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INFLUENCE OF NAILS SIZE AND LAYOUT TO OBTAIN THE REDUCTION COEFFICIENT OF MOMENT OF INERTIA FOR TIMBER BEAMS WITH COMPOSITE CROSS SECTION

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FRANCISCO A. R. LAHR[1*], ANDRÉ L. CHRISTOFORO[2], JULIANO FIORELLI[3]

ABSTRACT: A significant portion of rural building's roof is supported by timber trusses. Increasing distance between the trusses can be achieved by using composite cross sections ("T" and "I") for beams. With this, it is possible to reduce the number of columns and trusses, bringing significant savings in material and the desired cost reduction, mainly using wood from planted forests. The Brazilian Code ABNT NBR 7190:1997 establishes a coefficient (α_r) for the reduction of the theoretical moment of inertia (I_{teo}) of 0.95 and 0.85, for beams with "*T*" and "*I*" composite profile, respectively. However, no specification is showed about connections responsible for conferring such coefficients. This research aimed to investigate, based on static bending tests, analysis of variance and polynomial regression models (linear, quadratic, cubic), the precision of α_r coefficients set by the Brazilian Code for *Pinus* sp. beams. We considered to evaluate influence of factors such as: nails number (3, 5, 9, 17, 33), nails size (19×27 mm, 19×33 mm), profile types (*I*, *T*) and calculation form to obtain the equivalent modulus of elasticity (E_{deq}) used to determine the cited coefficients. The highest values of α_r were derived from beams with 17 and 33 nails, which provided similar results. The analysis also allowed admitting the adequacy, only to the beams with "T" section, of α_r stipulated by the Brazilian Code. For "I" section, α_r overestimates beams performance. Nails size and form of calculating E_{deq} were not significant in obtaining α_r. The quadratic model showed the best results, indicating that the amount of 25 nails can provides the highest values of α_r coefficients.

KEY WORDS: timber beams, composite cross section, steel connectors.

INTRODUCTION

Searches for production models that adopt sustainability standards have gained strength in recent years in the Brazilian civil construction. Therefore, there is the need to add technology to the wood coming from reforestation, especially when working with Pinus species. Defects resulting from fiber inclination, nodes, shrinkage due to drying, among others, require from the construction industry professionals to develop alternatives to minimize these suppressive effects and bring to consumer market products with quality and durability (FIORELLI, SORIANO & LAHR, 2012).

In this scenario the concept "engineered wood" emerged, terminology that defines its industrialization process. An example of this material is the composite beams formed by joining smaller wood pieces, allowing obtaining larger cross section of commercial dimensions (GÓES, 2002; GÓES & DIAS, 2005).

Such parts may be applied as purlins in roof structures, especially in rural buildings, which allow to significantly increasing distance between building columns, reducing the final cost of framework set (CALIL et al, 2003; CHRISTOFORO et al, 2013; GONÇALVES et al, 2013).

[1] EESC-USP, São Carlos, SP, Brasil.
[2] Universidade Federal de São Carlos (UFSCar), São Carlos - SP, Brasil.
[3] FZEA-USP, Pirassununga-SP, Brasil.
*Corresponding author. E-mail: frocco@sc.usp.br
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Composite beams may be produced by lumber pieces or obtained from wood-based panels such as plywood, in which the element junction is usually done by mechanical connections (MOURA et al., 2008; GÓES & DIAS, 2005; FAN, 2012) or adhesives.

For the design of composite parts, normative documents adopt two different alternatives methods: the analytical method and the mitigation coefficients method. The last one has gained greater acceptance in technical means after the publication of KIDWELL'S study (1897), among others.

The ABNT NBR 7190:1997 Brazilian Code (Timber Structures Design) adopts the method of mitigation coefficients, using reduction coefficient (α_r) of the theoretical moment of inertia of the composite beam cross section. According to this standard, the effective moment of inertia (I_{ef}) is obtained applying mitigation (α_r) of the theoretical moment of inertia (I_{teo}) by [eq. (1)], assuming to α_r : 0.95 for the cross sections composition made of two juxtaposed rectangular elements ("T" profile) and 0.85 for the sections formed by three rectangular elements ("I" profile).

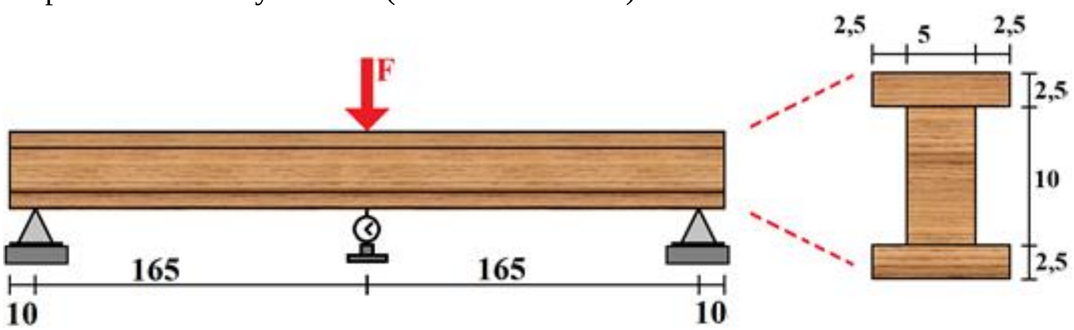
$$I_{ef} = \alpha_r \cdot I_{teo} \tag{1}$$

However, the ABNT NBR 7190:1997 Brazilian Code does not show specifications about the number of connections responsible for grating α_r coefficients, which are considered due only to geometry profile. In this sense, GÓES & DIAS (2005) evaluated, by four-points bending tests and using Angelim Pedra Verdadeiro (*Dinizia excelsa*), Cedrinho (*Erisma* sp.) and *Pinus hondurensis* wood species, the stiffness product in "*I*" profile beams integrally by nails, using EUROCODE-5:1993 normative documents and ABNT NBR 7190:1997. The authors conclude that the method used by the Brazilian Code is simple to estimate the effective moment of inertia of the composite section, however, inaccurate, unlike the method proposed by the EUROCODE-5: 1993, in which estimate stiffness in static bending follows successive accounts to be performed, but providing closer results of the ones obtained experimentally compared with the Brazilian Code.

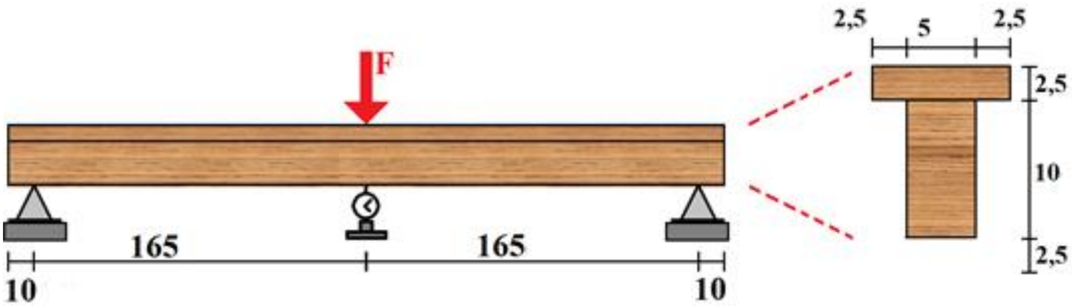
In order to collaborate with more information about the reduction of theoretical moment of inertia of composite cross section beam design, we aimed to investigate, in this study, α_r coefficients indicated by ABNT NBR 7190:1997 Brazilian Code, evaluating the following factors: geometry of the cross section profile; nails amount and size; and the method of calculating the equivalent elastic modulus of the parts number (profile) for determining the α_r coefficients.

MATERIAL AND METHODS

To carry out this study ten *Pinus sp.* Timber beams with "T" profile and ten other with "I" profile (Figure 1) were manufactured, all of them tested in three-points static bending, same structural model adopted by ABNT NBR 7190:1997 in obtaining modulus of elasticity (E) and strength modulus in static bending, in small clear specimens. For each profile type, five out of 10 beams had cross section composition using 19 × 27 (3.9 mm × 61 mm) nails with head and five others with profile formed by 19 × 33 (3.9 mm × 7.5 mm) nails with head.



(a)



(b)

FIGURE 1. Three-point static bending test in beams with "I" (a) and "T" (b) profiles [nominal dimensions in cm].

For the manufacture of the 20 beams, 30 wood pieces with 2.5 cm height, 10 cm width and 350 cm length; and 20 wood pieces with 10 cm of height, 5 cm of width and 350 cm of length were used. The profiles dimensions of the beams cross section are shown in Figure 1.

The static bending tests on each specimen (isolated pieces and composite section beams) were carried out in a non-destructive way and the displacements in the middle of the L span (distance between supports - 330 cm) were limited to the $L/200$, providing physical, linear and geometric behaviour for the evaluated pieces (ABNT NBR 7190:1997).We highlight that the relation between the support distance (L) and the cross section height (h) was above 21 ($L/h>21$), which enabled to despise the shear stresses in beam displacement calculations (LAHR, 1983; CHRISTOFORO et al., 2013; CHRISTOFORO et al. 2014).

For the composition of each "I" and "T" profile, three nails were employed initially, with later insertion of two (5 nails), four (9 nails), eight (17 nails) and sixteen (33 nails) arranged successively, and equidistantly from beams midpoints. For each number of nails (3; 5; 9; 17; 33) and dimensions (19 mm × 27 mm, 19 mm × 33 mm) static bending tests were carried out (five tests per piece), wherein the effective moment of inertia (I_{ef}) for those nails amount were obtained experimentally by eqs. (2) and (3).These equations were adapted from Strength Material Expression for determining displacements in the middle points of beams in three-point tests (Figure 1).

$$I_{ef1} = \frac{F \cdot L^3}{48 \cdot \sum_{i=1}^{n} \frac{E_i \cdot A_i}{A_i} \cdot \delta} \tag{2}$$

$$I_{ef2} = \frac{F \cdot L^3}{48 \cdot \sum_{i=1}^{n} \frac{E_i \cdot I_i}{I_i} \cdot \delta} \tag{3}$$

In eqs. (2) and (3), F is the force value obtained experimentally for the displacement measure (δ) in the middle of the span between the supports (L) equal to $L/200$ (1.65cm), n is the number of timber pieces that compose the cross-section, E is the modulus of elasticity for each piece , I is the moment of inertia of each piece in relation to the bending axis (which coincides with the geometrical centre of the cross-section beam) and A is the cross section area of each piece . The effective moment of inertia I_{ef1} and I_{ef2} obtained from eqs. (2) and (3) differ from each other only by the way in which modulus of elasticity in bending are calculated. In the calculation of I_{ef1}, the modulus of elasticity (or equivalent modulus of elasticity - E_{eq}) is obtained by weighting the areas of the pieces that make the section, and in I_{ef2} calculation, the equivalent modulus of elasticity is determinate by the weighting of the moment of inertia.

Thus, for each geometry of the cross section, size and nails number, with the eqs. (2) and (3), the effective moment of inertia were calculated, which divided by the theoretical moment of inertias (I_{teo}) of the sections provide the reduction coefficients (α_r) (Equation 1). In Figures 1a (*"I"* profile) and 1b (*"I"* profile), the theoretical moment of inertia calculated in relation the geometrical centres were: 2395.83 cm^4 and 1080.73 cm^4, respectively.

To investigate the influence of individual factors: number of nails (NN), dimensions of nails (DN), geometry of the composite section (Perf), form of calculation of the equivalent modulus of elasticity (E_{eq}) and interactions between both the values of the inertia reduction coefficients, variance analysis (ANOVA) at 5% significance level were used. The Anderson-Darling and Bartlett tests were used to verify residues normality and homogeneity of the residues variances between treatments (ANOVA requirements for validation). By the formulation of hypotheses in these tests, P-value greater than 0.05 (5%) implies that the distribution is normal and the variances between residues for treatment are equivalent, validating the ANOVA model. If a factor was considered significant by ANOVA, next Tukey test of multiple comparisons was applied for grouping the factor levels. In addition, the full factorial design ($2^3 5^1$) which includes 40 different treatments (combination of the four factors levels) by ANOVA, polynomial regression models (linear, quadratic, cubic) were used attempting to establish a correlation between α_r and number of nails (or the spacing between nails). From ANOVA of the regression models, also evaluated at confidence level 5%, P-value lower 5% level of significance, the P-value lower than 0.05 implies that the adjusted polynomial coefficient is significant in the regression model and not significant otherwise.

RESULTS AND DISCUSSION

Tables from 1 to 4 show mean values ($\overline{x}$) and variation coefficients (*Cv*) of the α_r coefficient for "*I*" and "*T*" cross sections with 19 mm x 27 mm and 19 mm x 33 mm nails.

TABLE 1. Results of α_r coefficients for "I" section beams with 19 mm × 27 mm nails.

	E_{eq} weighting by the cross section area of the pieces:				
Nails N°	3	5	9	17	33
Coefficient	α_r	α_r	α_r	α_r	α_{1r}
$\overline{x}$	0.360	0.419	0.551	0.672	0.655
Cv (%)	16.15	16.54	11.55	7.10	7.26
	E_{eq} weighting by the moment of inertia of the pieces:				
Nails N°	3	5	9	17	33
Coefficient	α_r	α_r	α_r	α_r	α_{1r}
$\overline{x}$	0.384	0.418	0.580	0.689	0.683
Cv (%)	15.15	14.12	9.17	3.49	4.07

- a_r – mitigation coefficient of the moment of inertia; $E_{eq\,–\,equivalent}$ modulus of elasticity.

TABLE 2. Results of α_r coefficients for "I" section beams with 19 mm × 33 mm nails.

	E_{eq} weighting by the cross section area of the pieces:				
Nails N°	3	5	9	17	33
Coefficient	α_r	α_r	α_r	α_r	α_{1r}
$\overline{x}$	0.419	0.453	0.590	0.704	0.702
Cv (%)	7.64	7.83	7.00	3.25	4.54
	E_{eq} weighting by the moment of inertia of the pieces:				
Nails N°	3	5	9	17	33
Coefficient	α_r	α_r	α_r	α_r	α_{1r}
$\overline{x}$	0.416	0.448	0.601	0.694	0.697
Cv (%)	7.79	6.95	5.18	3.73	1.72

- a_r – mitigation coefficient of the moment of inertia; E_{eq} – equivalent modulus of elasticity.

TABLE 3. Results of α_r coefficients for "T" section beams with 19 mm × 27 mm nails.

E_{eq} weighting by the cross section area of the pieces:					
Nails N°	3	5	9	17	33
Coefficient	α_r	α_r	α_r	α_r	α_{1r}
$\overline{x}$	0.566	0.593	0.723	0.846	0.854
Cv (%)	8.25	7.75	6.59	4.40	3.29
E_{eq} weighting by the moment of inertia of the pieces:					
Nails N°	3	5	9	17	33
Coefficient	α_r	α_r	α_r	α_r	α_{1r}
$\overline{x}$	0.605	0.634	0.747	0.889	0.883
Cv (%)	5.32	4.25	4.37	1.95	2.22

- a_r – mitigation coefficient of the moment of inertia; E_{eq} – equivalent modulus of elasticity.

TABLE 4. Results of the reduction coefficients for "T" section beams with 19 mm × 33 mm nails.

E_{eq} weighting by the cross section area of the pieces:					
Nails N°	3	5	9	17	33
Coefficient	α_r	α_r	α_r	α_r	α_{1r}
$\overline{x}$	0.610	0.635	0.777	0.891	0.885
Cv (%)	5.33	4.96	3.46	4.21	3.21
E_{eq} weighting by the moment of inertia of the pieces:					
Nails N°	3	5	9	17	33
Coefficient	α_r	α_r	α_r	α_r	α_{1r}
$\overline{x}$	0.619	0.642	0.777	0.894	0.898
Cv (%)	8.21	5.91	4.06	2.83	3.87

- a_r – mitigation coefficient of the moment of inertia; E_{eq} – equivalent modulus of elasticity.

In Tables from 1 to 4, we noted progressive increases in α_r values of 3 to 17 nails, being close to the reduction coefficient values for the profiles with 17 or 33 nails, which highlights not to be necessary the use of 33 nails to obtain highest α_r values.

"*T*" profiles are less dependent on the fixing elements than "*I*" profiles, the reduction coefficient values of profiles moments in "*T*" (0.898 - Table 4) were higher than the α_r mean values originating from "*I*" beams (0.704 - Table 2).

P-values of normality and homogeneity tests of variances between ANOVA residues were 0.173 and 0.458, respectively, validating the ANOVA model. Table 5 shows ANOVA results of the factors and interactions investigated in achievement of α_r.

TABLE 5. ANOVA results for α_r coefficient.

Factors and Interactions	P-value
Prof	0.000
ND	0.106
NN	0.000
Eeq	0.089
Prof x ND	0.831
Prof x NN	0.559
Prof x Eeq	0.267
ND x NN	0.899
ND x Eeq	0.073
NN x Eeq	0.995
Prof x ND x NN	0.928
Prof x ND x Eeq	0.748
Prof x NN x Eeq	0.886
ND x NN x Eeq	0.995
Prof x ND x NN x Eeq	0.980

- geometry of the composite section (Prof); nails dimension (ND); nails number (NN); equivalent modulus of elasticity (Eeq); interactions (x) between these factors.

From ANOVA results in relation to individual factors (Table 5), profile type of the cross section and nails number were significant in obtaining α_r coefficient (P-value<005). This did not occur with the nails dimensions and the way to obtain the equivalent modulus of elasticity, which provided similar results, and in relation to the factors interactions, none were significant in α_r calculation. Table 6 shows Tukey test results of the factors considered significants by ANOVA.

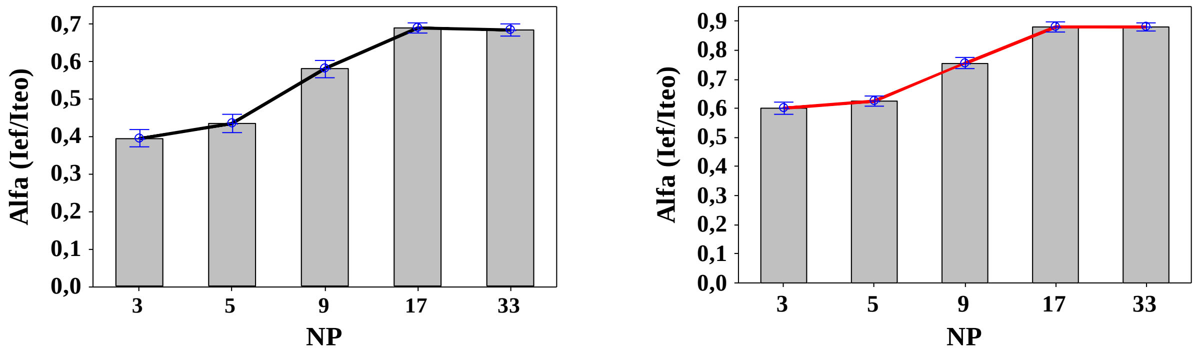
TABELA 6. Tukey test of the significant factors for α_r coefficients.

Prof	I	T	-	-	-
$\bar{x}$	0.557	0.749	-	-	-
Group	B	A	-	-	-
NN	3	5	9	17	33
$\bar{x}$	0.497	0.530	0.668	0.782	0.785
Group	C	C	B	A	A

Means followed by the same capital letters in lines do not differ from each other by Tukey test at 5% probability of error.

In Table 6, as previously discussed, "*T*" profiles show the highest reduction coefficient values of the theoretical moment of inertia, justified by using only the composition of two timber pieces with one line of nails, differently from "*I*" profile, in which three pieces and two lines of nails were used (GOES, 2002). In relation to nails number (or spacing), the highest α_r values were obtained from the compositions with 17 and 33 nails, which showed equivalent results to each other, followed by the composition of 9 and thereafter the composition of 5 and 3, showing the two last equivalent results.

Because of nails dimension and calculation form of the equivalent modulus of elasticity were not significants, α_r values were grouped in function of nails number and profile type. Figure 2 illustrates the α_r coefficient values found on nails number for each type of profile.



(a) (b)

FIGURE 2. Variation of α_r coefficients on nails number for beams profile "I" (a) and "T" (b).

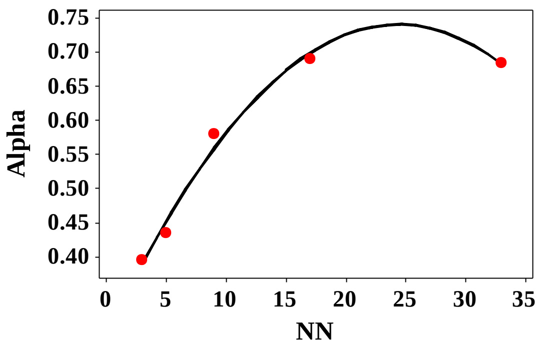
In Figure 2, we noted the α_r behaviour is the same for the two types of profiles, as expected. Values of α_r for "*I*" beams (0.70 – 17 or 33 nails) and "*T*" beams (0.88 – 17 or 33 nails) were both lower than those established by ABNT NBR 7190:1997 Brazilian Code (0.95 for two juxtaposed rectangular elements, 0.85 for three juxtaposed rectangular elements).

With the mean values of α_r Figure 2 by profile type and by the nails number, linear, quadratic and cubic polynomial regression models were used; enabling to relate α_r with nails number (NN) used in profile composition. Table 7 shows the results of regression models for the two profile types.

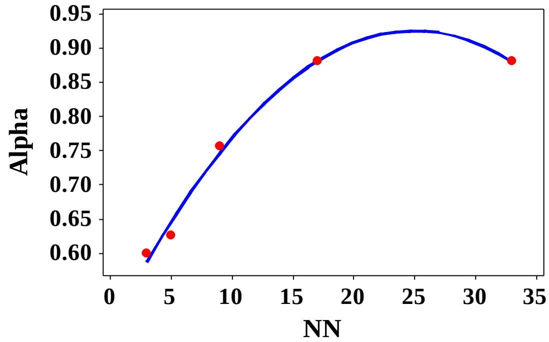
TABLE 7. Results of the regression models [α_r =f (NN)].

Adjustment	Equation	R^2 (adjust.)
	"*I*" profile	
Linear	$\alpha_r = 0.4310 + 0.009386 \cdot NN$	59%
Quadratic	$\alpha_r = 0.2846 + 0.03743 \cdot NN +$ $-0.000768 \cdot NN^2$	97.70%
Cubic	$\alpha_r = -0.2556 + 0.04706 \cdot NN +$ $-0.001495 \cdot NN^2 + 0.000014 \cdot NN^3$	96.30%
	"*T*" profile	
Linear	$\alpha_r = 0.6226 + 0.009393 \cdot NP$	64.30%
Quadratic	$\alpha_r = 0.4893 + 0.03493 \cdot NN -$ $-0.000700 \cdot NN^2$	98.20%
Cubic	$\alpha_r = 0.4974 + 0.03226 \cdot NN -$ $-0.000499 \cdot NN^2 - 0.000004 \cdot NN^3$	96.40%

In Table 7, for each type of cross-sectional profile, ANOVA results of the regression models showed the insignificance of the third-degree polynomials (P-value>0.05). Considering the values of the adjusted determination coefficient (R^2 adjust) for the other two adjustments admitted significant, second-degree polynomials showed as the best models in the study of the relation between α_r and NN for both types of profile. Figure 3 shows the curves obtained from the adjustments using the second-degree polynomial (Table 7) for the two investigated profiles.



(a)



(b)

FIGURE 3. Quadratic functions [$\alpha_r = f(NN)$] obtained for the "*I*"(a) and "*T*"(b) profiles.

In Figures 3a and 3b, we noted a possibility of existing a maximum point to α_r values. For "*I*" profile, the nails number that grants the highest α_r value (0.74) is 24.37 (25 nails), and for "*T*" profile, the nails number for α_r (0.93) is 24.95 (25 nails). For both profiles, the regression models indicate that 25 nails would be the ideal amount for the composition. So it is possible to achieve α_r value by NBR 7190: 1997 only for the "*T*" profile.

We noted that the use of the quadratic model is justified by the fact that α_r values do not grow indefinitely for a large and growing number of connectors (KIDWELL, 1987), because subsequent fixing of neighbouring nails may generate cracks in the nail piercing region in the wood, making less efficient the connection between pieces.

CONCLUSIONS

From the results of this research the following can be concluded:

- The α_r coefficient for *Pinus* sp. beams for both profiles showed successive increases in the use of 3 up to 17 nails, and equivalent results from 17 for 33 nails;

- From the factors investigated, only nails' number and type of the composite section profile were significant in obtaining the reduction coefficient;

- For composition with 17 and 33 nails, mean values of α_r for both profiles were lower than the values established by the Brazilian Code ABNT NBR 7190: 1997;

- The amount of 25 nails, equally spaced, can be accepted as providing the greatest value of α_r for both tested beams profiles;

- The α_r value (0.95) recommended by ABNT NBR 7190:1997 for "*T*" section beams can be considered compatible, the same does not apply to the "*I*" section beams since, as it turned out, the value of α_r (0.85) recommended by the Brazilian Code overestimates efficiency of the nailed connection. For this case, α_r should assume, at most, the 0.75 value.

REFERENCES

ABNT - ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. **ABNT NBR 7190**. Projeto de estruturas de madeira. Rio de Janeiro, 1997.

CALIL JR, C.; LAHR, F. A. R.; DIAS, A. A. **Dimensionamento de elementos estruturais de madeira**. Manole, 2003. 153p.

CHRISTOFORO, A. L.; ICIMOTO, F. H.; FERNANDES, C. V.; FIORELLI, J.; NEGRAO, J.; ROCCO, F. A. L. Shear and Longitudinal Modulus of Elasticity in Structural Profiled Round Timber Beams. **International Journal of Composite Materials**, Rosemead, v. 3, p. 87-91, 2013.

CHRISTOFORO, A. L.; MOLINA, J. C.; PANZERA, T. H.; ALMEIDA, D. H.; RIBEIRO FILHO, S. L. M.; SCALIANTE, R. M.; ROCCO LAHR, F. A. Módulo de elasticidade em vigas de madeira de dimensões estruturais pelo Método dos Mínimos Quadrados. **Revista Árvore**, Viçosa, MG, v. 37, p. 981-988, 2013.

CHRISTOFORO, A. L.; PANZERA, T. H.; SILVA, D. A. L.; FIORELLI, J.; ROCCO, F. A. L. Shear and Longitudinal Modulus of Elasticity in Structural Lumber Beams. **International Journal of Materials Engineering**, Rosemead, v. 4, p. 31-36, 2014.

COMITÉ EUROPÉEN DE NORMALISATION - **Eurocode 5**: design of timber structures. Brussels, 1993.

FAN, M. Performance of edgewise loaded wood based panels and their I-beam components. **Construction and Building Materials**, Washington, v. 30, p. 447-454, 2012.

FIORELLI, J.; SORIANO, J.; LAHR, F. A. R. Roof modular system in wood and particleboard (OSB) to rural construction. **Scientia Agrícola**, Piracicaba, v. 69, p. 189-193, 2012.

GÓES, J. L. N. **Análise de vigas de madeira pregadas com seção composta I**. São Carlos. 151f. Dissertação (Mestrado) – Escola de Engenharia de São Carlos, Universidade de São Paulo, São Carlos, 2002.

GÓES, J. L. N.; DIAS, A. A. Análise de vigas de madeira pregadas com seção composta I. **Cadernos de Engenharia de Estruturas**, São Carlos, v. 7, p. 57-77, 2005.

GONÇALVES, D.; ROCCO LAHR, F. A.; CALIL Jr, C.; DEMARZO, M. A.; CHRISTOFORO, A. L. Tree-shaped timber structural system. **Journal of Civil Engineering and Architecture**, New York, v. 7, p. 796-804, 2013.

KIDWELL, E. The efficiency of built-up wooden beams. **American Institute of Mining Engineers**, Pennsylvania, v.27, p.732-818, 1897.

MOURA, M. F. .F.; MORAIS, J. J. L.; DOURADO, N. New data reduction scheme for mode I wood fracture characterization using the double cantilever beam test. **Engineering Fracture Mechanics**, New York, v. 75, p. 3852-3865, 2008.

ROCCO LAHR, F. A. **Sobre a determinação de propriedades de elasticidade da madeira**. 1983. 216 f. Tese (Doutorado). Escola de Engenharia de São Carlos, Universidade de São Paulo, São Carlos, 1983.