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The problem of school enrollment rules: what can be changed in the largest city in Brazil

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School choice is an issue little studied in Brazil, despite its huge importance, especially in the USA. In this paper, using game theory, how students are allocated in municipality of São Paulo is analyzed. As students 'preferences are not taken into account, the São Paulo system does not meet the main qualities of an allocation mechanism: stability, non-manipulation and efficiency. Alternatively, the use of the Gale-Shapley mechanism is proposed. Simulations are performed confirming theoretical results and also indicating a huge potential for improvement in the system.

Keywords: education; subnational governments; school choice; game theory.

O problema das matrículas escolares: uma discussão sobre a melhoria das regras utilizadas em São Paulo

A adoção de mecanismos de escolha escolar é pouco estudada no Brasil, apesar do seu crescente uso em outros países, sobretudo, nos EUA. Neste artigo, com o uso de instrumental teoria dos jogos, o sistema de matrículas escolares do município de São Paulo é analisado. Por não usar as preferências dos alunos, verifica-se que o sistema não atende a nenhum dos conceitos desejados de um mecanismo de alocação: estabilidade, não manipulação e eficiência. Alternativamente, é proposta adoção do mecanismo de Gale-Shapley. Simulações são realizadas para corroborar os resultados teóricos, indicando enorme potencial de ganhos com a mudança do sistema existente.

Palavras-chave: educação; governos subnacionais; matrícula escolar; teoria dos jogos.

El problema de las reglas de matrícula escolar: una discusión sobre los cambios en la mayor ciudad de Brasil

La adopción de mecanismos de elección escolar es poco estudiada en Brasil. En este artículo, con el uso de la teoría de los juegos, el sistema de matrices escolares del municipio de São Paulo es analizado. Por no usar las preferencias de los alumnos, verificase que el sistema de São Paulo no atiende las principales cualidades de un mecanismo de asignación: estabilidad, no manipulación y eficiencia. Alternativamente, se propone utilizar el mecanismo de Gale-Shapley. Simulaciones son realizadas para corroborar los resultados históricos, exigiendo enormes potencialidades de ganancias con el cambio del sistema existente.

Palavras clave: educación; gobiernos subnacionales; registro escolar; teoría de los juegos.

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1. INTRODUCTION

The democratization of access to education has been a slow process in Brazil. Despite continued efforts, taking into account the students' preferences in school choice systems is a relatively new phenomenon, pioneered in the US in the mid-1990s. The lack of homogeneity in the school system results in students having preferences for certain schools, and factors such as quality of teaching staff and infrastructure are important inducers of attractiveness. However, in most of the school systems in Brazil, the only information used to lead the allocation process is the student's address. In these systems, the school where the student will be enrolled is conditioned by the income level, which is the decisive element for families when establishing themselves in a neighborhood. Thus, when taking into account the students' preference in the process of admission to the public education system, the democratization of access to school is expanded, since it is possible to lessen the relevance of the address and consequently, of income.

The introduction of school choice in the admission process, however, does not occur without obstacles. Due to the enormous size of the public school system, there is a need to develop sophisticated mechanisms that can simultaneously incorporate the criteria of priorities established by the government for student allocation, as well as students' school choice. In addition, as Roth (2002) and Roth and Sotomayor (1990) have shown, there is an inevitable trade-off: the best outcome for students in terms of satisfying their preferences is the worst at meeting the rules of priorities of government and vice versa. Thus, it is crucial to use appropriate game theory tools for a better understanding of the allocative process inherent in the school choice system, leading to the development of the best procedures. In general, the aim is to design rules that are tamper-proof, efficient and free of situations of injustice. This has already occurred with relative success in major US cities such as Boston and New York (Ergin and Sönmez, 2006; Abdulkadiroğlu et al., 2005).

It is important to emphasize that the school choice process also involves other fundamental questions. Unequivocally, better quality schools are needed. In addition, other phenomena such as cream-skimming by institutions, information asymmetry and unbalanced vocalization capacity are also important (Corcoran and Cordes, 2017; Hoxby, 2003). However, even if such problems are not solved by improving the rules of school choice systems alone, the gains from adopting better procedures are undeniable, especially by preventing urban segregation turning into school segregation, and by considerably expanding the system's democracy.

In Brazil, the introduction of school choice within the public system is recent, and the public administration literature on the subject is practically non-existent. Thus, the main objective of this article is to evaluate the school choice process in the public education system in the city of São Paulo, introducing an important research area, with potential to be applied in thousands of municipalities in the country. After this brief introduction, the problem of school choice will be modelled, introducing into an allocation mechanism the desirable concepts of: stability, non-manipulation and efficiency. In the third section, the early enrollment process in São Paulo is presented, discussing in detail its main problems. The following section will present proposed alternatives with special emphasis on the Gale Shapley algorithm, which will then be evaluated through simulations.

2. SCHOOL CHOICE SYSTEM

One of the major challenges faced in the public policy literature is the design of appropriate mechanisms to deliver public services (Roth and Sotomayor, 1990; Roth and Peranson, 1999; Roth, 2002). In the case of school choice problems, the most famous intervention in the literature is that of Boston, in the United States.[1] In 2005, local government redesigned school allocation procedures, adopting a student-optimal stable mechanism, inspired by Gale and Shapley's (1962) algorithm of Deferred Acceptance. A mechanism is the rule used for school allocation. This new model supplemented the so-called Boston mechanism, which was causing severe difficulties for parents and students.

The experience of the city of Boston shows that the adoption of a centralized procedure requires care. One of the qualities pursued in the school allocation mechanism is that it is free of manipulation. The dynamics induced by a manipulative procedure opens space for differentiation between good and bad players, because revealing true preference about schools is not always the best option. This makes the school choice a complex process, requiring parents to use an unobvious strategy.

It is expected from any mechanism to find a matching, that is, an allocation of students in schools, in which there is no possibility of unilateral improvements, without burden to third parties. In other words, it is assumed that the procedure produces Pareto Efficient allocations. Finally, another criterion is the elimination of justified envy, which is when a student, who has higher priority in a school, is deprived in favor of another student and ends up being allocated in a school that they consider inferior. A pairing between schools and students is stable when there is no justified envy (Abdulkadiroğlu and Sönmez, 2003).

The importance of stability to maintain centralized mechanisms in several markets was investigated by Roth (2002). In the case of Brazil, the student is responsible for blocking an unstable matching, because, the student with justified envy is allowed to appeal to justice in order to claim the violation of their rights. As a guarantee of legal stability of the school allocation mechanism, it is therefore desirable to have in place a stable mechanism.

3. THE PUBLIC EDUCATION SYSTEM IN THE CITY OF SÃO PAULO

The Public Education System in São Paulo is divided into three major cycles, named here: early childhood education (nursery/preschool); primary education (elementary/middle school); and secondary education (high school). The education services are maintained jointly by the municipal and state governments. Basically, the municipality offers early childhood education while the secondary education is delivered by the state government. Both municipality and the state offer primary education.

When it comes to numbers, the enrollment process of the Public School System in São Paulo is one of the largest allocation problems in the world, with 1,589,069 students enrolled in 2015. New York, for example, has approximately one million students (Abdulkadiroğlu et al., 2005). As for the education system, the city of São Paulo, capital of the state, is divided into 13 micro-regions, whose administration is shared by the state and the city hall. The mechanism is defined annually through

[1] Examples of the extensive literature about the school market in Boston may be found in Abdulkadiroğlu and collaborators, (2005), Abdulkadiroğlu and collaborators (2003), Chen and Sönmez (2006), Ergin and Sönmez (2006), among others.

resolutions, for example, for 2018, the ruling norms were in Resolution SE 33 and in Resolution SE 34, both of July 2017 and released by the State Secretariat of Education of São Paulo. Supervision of the public system is a responsibility of the regional department of education, while the municipal system is submitted to the municipal coordination of education, connected to the city's executive branch.

Because the city is administratively divided in districts, each student can only apply for a seat in an institution located in a micro-region in which they reside. Thus, within each district, the early enrollment process, or simply, the São Paulo school choice mechanism works in four steps:

Identification of places availability;
Students register
Compatibility of demand and supply
Enrollment

Step 1: Each institution informs its regional department of education about the number of seats available, identifying: the set of schools $I = \{i_1, \dots, i_n\}$ and the vector of capacities $Q = \{q_{S_1}, \dots, q_{S_J}\}$.

Step 2: Students go and enroll in the schools or in enrollment stations, providing: name, gender, date of birth, address, race and the name of the parents or guardian. Seniors in primary education (middle school) indicate three institutions in order of preference. Students of the last year of Early Childhood Education only inform registering data.

In turn, enrollment in primary education occurs in three phases:
Phase 1: registration of students coming from the public early childhood education system.
Phase 2: registration of children Aged 6 at time of registration) from outside the public early childhood education system.
Phase 3: registration of other children, from eight years old, for other grades.

Step 3: (Compliance of Demand)

3.1 For students in early childhood education, matching is determined according to the geographic proximity of the residence. The allocation is made simultaneously by the regional departments of education and by municipal coordination of education. The procedure used is as follows: students q_{S_j} living the closest to the institution s_j are allocated there. Students who remain without seats are allocated to schools in their nearest micro-region that still have seats.

3.2 Primary education students who are candidates for seats in secondary education schools, are divided according to their three options. If the student does not specify any, it is assumed the option for the school closest to their homes. Older students have preference. The q_S first students are allocated to school s. Others go to their second option or, if they did not specify a school, it is assumed the second one that is closest to their home. The mechanism ends when all students are allocated.

Step 4: For the enrollment to be completed, the final allocation of the students is included in the computerized system of the state government. The result is then announced.

3.1 CONSIDERATIONS ABOUT THE CURRENT MECHANISM IN SÃO PAULO

Procedures that do not take into account the preferences of a market share tend to generate large inefficiencies. Despite the elimination of the problem of queues and duplication of enrollments, the early enrollment process of São Paulo for primary education ends up generating situations where Pareto improvements are possible. For example, consider the following school problem with eight students $I=\{i_1,i_2,i_3,i_4,i_5,i_6,i_7,i_8\}$ and five schools $S=\{s_1,s_2,s_3,s_4,s_5\}$. The profile of preferences, priorities and the vector of capabilities being:

$$P(i_1)=(s_1,s_5,s_2,s_3,s_4) \quad P(i_5)=(s_2,s_4,s_5,s_3,s_1) \quad P(s_1)=(i_1,i_2,i_7,i_3,i_4,i_8,i_5,i_6)$$

$$P(i_2)=(s_5,s_4,s_3,s_2,s_1) \quad P(i_6)=(s_3,s_2,s_5,s_1,s_4) \quad P(s_2)=(i_8,i_6,i_4,i_3,i_5,i_2,i_7,i_1)$$

$$P(i_3)=(s_1,s_4,s_2,s_5,s_3) \quad P(i_7)=(s_1,s_3,s_4,s_5,s_2) \quad P(s_3)=(i_5,i_6,i_4,i_8,i_3,i_2,i_7,i_1)$$

$$P(i_4)=(s_3,s_2,s_4,s_1,s_5) \quad P(i_8)=(s_2,s_3,s_4,s_1,s_5) \quad P(s_4)=(i_2,i_4,i_5,i_3,i_7,i_1,i_8,i_6)$$

$$Q=\{q_{S_1}=2,q_{S_2}=2,q_{S_3}=2,q_{S_4}=2,q_{S_5}=1\} \quad P(s_5)=(i_7,i_3,i_2,i_1,i_4,i_5,i_8,i_6)$$

When the preference profile P (i) portrays the order of the schools according to the student's preference i, P (s) reveals the order of priorities of the students in the school s and, in turn, the vector of capacities Q(.) indicates the number of seats in the school. In addition, consider the following ordering of schools, at increasing distance from the individual's residence i:

$$i_1 \Rightarrow s_1,s_5,s_4,s_2,s_3 \quad i_5 \Rightarrow s_3,s_2,s_4,s_5,s_1$$

$$i_2 \Rightarrow s_4,s_5,s_1,s_2,s_3 \quad i_6 \Rightarrow s_2,s_3,s_4,s_1,s_5$$

$$i_3 \Rightarrow s_2,s_1,s_4,s_3,s_5 \quad i_7 \Rightarrow s_5,s_1,s_4,s_3,s_2$$

$$i_4 \Rightarrow s_3,s_2,s_4,s_1,s_5 \quad i_8 \Rightarrow s_2,s_3,s_4,s_1,s_5$$

Using the system of São Paulo, the resulting matching is:

$$\gamma(s_1)=\{i_1,i_2\},\gamma(s_2)=\{i_8,i_6\},\gamma(s_3)=\{i_5,i_4\},\gamma(s_4)=\{i_2,s_4\},\gamma(s_5)=\{i_7\}$$

Where γ indicates the students allocated in the school, i.e., $\gamma(s_1)=\{i_1,s_1\}$ shows that the student i_1 was allocated in the school, and a seat was not occupied – if it had been, another student would appear in the place of s_1. Note that if i_6 in s_2 and i_5 in s_3 changed their respective seats, they would be better off, without worsening anyone's situation. Besides, s_4 ends up failing to allocate one of its two places, although i_5 prefers s_4 to the school that they received (s_3), generating a situation of justified envy. In other words, São Paulo's school choice system for both secondary and primary education is not a stable or an efficient mechanism.

Indeed, in the specific case of allocation in secondary education, it is interesting to note that the algorithm resembles that of Gale-Shapley (Gale and Shapley, 1962), truncated in the third option in the students' preference list; however, from this point, students are allocated according to their priority, using the example discussed.

The extent of the consequences of preferences truncation in the third option is directly related to the structure of students' preferences.

To illustrate, consider the following school problem, with 10 students $I=\{i_1,i_2,i_3,i_4,i_5,i_6,i_7,i_8,i_9,i_{10}\}$ and five schools $S=\{s_1,s_2,s_3,s_4,s_5\}$. Without any loss of generality, the lists in this simplified example are truncated in the second option. The profile of priorities and the vector of capacities are:

$$
\begin{aligned}
P(s_1) &= (i_1,i_2,i_3,i_4,i_5,i_6,i_7,i_8,i_9,i_{10})\\
P(s_2) &= (i_2,i_4,i_{10},i_1,i_9,i_3,i_6,i_8,i_7,i_5)\\
P(s_3) &= (i_3,i_2,i_{10},i_4,i_9,i_7,i_8,i_6,i_1,i_5) \qquad Q=\{q_{S_1}=1, q_{S_2}=1, q_{S_3}=2, q_{S_4}=3, q_{S_5}=3\}\\
P(s_4) &= (i_{10},i_9,i_2,i_7,i_4,i_3,i_8,i_1,i_5,i_6)\\
P(s_5) &= (i_4,i_3,i_6,i_7,i_2,i_9,i_5,i_8,i_{10},i_1)
\end{aligned}
$$

Assume that the profiles of preference describe an unanimity on the part of the students around the institutions s_1 and s_2. That is, the students' preference for the school does not correspond to the geographical distance from their residence to the school, which is essential information in the criterion of public power. Therefore, the allocations selected by both mechanisms differ, and the result obtained by the early enrollment process is worse for the students than the result of the Gale and Shapley algorithm, which will be described next. The first one is written with two points, while the second one is marked in the preferences with a square:

$$
\begin{aligned}
P(i_1) &= \left(\overset{\bullet\bullet}{\boxed{s_1}},s_2,s_4,s_3,s_5\right) & P(i_6) &= \left(s_1,s_2,\boxed{s_4},\overset{\bullet\bullet}{s_5},s_3\right)\\
P(i_2) &= \left(s_1,\overset{\bullet\bullet}{\boxed{s_2}},s_5,s_4,s_3\right) & P(i_7) &= \left(s_1,s_2,\overset{\bullet\bullet}{\boxed{s_4}},s_5,s_3\right)\\
P(i_3) &= \left(s_1,s_2,\boxed{s_5},\overset{\bullet\bullet}{s_3},s_4\right) & P(i_8) &= \left(s_1,s_2,\boxed{s_3},s_4,\overset{\bullet\bullet}{s_5}\right)\\
P(i_4) &= \left(s_1,s_2,\boxed{s_3},s_5,\overset{\bullet\bullet}{s_4}\right) & P(i_9) &= \left(s_1,s_2,\boxed{s_5},s_3,\overset{\bullet\bullet}{s_4}\right)\\
P(i_5) &= \left(s_1,s_2,\boxed{s_4},\overset{\bullet\bullet}{s_5},s_3\right) & P(i_{10}) &= \left(s_1,s_2,\boxed{s_5},s_4,\overset{\bullet\bullet}{s_3}\right)
\end{aligned}
$$

In addition, another striking feature of the early enrollment process in São Paulo is that the algorithm produces the best possible result for institutions, since only their priorities are observed.

4. AN ALTERNATIVE PROPOSAL: GALE AND SHAPLEY

An alternative is the mechanism that uses the deferred acceptance algorithm, by Gale and Shapley (1962), with students making offers, which is used in Boston and New York. School priorities are considered as preferences. The mechanism consists of *n* steps:

- Step 1: Each student applies for their first choice. Each school tries to match its seats with the proponents, one at a time, respecting the school's order of priority. The other candidates are rejected. In general, in the n^{th} step:
- Step n: Each student rejected in the previous step applies for their n^{th} choice. Each school tries to match its seats among the proponents and those that were previously assigned, one at a time, respecting the school's order of priority. The other candidates are rejected.

The algorithm ends when no student proposal is rejected and each student is allocated on their last attempt. To exemplify its operation, consider the following school problem with three students $I=\{i_1,i_2,i_3\}$ and three schools $S=\{s_1,s_2,s_3\}$. The profile of preferences, priorities and the vector of capacities are:

$$P(i_1)=(s_2,s_1,s_3) \qquad P(s_1)=(i_2,i_1,i_3)$$

$$P(i_2)=(s_2,s_3,s_1) \qquad P(s_2)=(i_3,i_2,i_1)$$

$$P(i_3)=(s_1,s_2,s_3) \qquad P(s_3)=(i_2,i_3,i_1)$$

$$Q=\{q_{S_1}=1,q_{S_2}=1,q_{S_3}=1\}$$

The result produced is precisely the problem's only stable matching:

$$\mu(s_1)=\{i_1\},\mu(s_2)=\{i_3\},\mu(s_3)=\{i_2\}$$

In addition, the Gale and Shapley mechanism always produces matching without justified envy, which is student-optimal. However, the mechanism is not Pareto efficient, as it does not yet explore all possible improvements (Gale and Shapley, 1962). Another mechanism is the Top Trading Cycles (TTC) proposed by Abdulkadiroğlu and Sönmez (2003). The starting point of the mechanism is to consider the priorities of schools as opportunities, rather than preferences. Thus, if it suits them, both can agree and change their respective priorities. However, in view of the evident legal incompatibility of treating seats as goods to be exchanged (Abdulkadiroğlu and Sönmez, 2003), this mechanism will be used only for comparisons in the simulations, and will not be presented.[2]

5. SIMULATION OF RESULTS

The implementation of a new mechanism for the school choice system in the city of São Paulo means major changes for students and for the education system itself. Prior to the reformulation of the current procedure, it is desirable to carry out experiments to evaluate the extent of the proposed changes.

[2] Decisions to that effect can be found in judgments: 00572789, 00455050, 00441818, which can be consulted at the Court of Justice of São Paulo.

For this purpose, the following procedure was adopted:

a) In order to facilitate matchings, all the schools of the state and municipal systems were grouped according to the city's regional department of education they are related to. The schools are distributed and organized according to the city's districts, and the department of education of the city's administrative zone 'East 2' was chosen for the simulation because it is one of the most populated regions of São Paulo;
b) Within the department of education of administrative zone 'East 2', all the elementary schools with their respective capacities were listed, normalized by the number of seats in the larger school. Normalization was used so the proportions of the schools created in the simulation replicated the reality. For the largest institution a unit coefficient was assigned;
c) A random number generated from a Uniform Distribution (0,1) was assigned to each of the schools of the East 2 department. The schools were ordered according to this number, and the first 20% were chosen. More precisely from a total of 94 schools, 20 were drawn. This procedure aimed to reduce the number of schools of the simulation to better operate it;
d) The normalized capacity of the schools was multiplied by 10, with the number of seats defined as the largest integer associated, creating a total of 96 positions;
e) For the simulation of the students' priorities, a list of individuals was generated ordered by the lowest index: $i_1,\ldots,i_{96}$. Each student was assigned a random number generated from a Uniform Distribution (0,1). The student associated with the highest number took the first position in the priority list while the individual with the second highest number, the second position, and so on. For each of the 20 schools the procedure was repeated;
f) In case of a tie in the order of priorities, the lowest index of the school was assumed as the tie-breaking criterion. That is, if the student i has priority both in school s_k and s_l, with $k < l$, the student will have higher priority in s_k.

In the simulation, students' preferences were generated randomly. For this, a list of schools ordered by the lowest index ($s_1,\ldots,s_{20}$) was generated.[3] A random number generated from a Uniform Distribution (0.1) was associated with each establishment. The school associated with the largest number took the first position in the preference list while the one with the second highest number, the second position, and so on. For each of the 96 students the procedure was repeated.

As shown in Box I, of the 20 simulated schools, taking into account the most preferred school by students, 12 have a supply above capacity, one serves the demand perfectly, while the remaining seven units have a demand for seats that is higher than their capacity.

[3] Alternatively, preferences were simulated following a geographic concentration bias, i.e., a shorter distance means better positioning in the preference list. Nonetheless, the results remained the same qualitatively, which means that despite the preference profile for the students to be favorable to the current system, the alternative algorithm proposed performed better.

BOX I SIMULATION — SCHOOLS, PREFERENCES AND CAPACITY

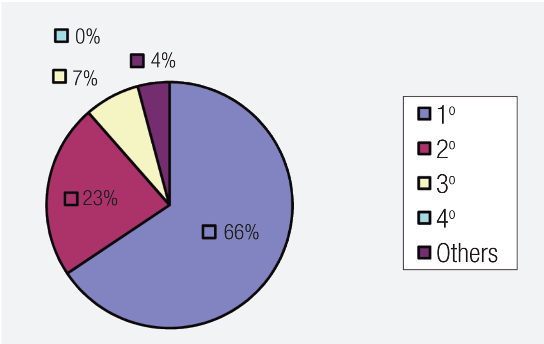
	1st option	Capacity		1st option	Capacity
S1	4	6	S11	5	7
S2	3	6	S12	5	7
S3	10	6	S13	7	5
S4	3	5	S14	3	5
S5	5	5	S15	4	5
S6	2	5	S16	5	5
S7	3	4	S17	7	4
S8	1	3	S18	9	4
S9	9	3	S19	3	4
S10	5	3	S20	3	4

Source: Elaborated by the author.

The comparison between the procedures used in São Paulo and the Gale and Shapley's students-optimal stable mechanism and the top trading cycles shows great disparities. The biggest difference is between the matching obtained by the early enrollment process for Elementary Education. In this case, approximately 88% of students end up receiving a seat in an institution that is not in the group of four most wanted schools. This percentage is drastically higher than that obtained by other mechanisms: 27% of the students using the algorithm by Gale and Shapley; 23% in the rule established by the school choice system for secondary education; and only 19% for the TTC, as it is possible to see in figures I, II, III and IV.

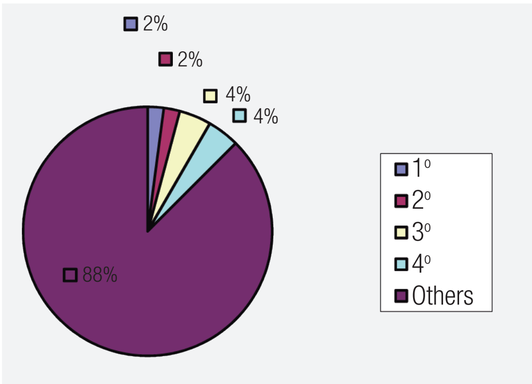
When analyzing the rate of students who receive the first choice, it turns out that the procedure used for allocation in primary education also presents the worst performance. Only 2% of the students got seats in their preferred institution, compared to 40% in the Gale and Shapley algorithm, 66% of the allocation procedure in secondary education and 69% in the TTC mechanism. This low performance of the Early Enrollment Process for Primary Education was already expected since the procedure ignores students' preferences, and focuses only on the priorities of the schools.

FIGURE I SECONDARY EDUCATION — ALLOCATION BY PREFERENCES



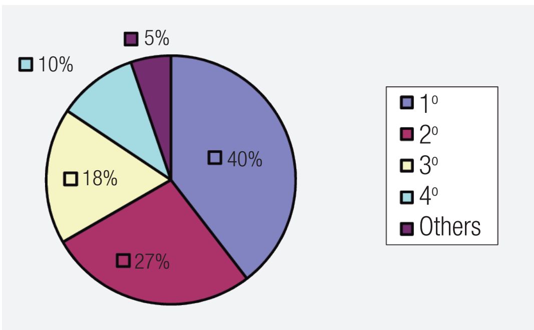
Source: Elaborated by the author.

FIGURE II PRIMARY EDUCATION — ALLOCATION BY PREFERENCES



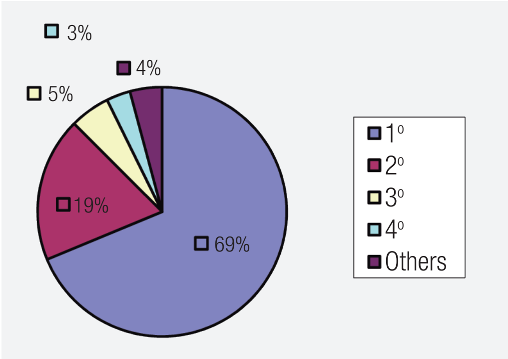
Source: Elaborated by the author.

FIGURE III GALE-SHAPLEY — ALLOCATION BY PREFERENCES



Source: Elaborated by the author.

FIGURE IV TTC — ALLOCATION BY PREFERENCES



Source: Elaborated by the author.

The performance of the current allocation mechanism for secondary education performed better, corroborating the theoretical results, since the algorithm used is a combination of the Gale-Shapley mechanism and the procedure used for primary education. Nevertheless, it was observed that students whose low priority did not allow them to be allocated to their third most wanted school, ended up receiving seats in unwanted institutions, as shown in Box II.

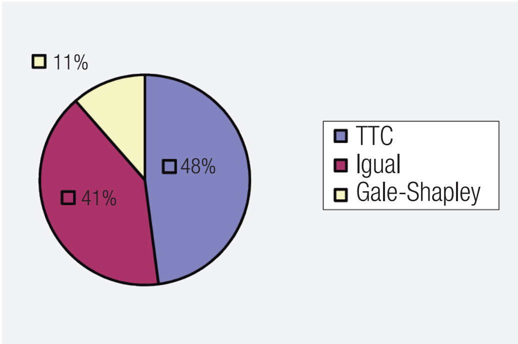
BOX II COMPARING THE DIFFERENT MECHANISMS

	TTC	GS	Primary Ed.	Secondary Ed.
1st	66	38	2	63
2nd	18	26	2	22
3rd	5	17	4	7
4th	3	10	4	0
Others	4	5	84	4
Worst allocation[1]	15th	8th	20th	16th

Source: Elaborated by the author.
[1] Position of the school on the list of preferences of the student who was enrolled in the worst school.

Finally, the simulation allows a comparison between the Gale and Shapley's student optimal stable mechanism and the TTC. As shown in the literature, TTC is a Pareto-optimal procedure (Abdulkadiroğlu and Sönmez, 2003). However, the matching obtained by the TTC is not better for all students than the matching obtained by the Gale and Shapley mechanism. Of the 96 students, 46 got a seat in a more wanted school when the TTC mechanism was applied, while 11 individuals have their best allocation in the matching obtained by Gale and Shapley.

FIGURE V GALE-SHAPLEY VERSUS TTC



Source: Elaborated by the author.

Therefore, the cost of this higher efficiency of the TTC mechanism appears precisely in the existence of justified envy. In fact, there are 64 cases of justified envy in the simulated matching involving 26 students and 18 schools,[4] or approximately 27% of the students. Therefore, the greater efficiency achieved by the TTC implies a burden in terms of justice, which can have important legal consequences, reducing the chances of success of the mechanism.

6. CONCLUSION

The introduction of school choice mechanisms that take into account the students' preferences is essential for a greater democratization of access to public education. The current early enrollment process in the city of São Paulo does not eliminate justified injustice, and it exhausts all possibilities of improvement without harming any student, which has been confirmed by comparison with alternative mechanisms, especially the Gale-Shapley. These findings indicate a potential for gains in redesigning the school choice system, introducing procedures that take into consideration the opinions of students and their families.

[4] The results are not presented due to limitation of space.

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