Analysis of efficiency of the regulated transportation coach operator by stochastic frontier of Cobb-Douglas and translog functions: the case of the interstate passenger coach transportation in Brazil

[Análise de eficiência estimando fronteiras paramétricas Cobb-Douglas e translog: o caso de empresas de transporte rodoviário interestadual de passageiros por ônibus no Brasil]

Francisco Giusepe Donato Martins, Francisco Gildemir Ferreira da Silva*,
Carlos Henrique Rocha, Marcelo Queiroz, Carlos Eduardo Freire Araújo

Tribunal de Contas da União (TCU), Brazil, Universidade Federal do Ceará (UFC), Brazil, Universidade de Brasília (UNB), Brazil, Universidade de Brasília (UNB), Brazil, MKMBr Engenharia Ambiental Ltda, Brazil,

Abstract

The theme about efficiency and productivity of companies that act in regulated industries have been extensively discussed in literature. In Brazil, there are just a few studies about the analysis of efficiency and productivity of interstate passenger coach transportation (trip). in this sense, this paper has three main objectives: to choose between the Cobb-Douglas and translog functions which is the best function to measure the technical efficiency on trip for the period of 2004-2006; to compare the efficiency scores among the companies of trip; and to make an analysis of the efficiency dynamic on trip and the factors that contributed for the gains and losses of the efficiency. It can be a strategy to competition in a long term. The results indicate a loss of efficiency during the years analyzed and this loss is probably due to the expansion of the capacity installed by acquisition of new buses, it can be a strategy to compete in quantity. The increase in production in 2005 based on 2004 is 4%; however, it was accompanied by an increase of 6% and 26% of annual distance traveled in km and employment of labor (drivers,) respectively, and 31% reduction in fleet use, with similar results for 2006.

Key words: efficiency; productivity; interstate transportation; regulation.

Resumo

O tema sobre eficiência e produtividade de empresas que atuam em indústrias reguladas tem sido extensivamente discutido na literatura. No Brasil há poucos estudos sobre análise de eficiência e produtividade de empresas de transporte rodoviário interestadual de passageiros por ônibus (TRIP). Nesse sentido, esse trabalho tem três objetivos principais: escolher entre a função Cobb-Douglas e Translog qual a melhor função para medir a eficiência técnica no TRIP para o período de 2004-2006; comparar os escores de eficiência entre as empresas de TRIP; e fazer uma análise da dinâmica da eficiência no TRIP e os fatores que contribuíram para ganhos ou perdas de eficiência. Os resultados indicam uma queda de eficiência durante os anos analisados e que tal queda se deve provavelmente a expansão da capacidade instalada por aquisição de novos ônibus, o que pode ser uma estratégia de longo prazo para competição em quantidade. Em termos relativos, o aumento da produção de 2005 em relação à de 2004 corresponde a 4%, porém, foi acompanhada do aumento de 6% e de 26% de distância anual percorrida em km e de emprego de mão de obra (motoristas), respectivamente, e pela redução de 31% da frota utilizada, com comportamento semelhante para 2006.

Palavras-Chave: eficiência; produtividade; transporte interestadual; regulação.

* Corresponding Author. Email: gildemir@gmail.com.

Recommended Citation

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1. Introduction

Transportation is essential to the production, to the spatial distribution and to the consumption of goods and services, apart from being an important instrument of distribution of incomes and interstate and national integration (Kawamoto, 1994). A transportation system can be defined, in economic terms, as a set of productive factors (inputs) that interact with each other to generate a determined result (outputs), as example the noise, trips, tones transported, distance covered, or simply the dislocation of people and goods. In this sense, the transportation system can be studied as a production function that relates the inputs and products, being considered the technological and operational specifications, with the purpose of permitting the evaluation of the reached productivity level. As a rule, an explicit mathematical expression is adopted, among many functions, to represent the production bound, being this function adjusted through statistic techniques, in a way to estimate the efficiency bound.

The parametric approach of stochastic bound consists basically in overcoming the deterministic bounds restrictions, through variables that capture the technical inefficiency of the firms control (Aigner e Chu, 1968; Meeusen e Van Den Broecker, 1977). There are two important preliminary stages in applying a parametric method in productivity studies. The first concerns to the choice of the mathematical expression of production. The second refers to the vectors of incomes and outcomes.

Concerning the production function, the Cobb-Doulgas type is largely used, because the simplicity of being linearized with the application of logarithms and its success in the estimation of American productions relations. Another function mainly applied in system transportation studies is the Translog function. The main difference between the Cobb-Douglas and Translog production functions is the functional form, specially Translog, which permits a larger flexibility that can conduct to more realistic efficiency scores in relation to the Cobb-Douglas model. The second form is statistically equivalent to the first one, then the resulting gains of the estimation justify the utilization of the Translog function to analyze the efficiency, but has the problem of multicollinearity to be estimate.

The themes about efficiency and productivity of companies that act in regulated industries have been extensively discussed in literature. The approach is also vast and diversified in terms of methodology adopted for analysis of efficiency and productivity (Coelli *et al.*, 2003), with applications in realization of transportation public services of passengers by bus. The studies are extremely rich, highlighting in the case of the transportation of passengers by bus: Merewitz (1977); Else (1985); Obeng (1985); Hensher (1987); White *et al.* (1992); Holvad *et al.* (2004).

There are few studies in Brazil about the analysis of efficiency and productivity of the interstate passenger coach transportation, like the studies of Novaes and Medeiros (2008) and of Araújo, Martins and Silva (2008). So far, Turolla *et. al.* (2008) has showed that gains of efficiency can be divided in some components – allocative and technical – that are very important to competition process in the market of passengers transport.

This paper has three main objectives. The first is to choose between the Cobb-Douglas and Translog functions, for the period of 2004-2006, which is the better functional form to measure the technical efficiency of companies that act in TRIP, using the parametric method of stochastic bound estimation. The second objective is to compare the efficiency scores among the TRIP companies, observing the dynamic of these scores during the years. The last is to make an analysis of the efficiency dynamic in TRIP and the factors that contributed for the gains and losses of efficiency.

The choice of the parametric models Cobb-Douglas and Translog will be based on the Hausman (1978) test[1] of specification. The selected period involves 159 buses companies that operate only in Brazilian interstate routes with journeys over 75 km. The information related to the production of the TRIP service were obtained in the electronic address of National Agency of Overland Transportation (ANTT) and they are allocated by lines and companies, in a way to permit the aggregation of information in business level. Therefore, the present paper was divided into six sections considering this introduction. The second section is a brief summary of concepts, properties and application to transportation systems of production functions. The third section shows a resume of the quantitative parametric techniques to

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[1]The Hausman test is test of specification based on the difference between an efficient estimator under the null and a non-efficient estimator. It compares two sets of estimates, one of which is consistent under both the null and the alternative and another which is consistent only under the null hypothesis. A large difference between the two sets of estimates is taken as evidence in favor of the alternative hypothesis of incorrect specification.

measure the efficiency with the economic theory, specifically the Cobb-Doulgas and Translog functions. The fourth section presents the adopted models and describes the main characteristics of the variables used according the obtained information. The fifth section presents the methodology, application and results and the differences of the followed approaches. Finally, on the sixth and last section the final considerations are presented.

2. Production Functions: Concepts, Properties and Application to Transport Systems.

On this section, concepts and properties of production functions and their application to transportation systems are discussed.

2.1. Production function: concepts, characteristics and properties.

Production function is a term used to represent the relation between the productive factors (natural resources, labor, capital, technology, business talent, etc) of a firm and the quantity of possessions and services produced in a determined period of time (Thompson & Formby, 2003). According to Samuelson e Nordhaus (1993), the production function is the maximum quantity of production that can be produced using some production factors considering a certain technological level. The production function can be represented by equation 1.

$$y = f(x)$$

(1)

Where x represents the quantity of inputs and y indicates the reached production in a determined period of time, from the used technology. The evaluation of the production function of one company permits a definition of the technical possibilities of production and the analysis of three important concepts: full product; medium; and marginal. The full product determines the total quantity of the product in physic units. The medium product corresponds to the rate between the full product and the total quantity of production factors - $y/x = f(x)/x$. The marginal product is equivalent to the additional product of a production factor, maintained the other constant factors, mathematically expressed by $dy/dx = df(x)/dx$ (Samuelson e Nordhaus, 1993).

A relevant characteristic concerns the returns of scale that reflect the answer of the full product when all the productive factors proportionally increase. According to Samuelson and Nordhaus (1993), the production can be increasing, decreasing or constant returns in scale when a proportional increase in all productive factors takes to an increase more than proportional, less than proportional or equally proportional of production, respectively. Knowing the functional form, we can optimize the usage of productive factors and the acquisition of scale economies, in a way to guarantee the maximum economic efficiency in production.

2.2. Inherent Economic Efficiency Concepts

The evaluation of the economic efficiency, with allocative, distributive and productive origins, has been concentrated in static aspects. This has conducted to a wider analysis from the concept of dynamic efficiency that embraces investments and capacity of technological innovation and not only the temporal course conditions or the criteria of allocative efficiency of Pareto that has great choice between allocations of limited resources to technologies and institutions roles. Under this scope, the term productivity appears and presents, according to Coelli *et al.* (1997), several tunes in its measuring form, which requires, therefore, a description of commonly used definitions, as: productivity; technical efficiency; allocative efficiency; technical changes; scale economies; Total Factor Productivity (TFP); production bound; feasible set of production.

Productivity is defined as a relation between the produced and the necessary inputs for the production. In the case of multiple inputs and products, the productivity concept is equivalent to TFP. The production bound represents the maximum production to each input level and reflects the state of the industry technology. Notice that the production bound guards a narrow relation with the definition of the production function. The company is technically efficient whether it is on the production bound, and it is inefficient whether it is bellow the curve. Every point below the curve are feasible because, it can be produced in given an input combination, nevertheless, above the curve it cannot produce with the relative input because it would overtake the industry production bound.

The time passage implies a change or a dislocation of the production bound with an enlargement or reduction of the production ensemble. According to Coelli *et al*. (1997), the

productivity growth is a consequence of the improvements in efficiency via technical changes or scale economies, which can also be the result of a combination among these factors. The techniques used to measure productive efficiency must simulate the behavior described on the previous section. From the computational methods, econometric models of production functions and stochastic frontiers can be highlighted.

2.3.1 Econometric models

In the econometric approach, we estimate parameters that are able to approximate a sample to a mathematical function and an arbitrary component and whose parameters are not influenced and efficient from the statistic point of view (Gujarati, 2000; and Pindyck and Rubinfield, 2004). This approach is called parametric. The estimated function is defined based on the economic theory of production. Generally, a Cobb-Douglas function is used, but there are more general functions like: CES, Translog and so on. The advantage of the econometric estimations is the possibility of testing the parameters' behavior and obtain answers related to the scale gains (identification and level), the production input relevance, the technological changes, among others. The disadvantage is the necessity of having big samples and with the minimum measure error terms, because, as the method being parametric, it should test the statistic validation of the parameters.

2.3.2 Stochastic Frontier

The stochastic frontier can be used as an alternative method to estimate bound functions. It is a parametric method proposed by Aigner *et al.* (1977) and Meeusen and Van den Broeck (1977), concomitantly, and it consists in a regression where the production value is limited by the arbitrary variable $(\exp(x_i+v_i))$. The expression $\exp(x_i)$ is the production function itself and v_i is a idiosyncratic term with 0 average and variance σ.

Indeed, the stochastic frontier is an econometric estimation of production function, but the interest lies on the error term. It represents the inefficiency of the company in case the functional form adopted is the most correct. In the end, we have the inefficiency score and it can be obtained indicatives of scale returns and full, medium and marginal products, and also marginal rates of substitution.

2.4. Application of Production Functions to Transportation Systems

The production of any transportation activity requires a combination of different productive factors (inputs: vehicles; equipments; labor; fuel; turnpikes; containers; terminals; roads, installations; among others), in function of a determined technology, with the intention of generate a result (outputs) that can be measured by distinct ways: trips; traveled distance in kilometers; transported passengers; passengers-kilometers transported; transported tones; offered seats; number of bustled containers; number of equivalent vehicles; among others. The transportation service production from the technological relation can be represented by equation 2.

$$y = f(K, E, L, F, N, t)$$

(2)

Where y corresponds to the maximum quantity of transportation that can be produced, taking into consideration the required quality and the adopted technology by each reference time unit; K is equivalent to infrastructure units; E refers to equipments or main and complementary vehicles; L is labor; F is the fuel used for the activity of the vehicles; N other relevant things like installation, ground, air or sea space usage; and t is the users time of transportation services (Rus *et. al.*, 2003).

In the transportation service production, the production unit is essential due to the possible different units, especially when the production function is adopted as a base to determinate costs, in a way that convenient units should be used to express input quantity, like: t (tonne); km (kilometer); number of vehicles (Kawamoto, 1994). Anyway, according to this author, none of these units used in isolation is sufficient to represent the variety of inputs involved in transportation production. In order to overcome these problems, units that represent a set of inputs are adopted, for the purpose of obtaining a production function that relates the quantity of offered service and necessary inputs with reasonable accuracy.

To know whether the period of reference analysis is short or long term is important. In the short-term some productive factors are going to be submitted to restrictions that stop their substitution easily being treated as steady factors, for example, the size of the vehicles in providing transportation services of passengers. Notice that in order to attend a development of request, the passengers' transportation companies by bus or by plane may have to

incorporate to the fleet a complete vehicle that will not be necessarily fully occupied. On the other hand, there are no steady factors in long term. According to the period of analysis, the properties of the production function are distinct.

Other relevant aspect is the possibility of multiple productions in transportation, in a way that the combination of productive factors generates not only one product, but several different products. In the case of passenger transportation service, it can be relevant to the analysis the consideration of each service and the different routes, in function of the pairs of the origin and destine cities. This procedure favors the evaluation of possible relations of complementarity and substitutability between inputs and products, which do not appear when the unique production factories are taken into consideration.

The concepts of economic efficiency and productivity in transportation are directly related to the production function. The terms of technical efficiency and productive efficiency refer to the fact of a company choosing the minimum quantities of productive factors to obtain the produced quantity, in a way that in this case all the points in an isoquant curve are efficient solutions, while the definition of economic efficiency considers only one of the points in an isoquant curve like being the great combination of productive factors (Rus *et al.* 2003). Having in mind these efficiency concepts, the production function can be represented to a passengers' transportation service company, according to equation 3.

$$y = f(K, L)$$
(3)

Where y corresponds to the maximum efficient quantity of transportation that can be produced; K is equivalent to the capital productive factor, represented by vehicles (buses); and L refers to the labor factor, represented, for example by the motorists.

The term productivity in transportation activities refers generally to indicators that measure the average productivity of the variable productive factors, in a way that the difference between efficiency and partial measures of average productivity presents a bigger evidence when a total production function in short term is used (Rus *et al.*, 2003), similar to the equation 3. According to these authors, the Cobb-Douglas function can be adopted as very dynamic production function in the general case of unique product companies with several inputs, while the Translog function is relevant in empiric analysis due to the multiple product

character of the activity of many companies, expressed by the different interactions among inputs.

3. Parametric Quantitative Techniques: Cobb-Douglas and Translog.

The Cobb-Douglas production function is largely used because its simplicity in being linearized with the logarithm application, being expressed by the equation 4:

$$y_i = A\prod_{k=1}^{k} x_{ik}^{\beta_k} \exp(\varepsilon_i)$$

$$(4)$$

Where y_i is the vector of firm products, $x_{ik}^{\beta_k}$ corresponds to the input vector, "A" e

β_k represent the parameters to be estimated and ε_i is the error-term. In other words,

parameter "A" measures the production scale, while the parameters β_k measure how the quantity of production interacts to the input variations (Varian, 2003). According to this author, the relation between the parameters, two by two, allows a definition of a technical rate of substitution (TTS) between two production factors, that is to say to predict the rate that the companies must substitute one input for another to maintain a constant production. Therefore, we have an assumption associated to technology that is in the same way we increase the quantity of factor 1 and adjust factor 2 to stay in the same isoquant, the TTS reduces (Varian, 2003). The TTS of factor 1 by factor 2 is given by the equation 5 (Coelli *et al*, 2003).

$$\sigma = \frac{\dfrac{du}{u}}{\dfrac{d(f_1/f_2)}{(f_1/f_2)}}$$

$$(5)$$

Functionally, the Cobb-Douglas function is very restrictive due to its mathematical properties: homogeneity of level 1; homogeneous marginal productivity of level 0, in other words, each input receives the value of its marginal product; way of expansion of linear production; the production is exhausted. Because it is a restrictive function, the mathematical treatment is easy, including for linearization.

Another applied function in transportation system studies is the transcendental logarithmic function (Translog), presented by Christensen *et. al.,* (1973) the same function is used by Oliveira, (2007) to describe the profit of coach operators using a game structure. This function is represented in the equation 6 by a succession of expansion of Taylor of second order of $\ln y$ in powers of $\ln x_i$:

$$\ln y = \beta_0 + \sum_{i=1}^{n} \beta_i \ln x_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} \ln x_i \ln x_j + \varepsilon_i \tag{6}$$

Where y is the products of firm vector, x corresponds to the input vector, β_i represents the parameter to be estimated, with $\beta_{ij} = \beta_{ji}$, and ε_i is the error-term.

The Translog function, according to Albuquerque (1985), is linear, has minimum quantities of parameters and does not impose separability and homogeneity as hypotheses, which allows establishing arbitrary values to the elasticity of substitution between any pair of inputs. Therefore, differently of the Cobb-Douglas function, the Translog function is more flexible, not being homogeneous and reducing to a Cobb-Douglas function if $\beta_{ij} = 0$ e $\beta_i > 0$. Besides that, econometrically, the term ε_i varies to different values of input. The disadvantage of the Translog function is its mathematic difficult to manipulate and the possibility, on econometrical estimations, of generating multicollinearity.

Concerning to the grade of homogeneity (r) of production functions and gains of scale, it can be obtained three situations: if r > 1, the function generates returns of crescent scales; if r = 1, there is constant returns to scale; and if r < 1, there are decreasing returns to scale. In the case of the Cobb-Douglas function, it is a homogeneous function with r equivalent to the sum of parameters β , while the elasticity of substitution is equal to the unit. The Translog function, though, is homogeneous of level r if $\sum_{i=1}^{n} \beta_i = r$ and if $\sum_{i=1}^{n} \beta_{ij} = 0$ to i = 1, 2,..., n. (Coelli et al, 2003).

For the case of $\beta_{ij} = 0$ and $\beta_i > 0$, the Translog function represented by the equation 6 reduces to the Cobb-Douglas function expressed by the equation 4. The main difference between the

production functions Cobb-Douglas and Translog is that the first one adopts constant returns to scale while the second allows a larger flexibility, what can conduct to more realistic efficiency scores to the ones obtained by the model Cobb-Douglas.

4. Applied Models to the Interstate Passenger Coach Transportation by Bus in Brazil: From 2004 to 2006.

The period of 2004-2006 involves 159 bus companies that operate only on interstate Brazilian routes with distance superior to 75 km. This data were obtained on the National Agency of Overland Transport (ANTT) homepage and they are related to the production of transported passengers (effective demand), passengers-kilometers, traveled distance in kilometers, fleet and quantity of drivers.

The data of production are allocated by lines and companies and they have been added on business level as an analysis object of productivity. According to the economy concepts applied to transportation (Hensher and Brewer, 2001; Rus, *et al.*, 2003), the information referred to passengers-kilometers can be adopted as billing proxy; the data about traveled distance in kilometers, though, are a good approximation of fuel consumption, while fleet and drivers refer respectively to the capital invested and to the labor used.

There had been established two production functions, one of the Cobb-Douglas type, and the other of the Translog type. The Cobb-Douglas model is expressed on equation 7:

$$\ln PASS\_KM = \ln \beta_0 + \beta_1 \ln KM + \beta_2 \ln ONIBUS + \beta_3 \ln MO + \varepsilon_i \tag{7}$$

Where PASS_KM is equivalent to the yearly production of passengers-kilometers, KM represents the total mileage traveled yearly by the company's fleet, ONIBUS corresponds to the quantity of the company's buses or the firm's fleet, MO is equivalent to the number of drivers and ε_i is the error-term.

The Translog model is the one represented by the equation 8:

$$\ln PASS\_KM = \ln \beta_0 + \beta_1 \ln KM + \beta_2 \ln ONIBUS + \beta_3 \ln MO + \beta_4 (\tfrac{1}{2} \ln^2 KM) + \beta_5 (\tfrac{1}{2} \ln^2 ONIBUS) + \beta_6 (\tfrac{1}{2} \ln^2 MO) +$$

$$\beta_7 \ln KM \ln ONIBUS + \beta_8 \ln KM \ln MO + \beta_9 \ln ONIBUS \ln MO + \varepsilon_i \qquad (8)$$

Where PASS_KM is equivalent to the yearly production of passengers-kilometers, used in the model as annual billing proxy of each TRIP company; KM represents the total traveled mileage annually by the company's fleet, serving as a consumption of fuel proxy; ONIBUS corresponds to the quantity of company's buses, being adopted as the invested capital proxy; MO is equivalent to the number of drivers of each company, being used as labor proxy; and ε_i is the error-term.

Descriptive statistics of the variables referred to the 159 TRIP companies analyzed on this paper are represented on Table 1. Despite of having huge disparity between the maximum values and the minimum of each variable used in the proposed models, we can consider that the production factors adopted by the firms are found in reasonable standards to be evaluated on the efficient production level. It can be noticed from Table 1 that the production level expressed by passengers-kilometers in 2005 overcame the levels of 2004 and 2006.

Table 1 – Descriptive statistics of the variables adopted on the models Cobb-Douglas and Translog.

Year	Descriptive Statistic	Billing (passengers-kilometers): pass-km	Diesel consumption (traveled distance): km	Invested capital (fleet): n.º buses	Labor: n.º drivers
2004	Total	24.041.551.986	1.250.063.329	10.811	18.715
	Maximum	3.999.092.419	168.915.464	1.266	3.299
	Minimum	15.021	776	1	1
	Average	151.204.729	7.862.034	68	118
	SD	412.094.190	19.160.605	148	333
2005	Total	24.272.833.692	1.214.905.876	10.266	19.637
	Maximum	3.701.558.037	145.630.148	1.063	3.018
	Minimum	11.933	776	1	1
	Average	152.659.331	7.640.917	65	124
	SD	401.057.467	17.431.370	140	338
2006	Total	22.844.340.782	1.211.393.572	11.260	18.543
	Maximum	3.485.091.922	135.510.753	1.146	2.384
	Minimum	11.300	776	1	1
	Average	143.675.099	7.618.827	71	117
	SD	373.831.408	17.042.773	148	292

SD= standard deviation. Each year has 159 observations concerning to the 159 companies.

The movements of the explanatory variables were going in the way of a reduction of fuel consumption year by year, reduction of fleet from 2004 to 2005 and an increase from 2005 to 2006, increasing and reduction of labor to the exercises 2004-2005 and 2005-2006, respectively. The behaviors suggest some mismatched information related to the distance traveled, however, it will neglect the possible error. There may be occurred a scale gain with fleet expansion from 2005 to 2006, supplemented by the reduction of labor to the same period that we will check on the following results.

5. Methodology and Results

5.1. Estimation of Stochastic Frontier

We used the STATA 10 software to adjust the production functions by the method of minimum squares, being analyzed, sequentially, the equivalence of the functions via Hausman's test. The result referred to the year of 2004 is on Table 2.

Table 2 – Result of the models for 2004

Parameters	With interception*				Without interception**			
	Translog		Cobb-Douglas		Translog		Cobb-Douglas	
	Coef	P>t	Coef	P>t	Coef	P>t	Coef	P>t
intercept	2,929	0,07	1,199	0				
lnKm	0,163	0,838	0,969	0	1,599	0	1,219	0
lnONIBUS	-1,224	0,221	0,029	0,63	-2,257	0,007	-0,067	0,266
lnMO	2,449	0,015	0,106	0,109	1,379	0,09	-0,064	0,278
1/2(lnkm)^2	0,171	0,385			-0,173	0,003		
1/2(lnONIBUS)^2	-0,087	0,709			-0,211	0,349		
1/2(lnMO)^2	0,047	0,886			-0,054	0,868		
(lnkm)*(lnONIBUS)	0,207	0,351			0,453	0,012		
(lnkm)*(lnMO)	-0,395	0,069			-0,153	0,375		
(lnMO)*(lnONIBUS)	0,074	0,728			-0,141	0,431		
Sum	0,017				-0,279			
R² Ajustado	0,9473		0,9448		0,9993		0,9992	

* Hausman's test: chi2(3) = 6.75 Prob>chi2 = 0.0804 for Ho: difference in coefficients not systematic;

** Hausman's test: chi2(3) = 28.77 Prob>chi2 = 0.0000 for Ho: difference in coefficients not systematic.

The result of the test for 2004, considering the interception, indicates that the models present similar answers, therefore, it would not be necessary to adopt a Translog model, having in

mind that additional information was not added. Still, on this model, heteroscedasticity was not identified on the models using the Breusch-Pagan test.

Concerning the result for 2004 without interception, the conclusion is similar. The Breusch-Pagan test cannot be executed because it is not possible after the suppression of the interception. However, when the Zsroeter's is executed, homoscedasticity can be identified among the variables on the Translog model.

The referred result of the year of 2005 is represented on Table 3.

Table 3 – Result of the models for 2005

Parameters	With interception*				Without interception**			
	Translog		Cobb-Douglas		Translog		Cobb-Douglas	
	Coef	P>t	Coef	P>t	Coef	P>t	Coef	P>t
intercept	0,462	0,781	1,331	0				
lnKm	1,373	0,092	0,933	0	1,596	0	1,211	0
lnONIBUS	-1,381	0,23	0,053	0,359	-1,461	0,188	-0,02	0,732
lnMO	0,826	0,533	0,14	0,045	0,593	0,561	-0,077	0,17
1/2(lnkm)^2	-0,116	0,553			-0,169	0,003		
1/2(lnONIBUS)^2	-0,013	0,952			-0,022	0,916		
1/2(lnMO)^2	-0,287	0,491			-0,339	0,359		
(lnkm)*(lnONIBUS)	0,247	0,305			0,264	0,253		
(lnkm)*(lnMO)	-0,038	0,897			0,015	0,945		
(lnMO)*(lnONIBUS)	-0,029	0,886			-0,042	0,826		
Sum	0,236				-0,293			
R² Adjusted	0,9367		0,9354		0,9991		0,999	

* Hausman's test: chi2(3) = 1.77 Prob>chi2 = 0.6209 for Ho: difference in coefficients not systematic;

** Hausman's test: chi2(3) = 28.21 Prob>chi2 = 0.0000 for Ho: difference in coefficients not systematic.

The result of the test for the exercise of 2005 with interception indicates that the models present different answers, being possible to adopt both models for comparison. The heteroscedasticity was still not identified on the models using the Breusch-Pagan test. The results for 2005 without interception, though, indicate that there is no distinction between the models, but, as it was observed for 2004, estimation gains were obtained on the Translog model. The Zsroeter's test identified homoscedasticity among the variables of this model.

The referred result of the year of 2006 is represented on Table 4.

Table 4 – Results of the models for 2006

Parameters	With interception*				Without interception**			
	Translog		Cobb-Douglas		Translog		Cobb-Douglas	
	Coef	P>t	Coef	P>t	Coef	P>t	Coef	P>t
intercept	0,186	0,913	1,545	0				
lnKm	1,574	0,056	0,879	0	1,663	0	1,196	0
lnONIBUS	-0,589	0,64	-0,018	0,479	-0,617	0,616	-0,029	0,3
lnMO	-0,002	0,999	0,291	0,005	-0,095	0,934	0,007	0,941
1/2(lnkm)^2	-0,185	0,349			-0,205	0		
1/2(lnONIBUS)^2	0,181	0,459			0,177	0,462		
1/2(lnMO)^2	-0,357	0,45			-0,378	0,384		
(lnkm)*(lnONIBUS)	0,116	0,666			0,122	0,64		
(lnkm)*(lnMO)	0,15	0,634			0,171	0,491		
(lnMO)*(lnONIBUS)	-0,147	0,493			-0,153	0,465		
Sum	0,653				-0,322			
R² Adjusted	0,9436		0,9375		0,9992		0,999	

*

Hausman's test: chi2(3) = 0.92 Prob>chi2 = 0.8198 for Ho: difference in coefficients not systematic;

** Hausman's test: chi2(3) = 42.93 Prob>chi2 = 0.0000 for Ho: difference in coefficients not systematic.

The result of the test for the exercise of 2006 with interception reveals that the models present distinct answers, being feasible to adopt both models for comparison. The heteroscedasticity was not identified on the models using the Breusch-Pagan test. The results of 2006 without interception show that there was no distinction between the models but, as it as observed to 2004 and 2005, there had been estimation gains on the Translog model. Zsroeter's test identified homoscedasticity among the variables on this model.

Comparing the obtained results, it can be concluded that the Translog model presented estimation gains related to the Cobb-Douglas model in all exercises, having in mind that the adjusted R² of this model was bigger than the one from the Cobb-Douglas model, with and without interception. Even though not all the present factors on the Translog models without interception are significant, possibly by multicollinearity, it is important to register that the objective is to measure the error-term component, in a way that we will neglect this problem and we will estimate the scores with the intention to analyze the inefficiency of the companies. In another paper, we suggest estimations with analysis and correction of the possible multicollinearity.

Therefore, we can say that the usage of the Translog model is better than the Cobb-Douglas model, because one provides advantages for empiric analysis due to the multiple product character of the activity of many companies, expressed by the different interactions among the inputs.

5.2. Estimation of the Efficiency Scores basing on the Translog Model

The efficiency patterns (scores) were obtained from the Translog model represented by the equation 7, having in mind the established conclusions based on the estimated results of the stochastic frontier presented on item 5.1. The scores were calculated in function of the standard error in a way of reflecting the economic inefficiency. In this sense, the nearest scores to zero indicate the most efficient firms, while the scores most distant from zero reveal the least efficient companies when they combine the productive factors to reach the maximum efficient production.

The scores were tabled by symmetric lanes in relation to zero position according to Table 5.

Table 5 – Distribution of companies by lane of efficiency scores

Lane of scores		Number of companies			% accumulated		
		2004	2005	2006	2004	2005	2006
1ª	0,05<0,00 : 0,00 >-0,05	35	34	47	22,01	21,38	29,56
2ª	0,10 <0,06 : -0,06>-0,10	43	32	30	49,06	41,51	48,43
3ª	0,15<0,11 : -0,11>-0,15	22	30	18	62,89	60,38	59,75
4ª	0,20<0,16 : -0,16>-0,20	24	23	26	77,99	74,84	76,1
5ª	0,25<0,21 : -0,21>-0,25	11	11	13	84,91	81,76	84,28
6ª	0,30<0,26 : -0,26>-0,30	11	15	5	91,82	91,19	87,42
7ª	0,35<0,31 : -0,31>-0,35	5	6	9	94,97	94,97	93,08
8ª	Last lane	8	8	11	100	100	100
	Total	159	159	159			

For analysis effects, it was taken into consideration as efficient to companies situated in the first lane of scores, from 0,05 to 0,00 and from 0,00 to -0,05. Table 4 indicates that the year of 2006 presented a larger number of efficient firms, in a total of 46, practically 30% of the total of the analyzed firms. The exercise with Translog and Cobb-Douglas models registered returns of increasing scales, with and without interception. Enlarging the analysis to the third lane of bigger technical efficiency, almost 60% from the 159 companies is reached.

The result, in graphic form, is found on Figure 1, reveals the existence of a movement on the range of each analyzed year in the sense of improving of the industry efficiency. This

indicates that there are components, technological, managerial and/or regulatory that favored an improvement in transporting people efficiently. This does not imply in improvement in terms of quality. However, this may indicate a larger profitability to the operators once that is an indicative of a better utilization of production inputs and that will be described during the analysis.
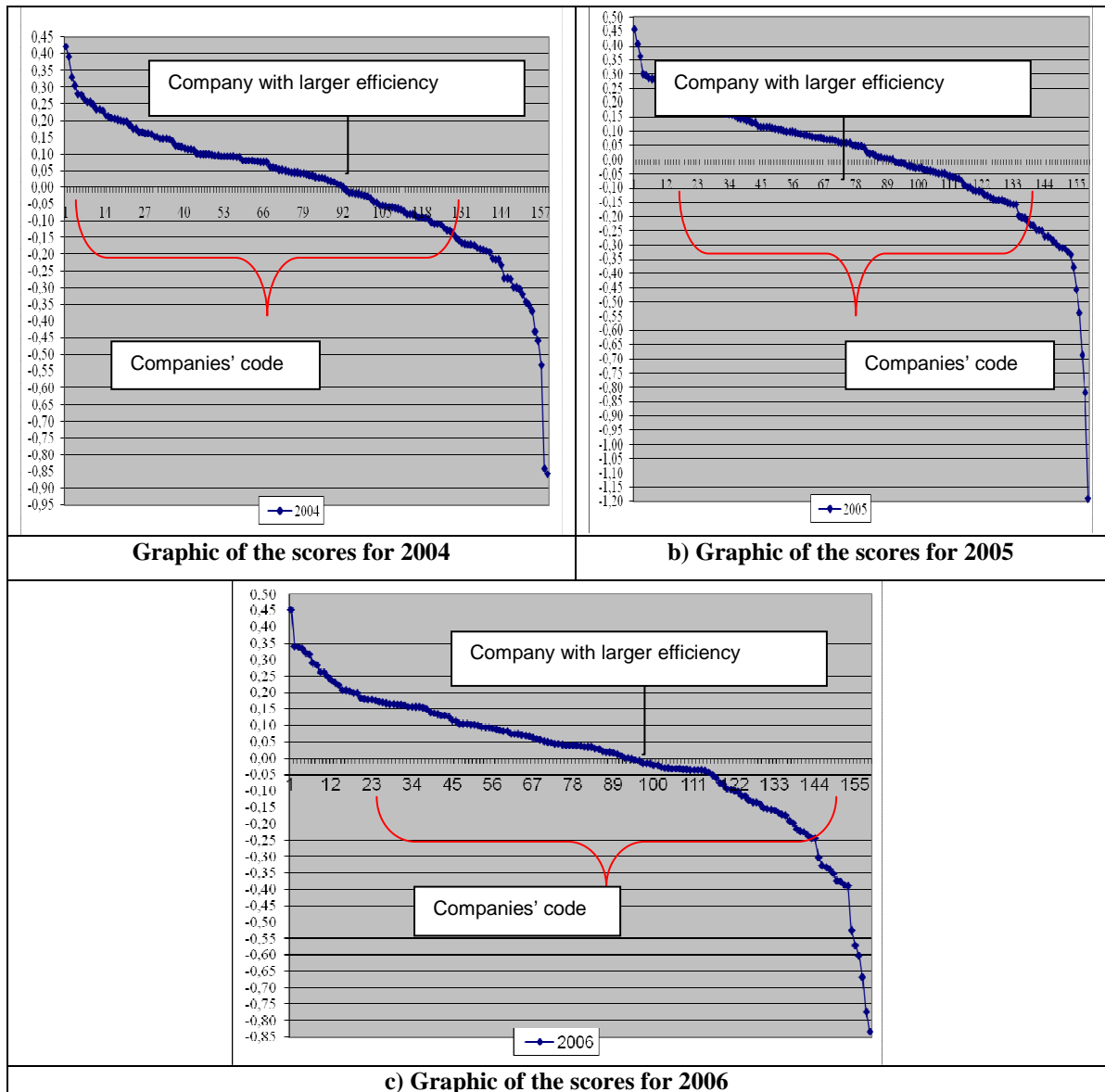

Graphic of the scores for 2004 | **b) Graphic of the scores for 2005**

c) Graphic of the scores for 2006

Figure 1 – Graphic result of the efficiency scores 2004-2006

It is interesting to notice that, year after year, the most efficient company is different and can be identified as being the one located on the cut point of the blue line above zero. The companies can migrate from one lane to another each year. The technical efficiency scores

can be distinct annually for each of the 159 analyzed firms. Figure 2 indicates, in terms of an associated dynamic to these modifications of position among the firms, year after year, that the companies with the lowest scores improved and the ones that showed the best scores declined. The classifications of the companies changed as well, the worst classified improved in their positions and the best classified lost their position.
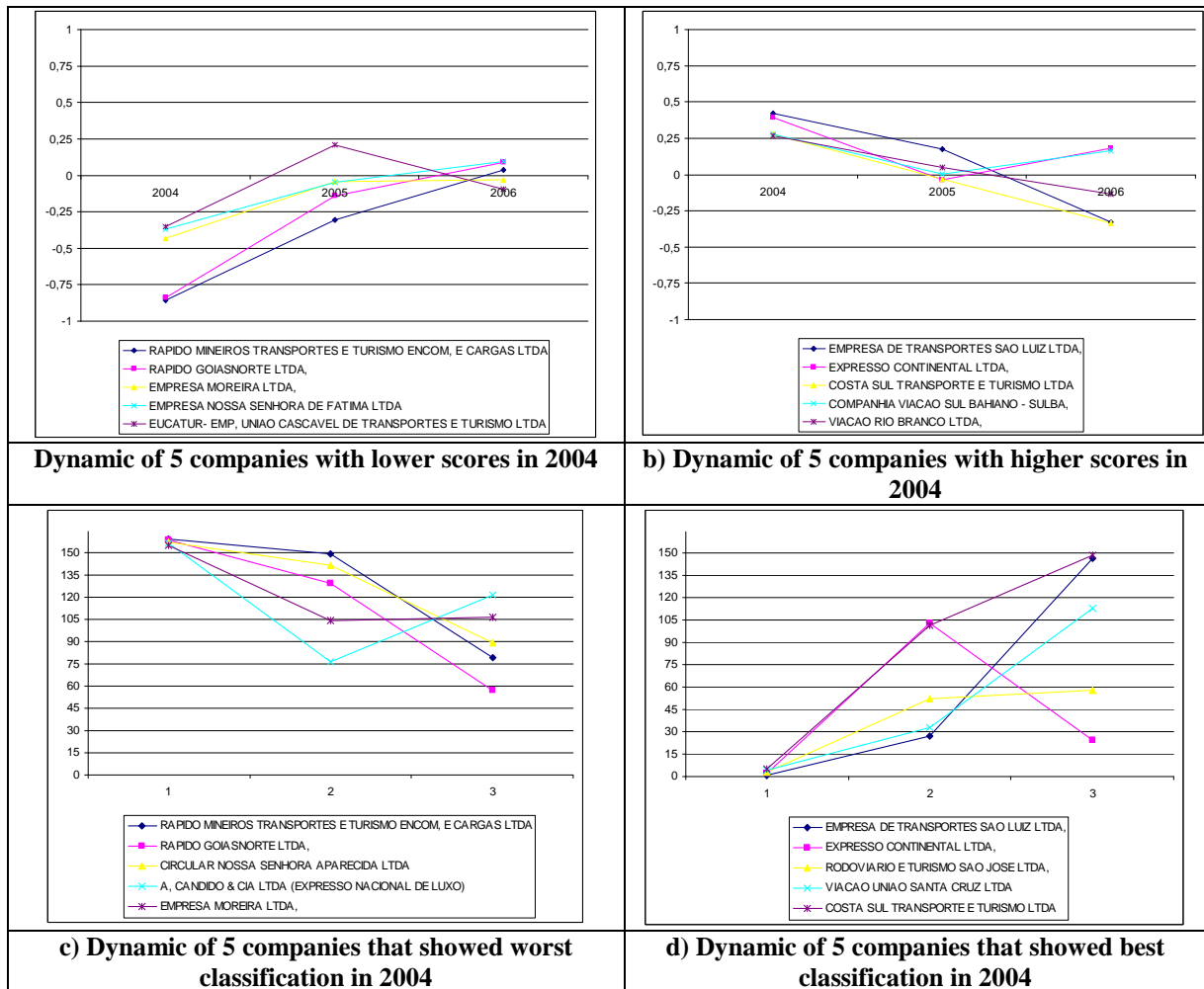

Dynamic of 5 companies with lower scores in 2004	**b) Dynamic of 5 companies with higher scores in 2004**
c) Dynamic of 5 companies that showed worst classification in 2004	**d) Dynamic of 5 companies that showed best classification in 2004**

Figura 2 – Dynamic of the scores and classification of the five best and five worst companies 2004-2006.

With this dynamic in view, we analyzed the wideness of this aspect for the companies that presented higher score in technical efficiency, for each included year, which are the ones situated on the first lane of scores on Table 5. It is important to highlight also that the total amount of different companies situated on the first line of scores on Table 5, considering the three exercises, is of 96 firms that are distributed year by year as represented on Figure 3. The Figure 3 represents a Venn Diagram related to all companies analyzed, the intersections

represents the quantity of the highest efficient companies in the set of two or three years. The quantity of technically inefficient companies is presented out of the intersection zones. It can be seen from Figure 3 that only one company shows on the first line of scores over the three years. It can also be noticed that 19 of the 96 companies situated on the first line of scores show simultaneously on both exercises, while 76 firms show only in one year of the period 2004-2006.
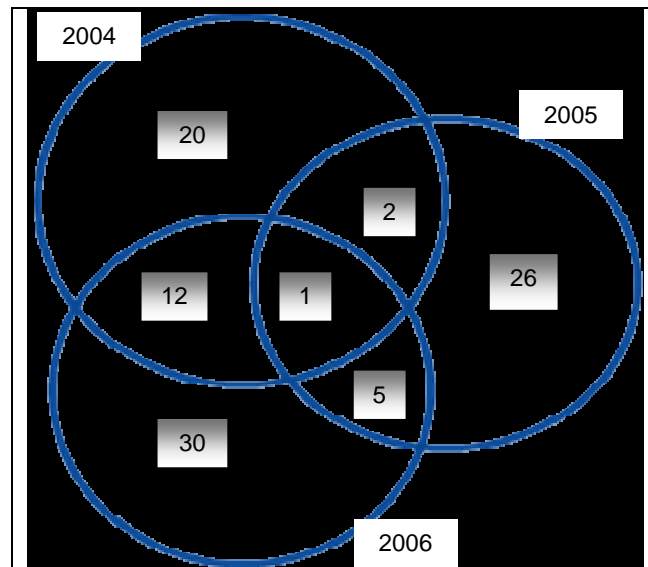

Figure 3 – Distribution of the companies of higher technical efficiency in each year.

One interesting aspect is the performance of all 96 companies situated on the first line of technical efficiency scores for the period of 2004-2006, in function of the adopted variables. The result is on Table 6.

Table 6 –96 companies performance situated on the first line of technical efficiency scores: 2004-2006

Year	Variables							
	pass-km	%	km	%	fleet	%	drivers	%
2004	5.303.848.443	-6	265.554.955	5	2.439	11	3.080	34
2005	2.645.642.388	89	133.501.574	110	1.474	83	2.000	107
2006	4.991.006.156	-x-	279.907.778	-x-	2.703	-x-	4.134	-x-
Total	12.940.496.987	-x-	678.964.307	-x-	6.616	-x-	9.214	-x-

(%) Increasing or decreasing in percentage considering 2006 as reference in relation to the other years. For example, 34= ((4.134-3.080)/3.080)*100.

According to Table 6, we can notice that the year of 2004 was of greater production in terms of passengers-kilometers (pass-km), followed by 2006. Analyzing in a comparative way, we can ascertain that the production of 2005 was practically 50% smaller than the production of

2004, but it was followed by a significant reduction of the production factors consumption, of 50%, 40% and 35%, being annual distance traveled in km, used fleet and employed drivers, respectively.

Comparing the years 2004 to 2005 and 2005 to 2006, it can be noticed from Table 6 that 47 companies of better score in 2006 registered a substantial increase of production factors, specially comparing to 2005, although they present a significant increase in production. In relation to 2004, these 47 companies had a reduction of 6% in production with the increase of consumption of all production factors analyzed. This reasoning was restricted only to the ten companies of higher technical efficiency score of each year. The result is expressed on Table 7.

Table 7 – Performance of the ten companies situated on the first line of technical efficiency scores: 2004-2006

Year	Variables							
	pass-km	%	km	%	fleet	%	drivers	%
2004	1.285.041.943	-40	65.732.089	-15	742	-54	800	-34
2005	1.340.767.496	-42	69.780.478	-19	513	-34	1.008	-47
2006	775.866.245	-x-	56.188.127	-x-	338	-x-	530	-x-
Total	3.401.675.684	-x-	191.700.694	-x-	1.593	-x-	2.338	-x-

(%) Increasing or decreasing in percentage considering 2006 as reference in relation to the other years. For example, 34 = ((4.134-3.080)/3.080)*100.

We can notice from Table 7 that in the year of 2005, in spite of the ten companies of best score registered a greater production in terms of billing expressed by passengers-kilometers (pass-km), they consumed a big absolute quantity of production factors, specifically the fuel represented by the annual traveled distance in km by the fleet and the labor corresponds to the number of employed drivers. In relative terms, the increase of the production of 2005 in relation to 2004 corresponds of 4%, however, it was followed by the increase of 6% and of 26%, of annual traveled distance in km and employment of labor (drivers), respectively, and by the reduction of 31% of the used fleet. It can also be noticed that the significant reduction of the production of 2006 comparing to 2004 and 2005, of 40% and 42% respectively, was followed by the reduction of the production factors, especially fleet and drivers, because they presented the biggest percentages of reduction.

5.3. Economic Measures based on the Translog Model without interception

In this section the homogeneity of the Translog function chosen is going to be evaluated, the marginal productivity of the production factors and the marginal rate of substitution. For the last one we will make only considerations referring to the 5 companies that most transported passengers and the 5 that least transported passengers, as we will present average statistics of the industry. We should make a consideration of this analysis because probably the estimation has multicolinearity, then, the economic measures could be biased and the results are only to show the power of the analysis. To better results we recommended a new modeling correcting the multicolinearity problem.

Concerning the degree of homogeneity, because of the fact that the chosen functions are Translog we did only the confirmation of the non homogeneity for the analyzed years using the Coelli's criteria (2003), where the degree r of homogeneity only occurs if $\sum_{i=1}^{n} \beta_i = r$ and if $\sum_{i=1}^{n} \beta_{ij} = 0$ to i = 1, 2,..., n. Like $\sum_{i=1}^{n} \beta_{ij} \neq 0$, so there is no homogeneity, complicating the task of evaluating the scale return.

The marginal productivity was calculated year by year obtaining the constant result of Table 8.

Table 8 – Marginal Productivity (PMg) of the Production Factors

Ano	PMg km	PMg Drivers	PMg Fleet
2004	1,59863	1,379867	-2,256932
2005	1,595728	0,593322	-1,460707
2006	1,654257	-0,607406	-0,069196

The marginal productivity of the production factors shows what was expected. The annual traveled distance in km is a fuel consumption Proxy, we expected that there would be a positive contribution to the production and, in case of negativity, if it had been waste; however, this would occur if it had not been renovation of fleet. On the other hand, the other two factors have different marginal productivity. In the case of marginal productivity of drivers (labor), we notice that it was decreasing, in other words, it probably happened a decrease of drivers, what is possible from the operational point of view. Besides that, it is

possible that the reduction of number of drivers was followed by the increase of work hours of the employed drivers.

Concerning the marginal productivity of fleet (invested capital), we can see negativity, but this has been tending to zero, even though the fleet of 2006 is bigger than the fleet of 2004. There had been possibly an expansion of the demand and the vehicles started to register a bigger occupation seat rate.

The marginal rate of substitution will demand a more careful treatment. We chose to make Tables 8 and 9 considering the 5 companies that most and least transported people year by year, to verify what the marginal rate of substitution of production factors indicate. The result, as can be seen table by table, is that when the number of passengers is small, the TMS of the companies is higher, occurring the contrary with the companies that transported more people. This in certain way contributes to the expected fact that the bigger companies have bigger control over their production inputs.

Tables 8 and 9 indicate that the drivers and fleet are considered in smaller proportion in the operator's decision. If not, the operator has a bigger concern with fuel consumption. But this is intimately connected to the fleet and to the driver's conduct while driving the steering wheel, therefore, it is possible that the results do not reflect the chain effect that the marginal substitution rate of the yearly traveled distance in km in other inputs.

We can see, beyond that, the ideal marginal substitution rate via comparison, providing to the regulative factors an instrument to foment a competition of the type yardstick. The scale of TMS fleet and TMS drivers is very low because of the nature of the proxies used. Remember that the proxies are number of buses, drivers and all distance tracked by the company.

Table 9 – Marginal substitution rate of the 5 companies which transported less passengers: 2004-2006 (left) and 5 Companies that most transported passengers: 2004-2006 (right)

2004						2004				
Company	Passengers	TMS KM	TMS FLEET	TMS DRI		Company	Passengers	TMS KM	TMS FLEET	TMS DRIVERS
A	233	0,08	-1,50E-04	9,19E-05		K	3391495	0,07	-7,14E-07	1,14E-06
B	1.071	0,16	-5,66E-06	6,92E-06		L	2.696.150	0,06	-1,58E-06	1,29E-06
C	1.797	0,33	-7,05E-05	8,08E-06		M	2.266.524	0,07	-1,14E-06	1,46E-06
D	1.841	0,12	-2,90E-05	6,70E-06		N	2.199.765	0,07	-6,34E-07	5,23E-07
E	2.200	0,15	-2,19E-06	4,01E-06		O	2.089.595	0,08	-1,05E-06	8,79E-07
2005						**2005**				
Company	Passengers	TMS KM	TMS FLEET	TMS DRI		Company	Passengers	TMS KM	TMS FLEET	TMS DRIVERS
A	156	0,1	-1,22E-04	4,97E-05		K	3222397	0,06	-4,19E-07	4,84E-07
F	565	1,79	-1,61E-04	6,55E-05		L	2783959	0,06	-6,76E-07	5,66E-07
G	1.801	0,05	-3,08E-06	3,75E-06		N	2.306.935	0,06	-4,19E-07	2,10E-07
H	1.926	1,15	-2,19E-05	5,93E-06		P	2.230.568	0,07	-6,95E-07	5,61E-07
E	2.231	0,11	-1,39E-06	1,70E-06		Q	2.112.388	0,08	-7,76E-07	5,85E-07
2006						**2006**				
Company	Passengers	TMS KM	TMS FLEET	TMS DRI		Company	Passengers	TMS KM	TMS FLEET	TMS DRIVERS
	121	0,11	-6,12E-06	-5,38E-05		K	3216436	0,06	-2,28E-08	-4,15E-07
I	1.528	1,09	-9,86E-07	-8,65E-06		L	2.529.796	0,07	-4,34E-08	-5,72E-07
F	1.601	0,53	-1,99E-06	-1,40E-05		M	2.169.012	0,07	-3,96E-08	-5,97E-07
E	1.645	0,18	-8,96E-08	-2,36E-06		Q	2.031.526	0,09	-3,71E-08	-5,75E-07
J	2.022	0,09	-4,69E-07	-4,11E-06		R	1.954.370	0,06	-1,34E-08	-3,58E-07

Finally, the statistics of the marginal substitution rates presented on Table 10 give a notion of the variability of data and of what occurs on the average of the sector of TRIP in Brazil. Roughly speaking, there is an installed capacity superior to the needed; this is reflected by the TMS of the fleet. The km TMS shows that there are few companies that use their fuel inefficiently. The TMS of the drivers indicate that the labor suffered great variation in the studied period and it is coherent to the smaller quantity of drivers employed in 2006 than in 2004.

Table 10 – Statistics of the Marginal substitution rates for the TRIP sector in Brazil

Year	Statistics	TMS KM	TMS FLEET	TMS DRIVERS
	Average	0,12	-4,21E-06	2,62E-06
	Maximum	1,27	-1,36E-07	9,19E-05
2004	Minimum	0,05	-1,50E-04	5,91E-08
	Average	0,13	-3,63E-06	1,54E-06
	Maximum	1,79	-6,59E-08	6,55E-05
2005	Minimum	0,04	-1,61E-04	1,61E-07
	Average	0,14	-1,55E-07	-1,30E-06
	Maximum	1,32	-4,35E-09	-1,32E-07
2006	Minimum	0,05	-6,12E-06	-5,38E-05

6. Final Considerations

The methodology presents deep link on the production theory and statistic base consolidated in econometric studies. The study is pioneer in the moment that escapes from the traditional approach of Cobb-Douglas parametric bound and applies a more general functional form using the stochastic frontier concept to measure technical inefficiency. With the estimation the inefficiency sources can be verified and actions can be showed in a way to improve the TRIP activity in Brazil.

As results, we have the efficiency profile of the TRIP for three years and the dynamic efficiency of the sector. A reduction of efficiency during the analyzed years was observed and it was identified that this probably is due to the expansion of the installed capacity of the companies by the acquisition of new buses. On the other hand, the concern of the operators with fuel consumption was assured, in a way that there were improvements in efficiency. Besides that, changes in the position of companies were verified year by year, the companies most and least efficient changed over the years. This suggests a strategic behavior in a meaning of a possible competitiveness among the companies, because, in one side, eventual substitutions of buses can come surrounded by the attraction of the demand. This is an assumption to be evaluated in another work with stratified data.

We can see also that the development of the 10 companies with bigger score strengthens this competitive behavior. It would be good to have an analysis route by route of the substitutability of the production factors facing the market's concentration or relative position of the leader year by year. This would corroborate to a regulatory analysis of the competitive behavior or cooperative behavior among the companies in specific lines, being a study issue and intervention of state in case it happens excess in market power.

Finally, this paper did not analyze the relations between the inputs, however, by the adopted functional form, this could be measured. We do not measure the possible existent multicolinearity, in a way that the correction and the analysis of the indirect effects between inputs are made in posterior works.

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