Erratum to "Bernstein-type Theorems in Hypersurfaces with Constant Mean Curvature" [An Acad Bras Cienc 72(2000): 301-310]

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ABSTRACT

An erratum to Lemma 2.1 in Do Carmo and Zhou (2000) is presented.

Key words: Riemannian manifold, eigenvalue, hypersurface, mean curvature.

ERRATUM

Replace Section 2 in Do Carmo and Zhou (2000) by the following. The resulting change in the lemma will not affect the rest of the paper.

2. A RESULT ON NODAL DOMAINS

In this section we prove a result on the nodal domains of $|\phi|$ which will be needed in our proof of main theorems. We first need to recall the definition of nodal domains.

Definition. An open domain D is called the nodal domain of a function f if $f(x) \neq 0$ for $x \in \operatorname{int} D$ and vanishes on the boundary of ∂D. We denote by $N(f)$ the number of disjoint *bounded* nodal domains of f.

Now we have the following lemma which follows directly from Proposition 2.2 below. We want to thank the referee who provided the clearer proof of Proposition 2.2.

Lemma 2.1. *Let M be a hypersurface in R^{n+1} with constant mean curvature H. Then*

$$\operatorname{ind}(M) \geq N(|\phi|). \tag{2.1}$$

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PROOF. Let $N = N(|\phi|)$ and $D_1, D_2, \cdots, D_N$ be the N nodal domains of $|\phi|$ and let

$$\varphi(u) = u^2 + \frac{n(n-2)}{\sqrt{n(n-1)}} Hu - nH^2.$$

Then from (1.5) and Proposition 2.2 below we have functions $f_1, f_2, \cdots, f_N$ with supports in D_1, $D_2, \cdots, D_N$ respectively such that

$$I(f_i, f_i) = \int_{D_i} (|\nabla f_i|^2 - \varphi(u) f_i^2) < 0.$$

Denote W the linear subspace spanned by $f_1, f_2, \cdots, f_N$. Since they have disjoint supports, they are orthogonal and thus the dimension of W is N. The index form $I(\cdot, \cdot)$ is negative definite on W so the Morse index is greater than or equal to N. □

PROPOSITION 2.2. *Let (M, g) be Riemannian manifold and $u \geq 0$ be a continuous function satisfying the following inequality of Simons' type in the distribution sense*

$$u^2\varphi(u) \geq a|\nabla u|_g^2 - u\Delta_g u\,, \tag{2.2}$$

where $a > 0$ is a constant and φ is a continuous function on R. If u has a relatively compact nodal domain D, then there exists a function f_D with support in D such that

$$\int_D (|\nabla f|^2 - \varphi(u) f^2) < 0.$$

PROOF. Suppose that u admits a relatively compact nodal domain D. Write $q := \varphi(u)$ and $v := \log u$ on D. Thus (2.2) can be written as

$$q \geq a|\nabla v|_g^2 - \Delta_g v - |\nabla v|_g^2.$$

Then for any Lipschitz function f with support in D and vanishing at ∂D, we have

$$\int_D (|\nabla f|^2 - qf^2) \leq -a\int_D f^2|\nabla v|^2 + \int_D |\nabla f - f\nabla v|^2.$$

Let $f = wu$, for some function w to be determined. We obtain

$$\int_D (|\nabla f|^2 - qf^2) \leq -a\int_D w^2|\nabla u|^2 + \int_D u^2|\nabla w|^2.$$

For all b such that $U/2 \leq b \leq U$, where $U := \sup_D u$, set

$$w_b(x) = \begin{cases} b & \text{as } u(x) \leq b, \\ u(x), & \text{as } u(x) > b. \end{cases}$$

Denote D_+ (resp. D_-) the set of points in D with $u(x) \geq b$ (resp. $u(x) \leq b$). A simple calculation leads to:

$$\int_D (|\nabla f|^2 - qf^2) \leq \int_{D_+} u^2 |\nabla u|^2 - \frac{aU^2}{4} \int_D |\nabla u|^2.$$

When b goes to U, the first term of right hand side tends to 0 (because $|\nabla u|^2$ is integrable), while the second term is fixed. It follows that $\int_D (|\nabla f|^2 - qf^2) < 0$ for all functions $f = w_b u$, when b is close to U. The conclusion is proved. □

REFERENCE

Do Carmo MP and Zhou D. 2000. Bernstein-type Theorems in Hypersurfaces with Constant Mean Curvature. An Acad Bras Cienc 72: 301-310.