{15} J_{15,15} \tag{18}$$
$$F_{15,15,1}(X_1,C_1,C_2) H_1^{-16,13}(X_1,C_1)$$

and

$$B_{2,0,1,0}(X_1,C_1,C_2) = \frac{\mu^4}{X^6} a_e^2 J_{2,0} F_{2,0,1}(X_1,C_1,C_2) \tag{19}$$
$$H_0^{-3,2}(X_1,C_2)$$

Considering Eq. 18 and Eq. 19 we have explicit functions relating $B_{(2p+k)mp(\acute{\alpha}m)}(X_1, C_1, C_2)$ and $F_{lmpq}(X_1, C_1, C_2)$ from the Eqs. 20 and 21:

$$\frac{dX_{1,15,15,1,1}}{dt} = \frac{4{,}15020466}{4X_1^8(C_2-3X_1)^{29}X_1^4}$$
$$\times 10^{25}.J_{15,15}.a_e^{15}.\mu^{17}\sqrt{\left(1-\frac{(C_2-3X_1)^2}{X_1^2}\right)^{13}} \tag{20}$$
$$\cos\left[\frac{1}{2}.\mathrm{arcos}\left(\frac{C_1+15X_1}{13X_1-C_2}\right)\right]^{28}\sin\varphi^*_{15,15,1,1}$$

$$\frac{d\varphi^*_{15,15,1,1}}{dt} = \frac{\mu^2}{X_1^3} - 15\omega_e - \frac{3.J_2 a_e^2\mu^4}{4(C_2-3X_1)^5 X_1^4\sqrt{1-\frac{(C_1+15X_1)^2}{(C_2-13X_1)^2}}}$$

$$\left\{(C_2^2-9C_2X_1+18X_1^2)\sqrt{1-\frac{(C_1+15X_1)^2}{(C_2-13X_1)^2}}+\left[-4+6.\cos\left(\frac{1}{2}.\arccos\left(\frac{C_1+15X_1}{13X_1-C_2}\right)\right)\right]+\right.$$
$$\left.3(C_1+C_2)X_1.\sin\left(\frac{1}{2}.\arccos\left(\frac{C_1+15X_1}{13X_1-C_2}\right)\right)\right\}+\frac{4{,}15020466\times 10^{25}.J_{15,15}.a_e^{15}.\mu^{17}}{4(C_2-3X_1)^{31}X_1^{16}\sqrt{1-\frac{(C_1+15X_1)^2}{(C_2-13X_1)^2}}}$$

$$\sqrt{1-\frac{(C_2-15X_1)^2}{X_1^2}}(C_2^2-26C_2X_1+168X_1^2)\cos\left(\frac{1}{2}.\arccos\left(\frac{C_1+15X_1}{13X_1-C_2}\right)\right)$$
$$\left[\sqrt{1-\frac{(C_1+15X_1)^2}{(C_2-13X_1)^2}}\cos\left(\frac{1}{2}.\arccos\left(\frac{C_1+15X_1}{13X_1-C_2}\right)\right)\sin\left(\frac{1}{2}.\arccos\left(\frac{C_1+15X_1}{13X_1-C_2}\right)\right)\right.$$
$$.(16C_2^4-1040C_2^3X_1+24333C_2^2X_1^2+245609C_2X_1^3+908544X_1^4)$$
$$\left.+14(13C_1-15C_2)X_1(C_2^2-26C_2X_1+168X_1^2)\ \right]\cos\varphi^*_{15,15,1,1} \tag{21}$$

The critical angle is

$$\varphi*_{15,15,1,1} = x_1 - 15\Theta_1 + 15\lambda_{15,15} \tag{22}$$

Finally, we can compute the time variations for the Keplerian elements: a, e, i, through the inverse transformations:

$$a=\frac{X_1^2}{\mu} \quad e=\sqrt{1-\frac{(kX_1-C_1)^2}{m^2\alpha^2X_1^2}} \quad i=\cos^{-1}\left[\frac{mX_1+C_1}{kX_1-C_2}\right] \tag{23}$$

where

$$C_1=\sqrt{\mu a}\left(\sqrt{1-e^2}\cos i - 1/\alpha\right)$$
$$C_2=\sqrt{\mu a}\left(k-m\alpha\sqrt{1-e^2}\right) \tag{24}$$

with C_1 and C_2 as integration constants.

RESULTS

In this section we present numerical results considering some initial conditions arbitrarily chosen. Several harmonics can also be considered and as an example, for the 15:1 resonance, it was considered the simultaneous influence of the harmonics J_2 e $J_{15,15}$.

Let us consider the case e=0.019, i=87°, φ^*=0 (critical angle) and the harmonics J_2 and $J_{15,15}$, where numerical values for the Harmonics coefficients are given by

JGM-3 (Tapley *et al.*,1996). The phase space for the dynamical system (18)–(19) is presented by Fig. 2. This space is topologically equivalent to that of the simple pendulum. The behavior of the motion is presented here in the neighborhood of the separatrix (*a* about 6930 km).

Figure 2 represents the temporary variation of the semi-major axis in the neighborhood of the 15:1 resonance. For one value considered for the semi-major axes, distinct behaviors for their temporary variations can be observed. The central circle represents a new region after an abrupt variation due to the resonance effect. The more the satellites approach the region, which we defined as a resonant one, the more the variations increase. It is remarkable the oscillation in the region between 1,420 and 1,440 days, which characterizes paths for which the effect of the resonance is maximum for the case, where e=0.01 and i=4°. This effect can be seen in Figs. 3 and 4 with more details.

A new region of libration can be seen in Figs. 5 and 6 when satellites are outside the neighborhood of the resonance. The stabilization of the orbit that appears after a period of 1,420 days after a maximum is related with discontinuity produced by the resonance.

By Figs. 7 and 8, we can see that the amplitude of variation of the semi major axis do not change in neighborhoods of the resonance. This libration motion remains for a long time. In both cases, in the resonance

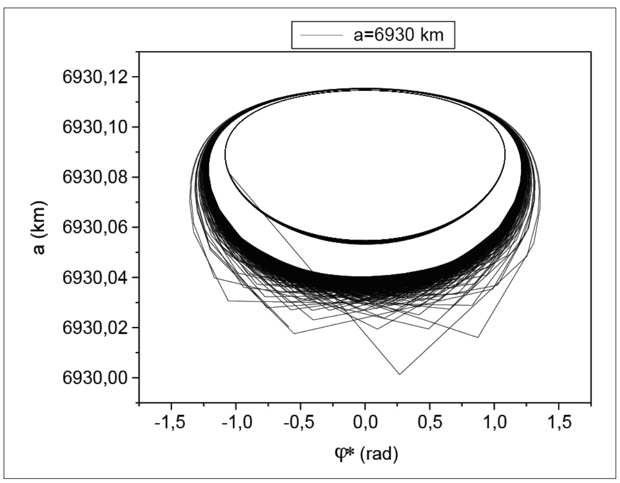

Figure 2. Semi-axis *versus* critical angle: e=0.019, i=87º e φ*=0.

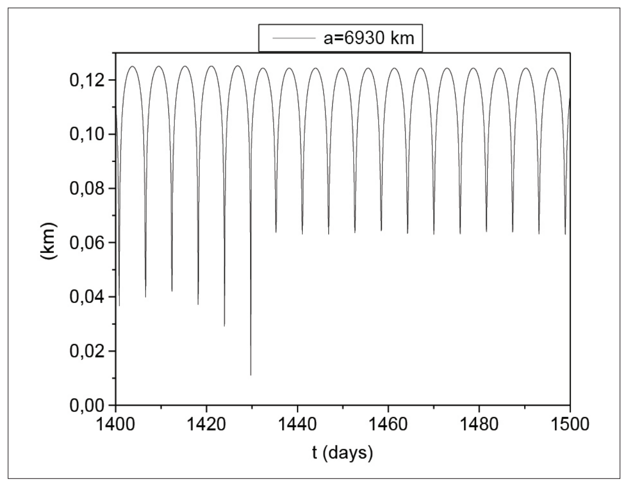

Figure 4. Amplification: Δa *versus* time: *e*=0.019, i=87º e φ*=0.

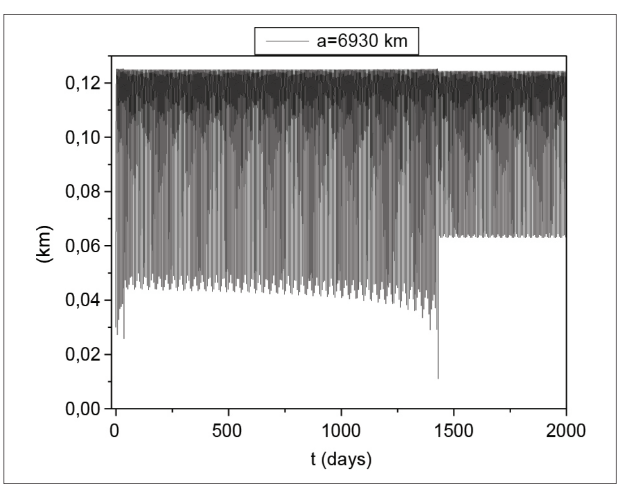

Figure 3. Variation of semi-major axis in the time: e=0.01, i=4°, φ*=0°.

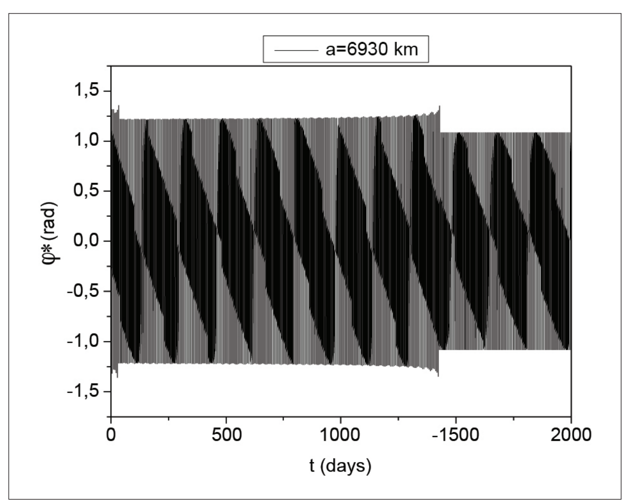

Figure 5. Variation of critical angle in the time: *e*=0.019, i=87º e φ*=0.

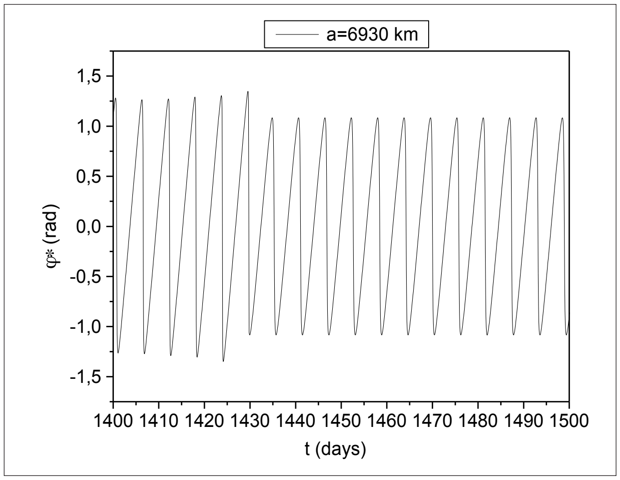

Figure 6. Amplification: Variation of critical angle in the time: e=0.019, i=87º e φ*=0.

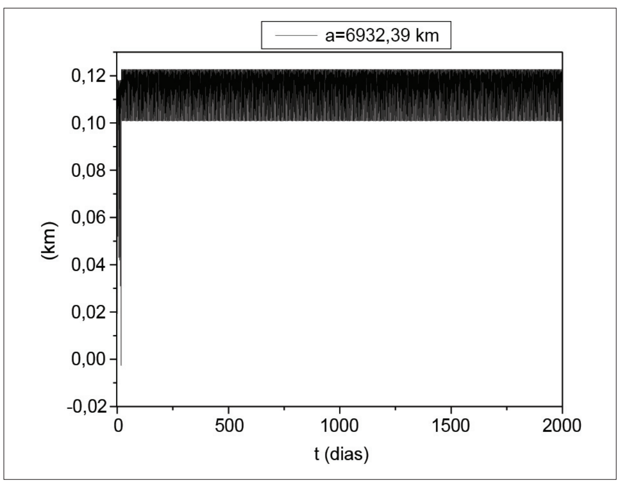

Figure 7. Variation of semi-major axis in the time: e=0.01, i=4°, φ^{*}= 0°.

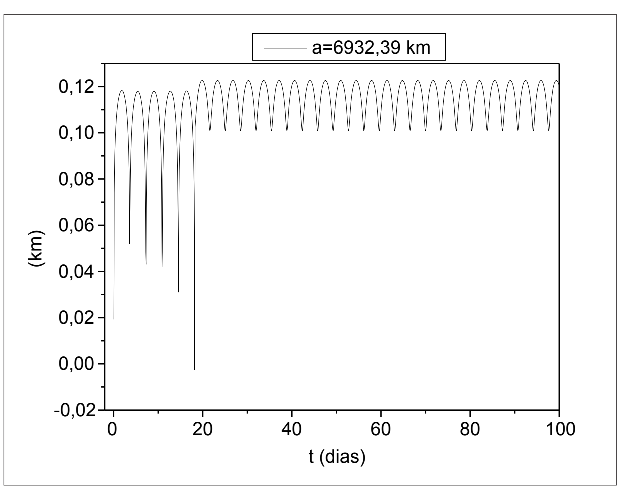

Figure 8. Amplification: Δa in the time: e=0.01, i=4°, φ^{*}= 0°.

region, we observed a region sensible to abrupt changes of orbital elements and the possible existence of chaotic region.

Several other initial conditions were considered and, as a sample, Table 2 contains the amplitude and period of the variations of orbital elements for hypothetical satellites considering low eccentricity, small and high inclinations, and influence of the harmonics J_{20} and $J_{15,15}$. As it can be observed, the 15:1 resonance can produce a variation of more than 100 m in the semi-major axis and this must be considered in practice when orbital elements are used in precise measurements.

CONCLUSIONS

A sequence of canonical transformations enabled us to analyze the influence of the resonance on the orbital elements of artificial satellites with mean motion commensurable with the rotation.

In this paper, an integrable kernel was found for the dynamical system describing the motion of an artificial satellite under the influence of the geopotential, and considering resonance between frequencies of the mean orbital motion and the Earth rotational motion. The theory, valid for any type of resonance p/q (p = mean orbital motion and q = Earth rotational motion), was applied to the dynamical behavior of a critical angle associated with the 15:1 resonance considering some initial conditions.

The motion near the region of the exact resonance is extremely sensitive to small alterations considered. This can be an indicative that these regions are chaotic.

This paper provides a good approach for long-period orbital evolution studies for satellites orbiting in regions where the influence of the resonance is more pronounced.

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Table 2. Maximum oscillation 15:1 resonance considering $J_2+J_{15,15}$.

a_o=6932,39 km	e	i	Δa (m)	Δe	$\Delta i_{máx}$(°)
a_o	0.019	4°	120	0.006	$2x10^{-4}$
a_o–2,39	0.019	4°	120	0.0065	$16.9x10^{-4}$
a_o–6,39	0.019	4°	118	0.0062	$11.1x10^{-4}$
a_o+2,53	0.019	4°	100	0.0058	$14.8x10^{-3}$
a_o	0.019	55°	125	0.0068	$2.9x10^{-3}$
a_o–2,39	0.019	55°	110	0. 0063	$4x10^{-3}$
a_o–6,39	0.019	55°	90	0.0055	$3.5x10^{-3}$
a_o+2,53	0.019	55°	125	0.0029	$2.8x10^{-3}$
a_o	0.019	63,4°	120	0.0068	$5.15x10^{-3}$
a_o–2,39	0.019	63,4°	100	0.0058	$4.4x10^{-3}$
a_o–6,39	0.019	63,4°	80	0.0048	$3.2x10^{-3}$
a_o+2,53	0.019	63,4°	100	0.0067	$5.7x10^{-3}$
a_o	0.019	87°	125	0.0057	$5.72x10^{-3}$
a_o–2,39	0.019	87°	120	0.0065	$7.16x10^{-3}$
a_o–6,39	0.019	87°	102	0.006	$5.7x10^{-3}$
a_o+4,61	0.019	87°	92	0.0055	$5.73x10^{-3}$

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