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ARTIFICIAL INTELLIGENCE TECHNIQUES APPLIED TO THE OPTIMIZATION OF MICRO-IRRIGATION SYSTEMS BY THE ZIMMERMANN-WERNER METHOD

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KEYWORDS

drip irrigation design, fuzzy linear optimization, flexibility restrictions, decrease in annual costs.

ABSTRACT

Optimal solutions derived from linear programming models depend entirely on input parameters, which may present some imprecision because they come from estimates. Fuzzy linear programming allows the incorporation of these uncertainties in linear models, which can include the flexibility of resources, costs, goals, and constraints. This paper aimed to show new optimal solutions for a model to minimize the equivalent annual cost of micro-irrigation systems on sloping terrains. The Zimmermann-Werner fuzzy linear programming method, whose objective function is diffuse due to the restrictions of the hydraulic network being dispersed, was used. Sixty models were created and all solutions were satisfactory, with an annual cost of the irrigation system lower than the original model. The lowest value was US\$ 238.74 ha⁻¹, which occurred on the 3% slope. A reduction was observed in the annual cost due to the increased use of pipes with a 50-mm nominal diameter in the secondary line. Thus, fuzzy linear programming provided better solutions with small modifications to the irrigation system, while maintaining all hydraulic network requirements for proper system operation.

INTRODUCTION

Water supply via irrigation aims to meet the water needs of crops in locations where the water depth provided by natural precipitation is not sufficient or has an irregular distribution (Jerónimo et al., 2015; Vitti et al., 2020). Water application in drip irrigation system is conducted at a high frequency and low volume, maintaining a high degree of moisture in a small soil volume, which has the plant root system (Arraes et al., 2019). Among all irrigation systems, trickle irrigation stands out due to its energy-saving aspect, the possibility of automation and fertigation, straightforward operation, water emission uniformity, and water preservation (Pereira et al., 2019).

The investment in a micro-sprinkler irrigation system has a high initial cost. It is essential to analyze the system components and costs to determine their economic viability (Oliveira et al., 2016). The optimization of the hydraulic network sizing is an important initial cost reduction option due to the number of variables to be

considered and the various possibilities of combinations, such as pipe diameter, discharge, and material costs (Mala-Jetmarova et al., 2018).

Several studies have reported the use of classical optimization techniques in choosing the most feasible and least costly sizing, such as linear programming (Galván-Cano & Exebio-García, 2020; Soler et al., 2016). This method presents an optimal solution that depends entirely on the input parameters, which are fixed, but resource availability constraints present some inaccuracies under actual situations (Zhang & Guo, 2018a).

Several fuzzy systems have been successfully applied in agricultural engineering, such as poultry production and industry companies (Pereira et al., 2019; Cremasco et al., 2010), cattle production (Gabriel Filho et al., 2011, 2016; Maziero et al., 2022), irrigation engineering (Viais Neto et al., 2019a, 2019b; Putti et al., 2017a, 2021, 2022; Boso et al., 2021a, 2021b; Matulovic et al., 2021; Gabriel Filho et al., 2022a, 2022b), optimization of

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agricultural implements (Góes et al., 2021), increased plant vitality (Putti et al., 2014, 2017b), and market of agricultural products (Gabriel Filho et al., 2015; Martínez et al., 2020).

Water demand amounts or pipe roughness coefficients can change over the water distribution network life span. Therefore, it cannot be constant, but crisp (Spiliotis & Tsakiris, 2012). Fuzzy linear programming is effective in keeping good control of the accuracy of the results and expressing the data with flexibility in the application.

Several applications of fuzzy linear programming methods can be found in agricultural engineering, especially land use planning in agricultural systems (Biswas & Pal, 2005), crop planning in agricultural management (Itoh et al., 2003), sustainable allocation of water for irrigation (Li et al., 2017), generation of water-allocation strategies to agricultural irrigation systems (Lu et al., 2011), water resource management (Lu et al., 2010), risk analysis for supporting sustainable watershed development (Tan et al. 2016), agricultural economic management (Wang, 2022), optimization of agricultural planting structure (Yang et al., 2020), crop area planning (Zeng et al., 2010), and agricultural water management considering ecological water requirement (Zhang & Guo, 2018b).

The optimization of a micro-sprinkler irrigation system hydraulic network involves hydraulic calculations and selection of pipe diameters so that the total cost is minimal and the network works properly (Baiamonte, 2018).

Revelli & Ridolfi (2002) used this technique to analyze the hydraulic behavior of hydraulic pipe networks in imprecise parameters, such as the diameter and roughness

coefficient of old pipes. Conversely, Bhave & Gupta (2004) proposed a fuzzy approach to deal with uncertainty in nodal water demands aiming for the optimization of water distribution networks.

Kanakakis et al. (2014) used fuzzy linear programming for the analysis and performance of possible future changes by users, increasing the emitter pressure load of a pressurized irrigation network on demand, but also minimizing the minimum cost of the piping. This study showed that the method presents flexibility in the future adaptations required for the higher-pressure head at the hydrants, working even better than linear programming, mainly in cases in which the pressures of hydrants were not previously selected at the design stage.

Fuzzy linear programming is a tool to be considered in the treatment of uncertainties. Thus, this study aimed to determine and evaluate by the Zimmermann-Werner method the performance of a range of new solutions for the reformulated model, minimizing the equivalent annual cost of the micro-irrigation system, imposing objective function and diffuse constraints.

MATERIAL AND METHODS

The micro-irrigation system design proposed by Saad & Mariño (2002) assumes that the irrigated area is rectangular and has a uniform slope, with leveled lateral and submain lines. For this design, downhill manifolds (feeder pipes) must have a uniform slope and the mainline must be uphill (Figure 1).

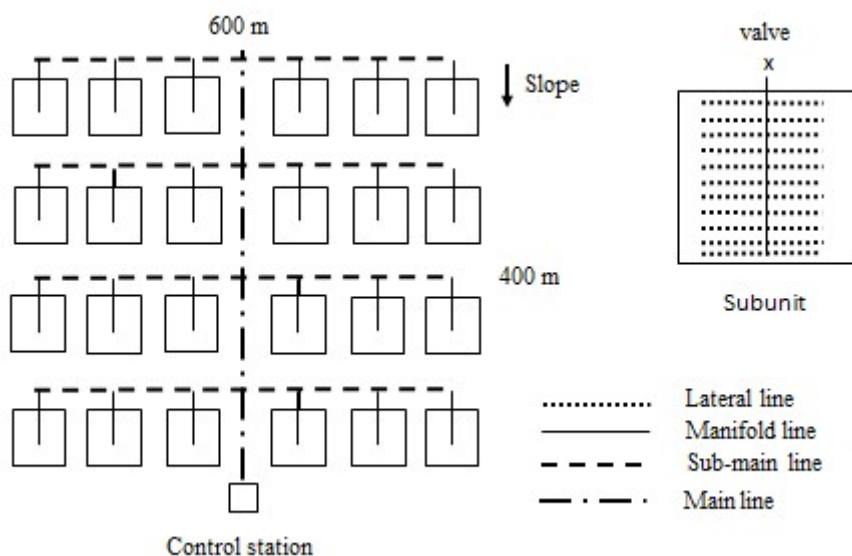


FIGURE 1. Irrigation system layout with 24 operating units. Source: Saad e Mariño (2002).

Polyvinyl chloride (PVC) is used as the material of manifolds, submain lines, and mainlines, as they present multiple diameters. Lateral lines are made out of polyethylene, with a single diameter. Also, subunits, composed of a valve, manifolds, lateral lines, and micro-sprinklers, have the same dimension, hydraulic network, and outlet pressure (Waller & Yitayew, 2016; Bernardo et

al., 2019). This study focused only on negative slopes of 3, 6, and 9% because they allow for different pressure profiles along the manifold line according to the slope gradient (Silva Junior & Saad, 2021).

The objective function to be minimized is the equivalent annual cost of the micro-irrigation system, in U.S. dollars, and is presented as:

$$\text{Minimize } C = \{N_s[N_l(N_m P_m + L P_{pe}) + P_v] + C_{cp} + C_f + 1605.08\}CRF + N_s CRF \sum_{j=1}^J \sum_{i=1}^I P M_i L M_{j,i} + 2CRF \sum_{k=1}^K \sum_{r=1}^R P S_r L S_{k,r} + CRF \sum_{k=1}^K \sum_{v=1}^V P N_v L N_{k,v} + \left(7.764CRF + \frac{10.787 Q_s N_s I_d I_h E}{I_f \eta}\right) H_T$$

in which:

C is the equivalent annual cost of the irrigation system (US\$);

N_s is the number of subunits;

N_l is the number of lateral lines per subunit;

N_m is the number of micro-sprinklers per lateral line;

P_m is the micro-sprinkler price (US\$/unit);

L is the lateral line length (m);

P_{pe} is the polyethylene pipe price (US\$/m);

P_v is the valve price (US\$/unit);

C_{cp} is the control panel cost (US\$);

C_f is the filter system cost (US\$);

CRF is the capital recovery factor;

$P M_i$ is the price of a PVC pipe with the diameter i used in the manifold line (US\$/m);

$L M_{j,i}$ is the length (m) of the PVC pipeline with diameter i used in section j of the manifold line;

$P S_r$ is the price of the PVC pipe with diameter r used in the submain line (US\$/m);

$L S_{k,r}$ is the length (m) of the PVC pipeline with diameter r used in submain line k ;

$P N_v$ is the price of the PVC pipe with diameter v used in the mainline (US\$/m);

$L N_{k,v}$ is the length of the PVC pipeline with diameter v used in section k of the mainline;

Q_s is the subunit discharge (m³/s);

I_d is the number of irrigation days during the season;

I_h is the number of irrigation hours per set of subunits working simultaneously;

E is the electricity price (US\$/kWh);

I_f is the irrigation frequency (days);

η is the pump efficiency, and

H_T is the total operating head (m).

The decision variables are $L M_{j,i}$, $L S_{k,r}$, $L N_{k,v}$, and H_T . The nominal diameters were ND35 and ND50 for manifold lines and ND50, ND75, and ND100 for submain and main lines at the nominal pressure PC40 or PC80.

The portions $N_s N_l N_m P_m$, $N_s N_l L P_{pe}$, $N_s P_v$, $7.764 H_T + 1605.8$ for $50 \text{ m} \leq H_T \leq 80 \text{ m}$, $N_s \sum_{j=1}^J \sum_{i=1}^I P M_i L M_{j,i} + 2 \sum_{k=1}^K \sum_{r=1}^R P S_r L S_{k,r} + \sum_{k=1}^K \sum_{v=1}^V P N_v L N_{k,v}$, and $(10.787 Q_s N_s I_d I_h E H_T) /$

$(I_f \eta)$, in the objective function represent, respectively, the total micro-sprinkler cost (US\$), polyethylene pipeline cost (US\$), valve cost (US\$), pump system cost (C_p , US\$), polyvinyl chloride (PVC) pipeline cost (C_{PVC} , US\$), and the annual pumping cost (C_{pp} , US\$).

Constraints (1) to (4) guarantee that the dimensions of the irrigation system are met:

$$\sum_{i=1}^I L M_{1,i} = 0.5 S_L \quad (1)$$

$$\sum_{i=1}^I L M_{j,i} = 0.5 S_L, j = 2, \dots, J \quad (2)$$

$$\sum_{r=1}^R L S_{k,r} = S_k, k = 1, \dots, K \quad (3)$$

$$\sum_{v=1}^V L N_{k,v} = N_k, k = 1, \dots, K \quad (4)$$

in which:

S_L is the spacing between lateral lines (m);

S_k is the length of submain line k (m), and

N_k is the length of section k (m).

Constraints (5) to (8) design all the possible pressure profiles for manifolds that can occur under the downhill condition, as detailed in Wu (1986). These constraints assure that the difference between the maximum and minimum load is lower than or equal to the maximum variation of pressure head allowed in the manifold line, regardless of where the points are located.

$$1.05 \sum_{i=1}^l \sum_{q=1}^j JM_{q,i} LM_{q,i} \leq DM + (j - 1 + 0.5)S_L dz, j = 1, \dots, J \quad (5)$$

$$-1.05 \sum_{i=1}^l \sum_{q=1}^j JM_{q,i} LM_{q,i} \leq DM - (j - 1 + 0.5)S_L dz, j = 1, \dots, J \quad (6)$$

$$1.05 \sum_{i=1}^l \sum_{q=g+1}^l JM_{q,i} LM_{q,i} \leq DM - (g - l)S_L dz, g, l = 1, \dots, J \text{ and } g < l \quad (7)$$

$$-1.05 \sum_{i=1}^l \sum_{q=g+1}^l JM_{q,i} LM_{q,i} \leq DM + (g - l)S_L dz, g, l = 1, \dots, J \text{ and } g < l \quad (8)$$

in which:

DM is the allowed maximum value of pressure difference in the manifold line (m). The DM value in eqs (5) to (8) is data calculated by adopting the design criteria of the desired emission uniformity (EU) in the subunit (Silva & Junior & Saad, 2021).

The determination of the average pressure in the manifold line (H_{av} , mca) is accomplished by the constraint:

$$\begin{aligned} & (-1.05J + 0.6615J) \sum_{i=1}^l JM_{1,i} LM_{1,i} + [-1.05(J - 1) + 0.6615J] \sum_{i=1}^l JM_{2,i} LM_{2,i} + \dots + \\ & + [2(-1.05) + 0.6615J] \sum_{i=1}^l JM_{J-1,i} LM_{J-1,i} + (-1.05 + 0.6615J) \sum_{i=1}^l JM_{J,i} LM_{J,i} + \\ & + \frac{N_l}{2} H_{av} = J[h_w + 0.75hf_l + 0.5dz(JS_L - M)] \end{aligned} \quad (9)$$

in which:

$JM_{j,i}$ is the head loss gradient (m/m) in the PVC pipe with diameter i used in section j of the manifold line;

h_w is the micro-sprinkler working pressure (mca);

hf_l is the pressure head loss in the lateral line (mca);

dz is the slope gradient (m/m), and

M is the manifold line length (m).

The total operating head (H_T) is determined so that it meets the subunit that operates under the most critical condition. Hence, for constraint (10), H_T is equal to the sum of H_{uo} with the load losses in the last section of the submain and mainlines, the control station (H_{es}), suction lift (H_{su}), and the difference between the most distant subunit level.

$$\begin{aligned} & 0.6615 \sum_{j=1}^J \sum_{i=1}^l JM_{j,i} LM_{j,i} + 1.05 \sum_{k=1}^K \sum_{v=1}^V JN_{k,v} LN_{k,v} + 1.05 \sum_{r=1}^R JS_{K,r} LS_{K,r} - H_T = \\ & = -[h_w + 0.75hf_l + H_v + H_{es} + H_{su} + (KN - 0.5M)dz] \end{aligned} \quad (10)$$

in which:

$JN_{k,v}$ is the head loss gradient (m/m) in the pipe with diameter v used in section k of the mainline;

$JS_{K,r}$ is the head loss gradient (m/m) in the PVC pipe with diameter r used in the submain K ;

H_v is the head loss in the valve (mca);

H_{es} is the head loss in the control station (mca);

H_{su} is the suction lift (mca);

K is the total number of sections in the mainline or the total number of submain lines, and

N is the mainline length (m).

The submain line is designed considering the available pressure in the outlet of the mainline and the pressure requested in the inlet of the subunit:

$$0.6615 \sum_{j=1}^J \sum_{i=1}^I JM_{j,i}LM_{j,i} + 1.05 \sum_{t=1}^K \sum_{v=1}^V JN_{t,v}LN_{t,v} + 1.05 \sum_{r=1}^R JS_{k,r}LS_{k,r} - H_T \leq -[H_{es} + H_{su} + 0.75hf_l + h_w + H_v + (kN - 0.5M)dz], k = 1, \dots, K \tag{11}$$

in which:

$JS_{q,r}$ is the head loss gradient (m/m) in the PVC pipe with diameter r used in the submain lines $1, \dots, K - 1$.

The no negativity of the decision variables is guaranteed in the following constraint. H_T is inserted in a lower (50 mca) and upper (80 mca) limiting, which is a specific condition for determining the pump cost through a regression as a function of the product between the total operating head and the discharge (Saad & Mariño, 2002):

$$LM_{j,i} \geq 0, LS_{k,r} \geq 0, LN_{k,v} \geq 0, H_{av} \geq 0 \text{ and } 50 \text{ m} \leq H_T \leq 80 \text{ m.}$$

The hydraulic network layout and the operation conditions are required to be previously defined for using the presented model. The input parameters of a micro-irrigation system that irrigates a citrus orchard located in

Limeira, São Paulo, Brazil, were used. The orchard has an area of 600×400 m and a uniform slope in the direction of the shortest length. Table 1 shows the solution found for input dates present in Saad & Mariño (2002).

TABLE 1. Output parameters of the model in function of the three slopes.

Item	Slope (%)		
	3	6	9
Manifold line	66.5 m ND50 28 m ND35	66.5 m ND50 28 m ND35	41.3 m ND50 53.2 m ND35
Submain lines			
Section 1	172.15 m ND75 77.85 m ND50	86.6 m ND75 163.4 m ND50	1.1 m ND75 248.9 m ND50
Section 2	197.8 m ND75 52.2 m ND50	131.8 m ND75 118.2 m ND50	65.8 m ND75 184.2 m ND50
Section 3	243.7 m ND75 6.3 m ND50	210.7 m ND75 39.3 m ND50	177.7 m ND75 72.3 m ND50
Section 4	250 m ND100	250 m ND100	250 m ND100
Main line			
Section 1	98 m ND100 (PC80)	98 m ND100 (PC80)	98 m ND100 (PC80)
Section 2	98 m ND100 (PC80)	98 m ND100 (PC80)	98 m ND100 (PC80)
Section 3	98 m ND100 (PC40)	98 m ND100 (PC40)	98 m ND100 (PC40)
Section 4	98 m ND100 (PC40)	98 m ND100 (PC40)	98 m ND100 (PC40)
H_T	50.6 m	60.9 m	72.6 m
H_{av}	17.1 m	17.2 m	17.6 m
Annual cost (US\$/ha)	240.43	251.52	263.54

Note: DN = nominal diameter and PC = pressure class.

Zimmermann-Werner method

Optimum solutions from a linear programming model depend entirely on input parameters, which correspond to the values of the technological matrix, demand vectors, and costs (Camargo, 2018). It can be seen in its formulation (Model 1), in vector notation, which aims to maximize objective functions $f(x)$, respecting the constraints $Ax \leq b$ and $x \geq 0$.

$$\begin{aligned} \text{Model 1: Maximize } f(x) &= c^T x \\ \text{Subject to} & \\ & Ax \leq b \\ & x \geq 0 \end{aligned}$$

in which:

$$A \in \mathbb{R}^{m \times n}; c, x \in \mathbb{R}^n; b \in \mathbb{R}^m.$$

The values A , b , and c in most optimization models of real problems are not known exactly, as they come from

estimates and projections and have a certain variability.

Assuming that the optimum solution of Model 1 is z_0 and that the objective function can adopt an aspiration level z_1 higher than z_0 as long as the constraints suffer minor violations in a way that guarantees the feasibility of z_1 .

According to Zimmermann (1996), z_1 is denominated as the goal of the objective function and determined by:

$$\begin{aligned} \text{Model 2: Maximize } c^T x & \\ \text{Subject to} & \\ & Ax \leq b + t \\ & x \geq 0 \end{aligned}$$

in which:

$t \in \mathbb{R}^m$ is the vector with the aspiration levels that each constraint can suffer and must be imposed by the specialist. They are named tolerance constraints $Ax \leq b$.

The following fuzzy linear programming problem will be solved based on these hypotheses:

$$\begin{aligned} \text{Model 3: Determine } x \\ \text{Subject to} \\ c^T x \gtrsim z_1 \\ Ax \gtrsim b \\ x \geq 0 \end{aligned}$$

$$\mu_1(c^T x) = \begin{cases} 1 & z_1 < c^T x \\ \frac{c^T x - z_0}{t_1} & z_0 \leq c^T x \leq z_1 \\ 0 & c^T x < z_0 \end{cases}$$

where $t_1 = z_1 - z_0$ and

$$\mu_j(a_k^T x) = \begin{cases} 1 & a_k^T x < b_k \\ \frac{b_k + t_j - a_k^T x}{t_j} & b_k \leq a_k^T x \leq b_k + t_j \\ 0 & b_k + t_j < a_k^T x \end{cases}$$

for $k = 1, \dots, m$ and $j = 2, \dots, m + 1$.

We interpreted that the i -th constraints (including the objective function) are strongly violated in the case of $\mu_i(x) = 0$. Moreover, the i -th constraint is well satisfied if $\mu_i(x) = 1$ and the i -th constraint is within an acceptable range of violation if $\mu_i(x)$ is monotone increasing (or decreasing) in the range]1,0[(Zimmermann, 1996).

Multiplying the constraint equivalent to the objective function in Model 3 by -1 in both members, we get $-c^T x \lesssim -z_1$, assuming $B = \begin{pmatrix} -c^T \\ A \end{pmatrix}$ and $d = \begin{pmatrix} -z_1 \\ b \end{pmatrix}$, with $B \in \mathbb{R}^{(m+1) \times n}$ and $d \in \mathbb{R}^{m+1}$, and Model 3 becomes:

$$\begin{aligned} \text{Model 4: Determine } x \\ \text{Subject to} \\ Bx \lesssim d \\ x \geq 0 \end{aligned}$$

Definition (Bellman & Zadeh, 1970): a diffuse objective \tilde{G} and a diffuse constraint \tilde{C} are set in a space of X . Then, \tilde{G} and \tilde{C} combined form a decision \tilde{D} , which is a fuzzy set, resulting from the intersection of \tilde{G} and \tilde{C} , that is, $\tilde{D} = \tilde{G} \cap \tilde{C}$ and, correspondingly, $\mu_{\tilde{D}} = \min\{\mu_{\tilde{G}}, \mu_{\tilde{C}}\}$. The resulting decision with n objectives $\tilde{G}_1, \tilde{G}_2, \dots, \tilde{G}_n$ and m constraints $\tilde{C}_1, \tilde{C}_2, \dots, \tilde{C}_m$ is the intersection of objectives and restrictions, that is:

$$\tilde{D} = \tilde{G}_1 \cap \tilde{G}_2 \cap \dots \cap \tilde{G}_n \cap \tilde{C}_1 \cap \tilde{C}_2 \cap \dots \cap \tilde{C}_m$$

Hence,

$$\mu_{\tilde{D}} = \min\{\mu_{\tilde{G}_1}, \mu_{\tilde{G}_2}, \dots, \mu_{\tilde{G}_n}, \mu_{\tilde{C}_1}, \mu_{\tilde{C}_2}, \dots, \mu_{\tilde{C}_m}\}$$

According to the previous definition, the pertinence function of the fuzzy decision set in Model 4 is:

$$\mu_{\tilde{D}} = \min_i \{\mu_i(x)\}.$$

The maximizing solution for the fuzzy decision set will be:

$$\max_{x \geq 0} \mu_{\tilde{D}}(x) = \max_{x \geq 0} \min_i \{\mu_i(x)\} \quad (12)$$

that is, the one that will have the highest pertinence function.

The symbol \gtrsim (\lesssim) represents the flexibility in restrictions \leq (\geq).

Importantly, the objective function in the model above has been transformed into a new constraint. Each of the $(m+1)$ lines of Model 3 are represented by a fuzzy set, with a membership function $\mu_i(x)$, where $i = 1, \dots, m + 1$, defined, according to Zimmermann (1996), as:

Taking $\lambda = \mu_{\tilde{D}}(x)$, Model 5 has the vector (λ_0, x_0) as the solution. According to Zimmermann (1996), x_0 is the solution to Problem (12) and λ_0 is the degree of risk associated with carrying out the violations on Model 4.

$$\begin{aligned} \text{Model 5: Maximize } \lambda \\ \text{Subject to} \\ \lambda t_i + B_i x \leq d_i + t_i, i = 1, \dots, m + 1 \\ x \geq 0, \lambda \in [0,1] \end{aligned}$$

The solution of Model 5 for $\theta = 1 - \lambda$ is equivalent to that found in Model 6.

$$\begin{aligned} \text{Model 6: Minimize } \theta \\ \text{Subject to} \\ c^T x \geq z_1 + \theta t_1 \\ (Ax)_k \leq b_k + \theta t_j, k = 1, \dots, m, j = 2, \dots, m + 1 \\ x \geq 0, \theta \in [0,1] \end{aligned}$$

There is $(Ax)_k \geq b_k - \theta t_k$ in the case of restrictions of the type $(Ax)_k \gtrsim b_k$ for any k .

Each fuzzy linear programming model was solved by the algorithms developed in the Matrix Laboratory (MatLab) 7.0.lnk computational program, version 7.0.0.19920, with the solve LinProg and Simplex Dual Method.

RESULTS AND DISCUSSION

Importantly, the objective function in the model is minimized and the constraints are composed of linear inequalities with a lower-than-or-equal-to sign (\leq) so that Model 6 is formulated as follows:

$$\begin{aligned} \text{Model 7: Minimize } \theta \\ \text{Subject to} \\ c^T x \leq z_1 + \theta t_1 \\ (Ax)_k \leq b_k + \theta t_j, k = 1, \dots, m, j = 2, \dots, m + 1 \\ x \geq 0, \theta \in [0,1] \end{aligned}$$

The purpose of the objective function (z_1) is determined by the Model 8 solution.

$$\begin{aligned} &\text{Model 8: Minimize } c^T x \\ &\text{Subject to} \\ &c^T x \leq b + t \\ &x \geq 0 \end{aligned}$$

The tolerance of the objective function is obtained through z_1 by $t_1 = z_0 - z_1$.

Fuzzy tolerances of constraints are determined by $t_j = p_w b_k$ for $x \geq 0$, w fixed and $k = 1, \dots, m$, where $p_w = w(10^{-2})$, with $w = 1, \dots, 30$ and $w \in \mathbb{Z}$, the percentage in decimal. For example, $p_{10} = 0.1$ when fixing $w = 10$; and the tolerances of restrictions are given by $t_j = 0.1b_k$ for $x \geq 0$ and $k = 1, \dots, m$.

Notice that the tolerances occur only in restrictions with inequalities. Thus, Restrictions 5 to 8 and 11 have tolerance in the model. In other words, the required pressure heads in the flexible approach are raised by the value t_j , which is the fuzzy tolerance.

The application of the Zimmermann-Werner method occurs at levels of 3, 6, and 9% slope. Therefore, 30 models of linear fuzzy programming are built when fixing a slope, that is, 30 models with the Model 7 formulation.

For the 3% slope, θ equaled 0.5 for the all the considered range w , while for the 6% slope, $\theta = 0.5$ for the range $[1, \dots, 25]$, $\theta = 0.49$ for $[26, \dots, 28]$, $\theta = 0.48$ for $w = 29$, and $\theta = 0.48$ for $w = 30$. It means that the value of the annual cost of the irrigation system is intermediate to z_1 and z_0 in the solution of all linear fuzzy programming models for both slopes.

The lengths (according to their respective diameters) of the submain lines 1, 2, and 3 on these slopes were the

only variables that changed with the application of the Zimmermann-Werner method. The results for the mainline, submain line 4, and manifold lines were the same as shown in Table 1, as observed for H_T and H_{av} because they do not depend on the length of the modified lines.

The 3% slope (Figure 2) showed that the submain line section 1 has a pipe length with a nominal diameter of 50 mm (CDN50), less than the pipe length with a nominal diameter of 75 mm (CDN75), up to the percentage 0.27 ($p_{27} = 0.27$), in which $CDN50 = \$124.74$ m and $CDN75 = \$125.26$ m. CDN50 becomes higher than CDN75 from this point on.

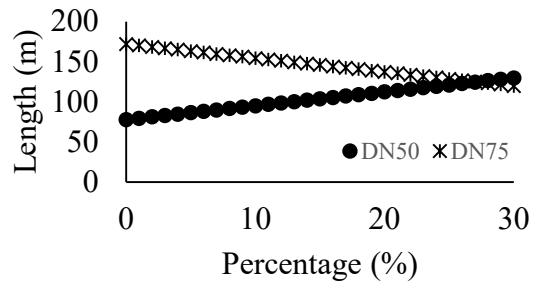
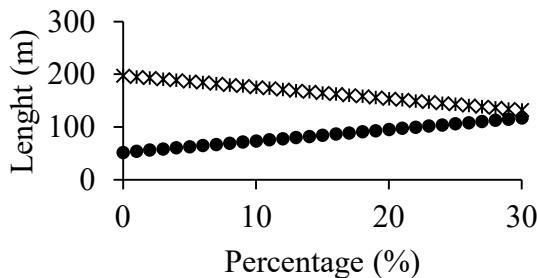
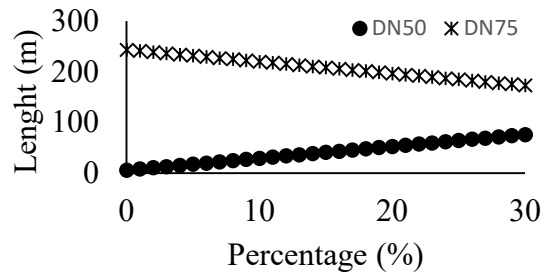


FIGURE 2. Length (m) of the submain line 1, nominal diameter 50 mm (ND50) and 75 mm (ND75), for 3% slope, as function of the tolerance percentage (%).

Figure 3(a) section 2 shows that $CDN50$ is lower than $CDN75$ in the entire considered percentage range. The same is true for section 3, as shown in Figure 3(b).



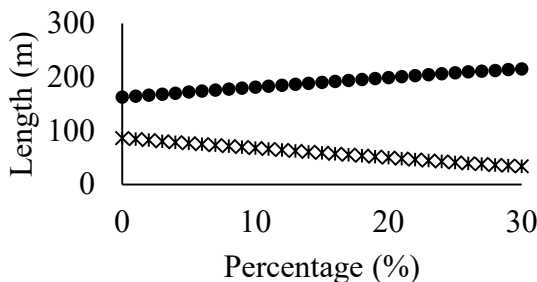
(a)



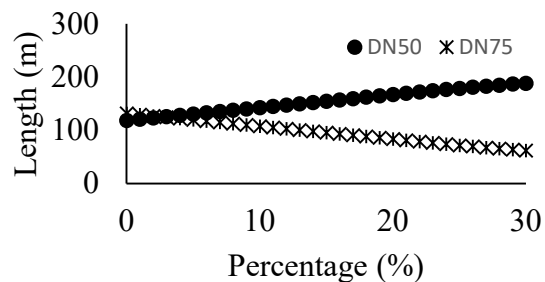
(b)

FIGURE 3. Length (m) of the (a) submain line 2, and (b) submain line 3, nominal diameter 50 mm (ND50) and 75 mm (ND75), for the 3% slope, as function of the tolerance percentage (%).

The 6% slope (Figure 4(a)) shows that $CDN50$ is higher than $CDN75$ in the submain line section 1 in all percentage ranges. Moreover, Figure 4(b) section 2 shows that $CDN50$ is lower than $CDN75$ only for $p_1 = 0.01$, $p_2 = 0.02$, and $p_3 = 0.03$, but the opposite occurs from this point on.



(a)



(b)

FIGURE 4. Length (m) of the (a) submain line 1, and (b) submain line 2, nominal diameter 50 mm (ND50) and 75 mm (ND75), for the 6% slope, as function of the tolerance percentage (%).

Finally, Figure 5 section 3 shows that CDN50 is higher than CDN75 in the entire percentage range.

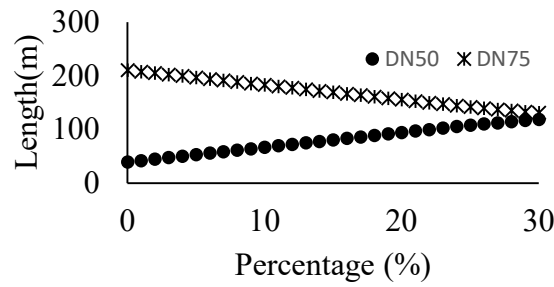


FIGURE 5. Length (m) of the submain line 3, nominal diameter 50 mm (ND50) and 75 mm (ND75), for the 6% slope, as function of the tolerance percentage (%).

Figures 2 to 5 allow concluding that CDN50 is increasing in every percentage range in the submain lines 1, 2, and 3 for the 3 and 6% slopes. In contrast, the CDN75 behavior is decreasing. Thus, the lowest PVC pipeline cost (C_{PVC}) is observed with an increase in CDN50 and a

decrease in CDN75 under these sections because the PVC pipe price is lower for the nominal diameter of 50 mm.

Tables 2 and 3 show the CDN50 and CDN75 values for $p_1 = 0.01$ and $p_{30} = 0.30$ due to the submain line on the 3 and 6% slopes, respectively.

TABLE 2. Length (m) of pipes with nominal diameter of 50 mm and 75 mm, for the tolerances percentage of 1% and 30%, as function of the submain lines, for the 3% slope.

Submain	Nominal diameter (mm)	Length (m)	
		1%	30%
1	50	79.58	129.95
	75	170.41	120.05
2	50	54.37	117.46
	75	195.63	132.54
3	50	8.66	76.543
	75	241.34	173.46

TABLE 3. Length (m) of pipes with nominal diameter of 50 mm and 75 mm, for the tolerances percentage of 1% and 30%, as function of the submain lines, for the 6% slope.

Submain	Nominal diameter (mm)	Length (m)	
		1%	30%
1	50	165.17	215.53
	75	84.82	34.47
2	50	120.63	188.11
	75	129.37	61.89
3	50	42.08	118.78
	75	207.91	131.25

The higher the percentage value, the higher the CDN50 in the submain line sections 1, 2, and 3. Consequently, the higher the CDN50, the lower the C_{PVC} on both slopes (Figure 6).

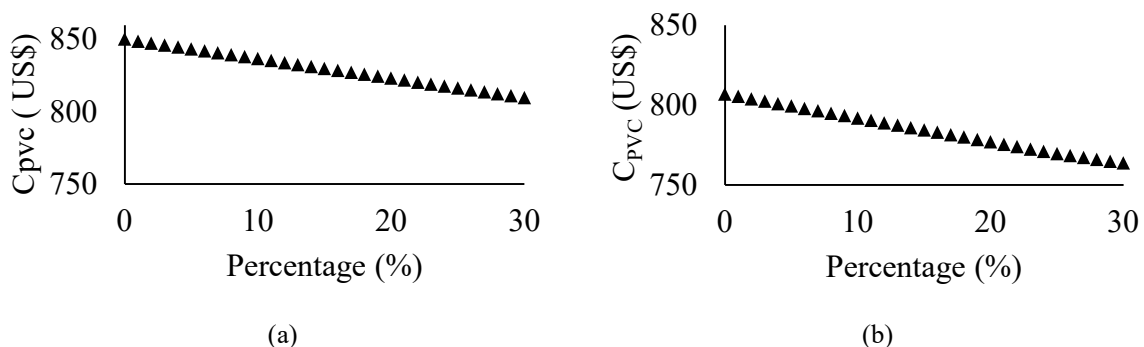


FIGURE 6. PVC pipeline cost (C_{PVC} ; US\$), for the (a) 3% slope, and (b) 6% slope, as function of the tolerance percentage (%).

The maximum and minimum costs for the 3% slope are US\$ 849.11 and US\$ 809.91, respectively. The highest cost for the 6% slope is US\$ 805.53 and the lowest cost is US\$ 763.86.

The reduction in C_{PVC} due to an increase in the percentage is associated with a decrease in the annual cost

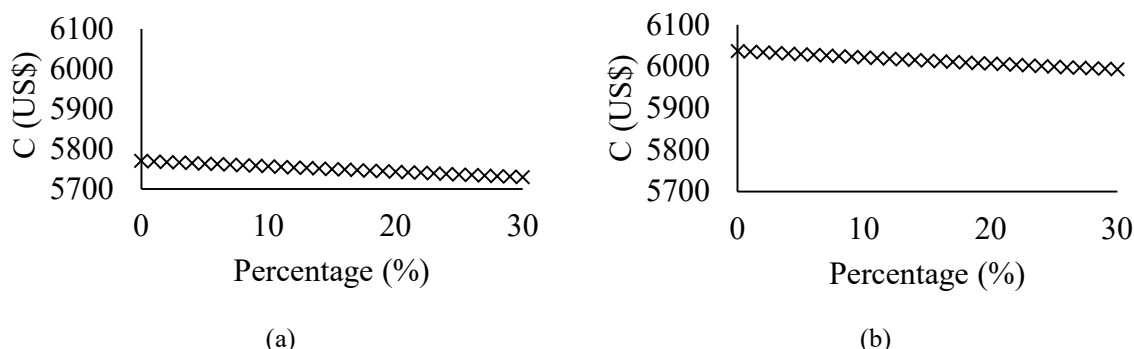


FIGURE 7. Annual pumping cost of the irrigation system (C ; US\$), for the (a) 3% slope, and (b) 6% slope, as function of the tolerance percentage (%).

The decreasing value behavior of the objective function on both slopes is justified by its only term dependent on the decision variable that changes. C_{PVC} is the only term that interferes with the decrease in C , as it is the only C factor that depends on the length of the submain line pipes, providing a reduction of up to US\$ 40.55 for the 3% slope and US\$ 43.16 for the 6% slope compared to the non-fuzzy model.

The results in the application of the Zimmermann-Werner method were not satisfactory for the 9% slope because the mainline section 3 in all percentage values (Table 4) uses a certain amount (meters) of the pipe with a nominal diameter of 125 mm, but sections 1, 2, and 4 use only the nominal diameter of 100 mm. It makes the irrigation system operationally impracticable, as usually, the diameter decreases with a decrease of the flow in each stretch. For this reason, a restriction needs to be added to the non-fuzzy model so that the method can be applied without this operational condition occurring.

TABLE 4. Length (m) of pipes with nominal diameter of 100 mm and 125 mm for the tolerances percentage (%), in the mainline, section 3, for the 9% slope.

Percentage (%)	Length (m)	
	100 mm	125 mm
1	88.28	9.72
2	62.04	35.96
3	36.93	61.06
4	12.07	85.92
5-30	0	98

Spiliotis & Tsakiris (2007) used the Zimmermann-Werner method to solve a model of the integer programming aiming at minimizing the total cost of pressurized irrigation networks subject to head constraints at the hydrants and length constraints related to the branches of the network. The solution found by the authors also resulted in economic gains with an increase in the pipe length in branches of smaller diameters, satisfying the

of the irrigation system (C) on both slopes. In this case (Figure 7), the highest and lowest costs on the 3% slope are US\$ 5769.08 and US\$ 5729.88, respectively, while the highest and lowest costs for the 6% slope are US\$ 6035.09 and US\$ 5993.43, respectively.

hydraulic conditions required by the system. However, it is a simpler model, as it does not consider the pumping system, with decision variables only for the length, according to their respective diameter. Fuzzy constraints occurred in minimum allowable pressure head, while the other restrictions were crisp numbers. The method application with the absence of the strictest control of a supreme, maximum allowable pressure head will have a solution involving small diameters followed by high-pressure losses. It is not the model described in this study, as the irrigation network is a case of a more complex network and including the pump system, allowing inequalities to be fuzzy.

CONCLUSIONS

The application of the Zimmermann-Werner method allowed obtaining a range of new solutions that reduce the annual cost of the irrigation system, without changing the average pressure in the manifolds and the total operating head. Furthermore, a relationship was observed between the annual cost of the irrigation system and the price of PVC pipes, which decreased with an increase in pipe length for the nominal diameter of 50 mm in the submain sections 1, 2, and 3 on the 3 and 6% slopes.

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