LARGE DISCREET RESOURCE ALLOCATION: A HYBRID APPROACH BASED ON DEA EFFICIENCY MEASUREMENT

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Abstract

Resource allocation is one of the traditional Operations Research problems. In this paper we propose a hybrid model for resource allocation that uses Data Envelopment Analysis efficiency measures. We use Zero Sum Gains DEA models as the starting point to decrease the computational work for the step-by-step algorithm to allocate integer resources in a DEA context. Our approach is illustrated by a numerical example.

Keywords: resource allocation; DEA; zero sum gains; step-by-step allocation algorithm.

Resumo

A alocação de recursos é um dos problemas clássicos da Pesquisa Operacional. Neste artigo é proposto um modelo híbrido para alocar recursos, que faz uso de medidas de eficiência calculadas por Análise de Envoltória de Dados (DEA). São usados modelos DEA com Ganhos de Soma Zero como ponto de partida para reduzir o esforço computacional do algoritmo seqüencial para alocação de recursos discretos em DEA. A abordagem aqui proposta é aplicada a um exemplo numérico.

Palavras-chave: alocação de recursos discretos; DEA; ganhos de soma zero; algoritmo seqüencial.

1. Introduction

Data Envelopment Analysis (DEA) models measure the efficiency of productive units (known as *Decision Making Units* – DMU's). Usually, this measure evaluates the units and shows the better management practices that become benchmarks for the other units. The efficiency measure can also be taken as an index of good use of resources and, thus improve its allocation. It is clear that more resources should be allocated to the units that make better use of them. This procedure is of particular interest whenever there is a central decision making controller that allocates scarce resources severely limited in number to the DMU's.

Using DEA models to achieve this has been a study subject in the literature as shown, for instance in Athanassopoulos (1995, 1998), Thanassoulis (1996), Cook & Kress (1999), Yan *et al.* (2002), Beasley (2003), Gomes (2003), Korhonen & Syrjänen (2004), Lozano & Villa (2004), Amirteimoori & Kordrostami (2005), Ertay & Ruan (2005), Marinescu *et al.* (2005), Soares de Mello *et al.* (2006), Avellar *et al.* (2007), Gomes & Lins (2008), among others. A revision of the "DEA resource allocation" theme can be found in Soares de Mello *et al.* (2006).

Here, two approaches rise from the others: The DEA with Zero Sum Gains (DEA-ZSG) model (Gomes, 2003; Lins *et al.*, 2003; Gomes *et al.*, 2003, 2004, 2005; Gomes & Lins, 2008) and the sequential algorithm to allocate discreet resources in DEA models (Soares de Mello *et al.*, 2006). Both models assume that the sum of resources is constant and do not assume a pre definition of the efficient frontier. They are, therefore, non-parametric models.

Classic DEA models assume that the quantity of consumed resources, as well as each unit's output depend only upon that unit's decisions. So, units are independent. On the contrary, DEA-ZSG models assume the hypothesis of output and resource consumption not being fully independent. This arises whenever resources are shared or when a larger production from one unit compels another to have a lesser output, the classic example being that of Olympic medals being won (Lins *et al.*, 2003). An additional constraint to optimization problems used to determine classic DEA efficiency is required when the dependency hypothesis is assumed. This constraint compels the sum of a given resource (or output) to be constant and raises non-linearity in the optimization problems. In principle, this increases the computational effort, which is already usually large to solve DEA models (Dulá, 2002; Biondi Neto *et al.*, 2004).

This added constraint implies that when an inefficient DMU tries to reach its target along the efficiency frontier, the position of the remainder DMU's changes so the sum of resources (or outputs) is kept constant. Therefore, as opposed to what occurs in the classic DEA models, the efficient frontier moves whenever a DMU tries to reach it. This allows DEA-ZSG models to be used to reallocate resources. All that is needed is to move to the frontier all inefficient DMU's. Necessarily, the frontier is displaced. After this procedure, all DMU's will belong to the efficiency frontier. However, this method requires continuous variables, which may not be the case.

Should discreet resources be reallocated, the sequential algorithm for this objective can be used. In this algorithm, the resources are reallocated, step-by-step, to each set of efficient DMU's but only one unit at a time. Once finished, it is assumed that all resources have been effectively reallocated and new DMU efficiency indexes are computed. The algorithm is repeated until all resources are reallocated. Thus the algorithm becomes a sequential one to determine resource allocation on the basis of the DEA efficiency measures. For a detailed description of the algorithm see Soares de Mello *et al.* (2006). The major disadvantage of this procedure is that it becomes very slow when the integer resource to reallocate is large and the number of efficient DMU's is small. The lesser the number of efficient DMU's, the slower the process as, in this case, any iteration causes only a small resource reallocation. This leads to many iterations being required.

A comparative analysis of both the resource allocation sequential algorithm and the DEA-ZSG model shows they are based on similar assumptions: the efficiency frontier is not specified in a functional way and the total of a given variable is limited. However, whilst in the DEA-ZSG models the existing total of a given variable is reallocated, in the sequential algorithm new resources are allocated to the DMU's. Another difference is that DEA-ZSG models deal with continuous variables while the sequential algorithm deals with discreet variables.

Taking advantage of the existing similarities, this paper proposes the joint use of the two approaches as a means to overcome the slow processing of the resource allocation sequential algorithm. A DEA-ZSG is proposed to start off with as a means to speed up resource allocation. The hybrid approach herein proposed is applied to a numerical example adapted from the Korhonen & Syrjänen (2004) case study.

2. DEA Zero Sum Gains Models

The classic DEA models, either the CCR (Charnes *et al.*, 1978) or the BCC (Banker *et al.*, 1984) and their variations, assume total freedom of production, i.e., a given DMU's production does not interfere with the production of the other DMU's. The same assumption can be extended to the use of resources. However, in many cases there is no such freedom. In competitions, for instance, to improve a competitor's ranking requires an equivalent loss for the other competitors. If a variable is linked to the rankings obtained (Lins *et al.*, 2003; Villa & Lozano, 2004; Gomes & Avellar, 2005), this has to be taken into account.

In the so called Zero Sum Gains DEA models (DEA-ZSG) (Gomes, 2003; Lins *et al.*, 2003; Gomes *et al.*, 2003, 2004, 2005; Gomes & Lins, 2008), an inefficient DMU trying to reach the frontier by reducing its inputs (or increasing its outputs) shall impart to the others that increase or decrease so the sum is constant. So, DEA-ZSG models have a direct application in the resource (or output) allocation or reallocation studies whenever a constant sum is a modeling constraint.

DEA-ZSG models are similar to zero sum games in which others must lose whatever one given player wins. In other words, the net sum of wins must be zero. As opposed to traditional models, the way followed by a DMU to reach its target on the frontier may imply changes in the shape of the efficient frontier.

There are several ways or strategies for an inefficient DMU to go after its target under these conditions. Strategies to search targets are proposed in Lins *et al.* (2003), the proportional reduction strategy being of special interest: DMU's searching efficiency (trying to reach the frontier) must shed input units. So the sum is kept constant, the inputs acquired by other DMU's must be proportional to their input level. This means that the lower the input level of a DMU, the lesser the inputs it acquires. What has just been said applies to the outputs: the higher the output level, the higher the outputs it looses.

There is always the possibility of more than one DMU trying to maximize efficiency. This can either be done in competition or cooperation. The case in which DMU's create a cooperative group is the most interesting case in ZSG modeling. In the DEA-ZSG paradigm, cooperative search for efficiency means that the DMU's belonging to the group try to allocate a given quantity of input only to those DMU's that do not belong to the group, the same applying when the group tries to withdraw a given quantity of outputs only from those DMU's that do not belong to the group, either.

In the general case of multiple DMU's acting in cooperation, the DEA-ZSG model becomes a Multi-objective Nonlinear Programming Problem (Gomes *et al.*, 2003). Problems like this tend to lead often to the use of metaheuristics owing to the large number of variables and DMU's. However, for the proportional reduction strategy, Gomes *et al.* (2003) prove that the model is reduced to a Mono-objective Nonlinear Programming Problem in accordance with the Proportional Efficiencies in the Proportional Strategies Theorem. The Theorem establishes that if various cooperating DMU's search targets following proportional strategies, the efficiencies of the DMU's in the DEA-ZSG model are directly proportional to their efficiencies in the classical model.

Should all inefficient DMU's gather in a sole cooperative group and search for their efficiency in the classic DEA efficiency frontier, the use of DEA-ZSG will lead to the complete constant sum input (or output) reallocation. After this reallocation, all DMU's will belong in the efficient frontier, i.e., they all will be 100% efficient.

This new DEA frontier, herein named uniform DEA frontier or maximum efficiency frontier, will be located at lower levels than those of the DEA classic model frontier. This happens because efficient DMU's lose efficiency, as they end up having have more input units and/or less output units. This is so to compensate for the inverse movement in the previously inefficient DMU's in order to keep constant the sum of either the inputs or outputs. This maximum efficiency maybe seen as "ideal" by regulating organs as the decision maker will be presented with an input and/or output reallocation that makes all units be 100% efficient.

To build directly a uniform frontier in which inefficient DMU's are joined in a single cooperative grouping *W*, Gomes *et al.* (2003) have proved the Target Determination Theorem. This theorem establishes that "the DMU target being studied in the proportional strategy DEA-ZSG model equals the classic target multiplied by the reduction coefficient". Together with the Proportional Efficiencies in the Proportional Strategies Theorem, the Target Determination Theorem leads to the solution of the Non Linear Programming Problem being a single equation.

Thus, for both the CCR and BCC input oriented models, equation (1) is valid. In it, h_{Ri} and h_i are the respective efficiencies of the DEA-ZSG and classic DEA models; W is the cooperating DMU's group; $r_{ij} = h_{i-I} / h_{j-I}$ is the proportionality factor resulting from the use of the input oriented proportional strategy. Equation (2) is the corresponding equation for output-oriented models in $q_{ij} = h_{i-O} / h_{j-O}$ is the proportionality factor.

$$h_{Ri} = h_i \left(1 + \frac{\sum_{j \in W} \left[x_j \left(1 - r_{ij} \; h_{Ri} \right) \right]}{\sum_{j \notin W} x_j} \right) \tag{1}$$

$$h_{Ri} = h_i \left(1 - \frac{\sum_{j \in W} \left[y_j \left(q_{ij} h_{Ri} - 1 \right) \right]}{\sum_{j \notin W} y_j} \right) \tag{2}$$

It should be pointed out that other DEA models that use the constant sum constraint are to be found in the literature. The works of Avellar *et al.* (2007, 2005) and Lozano & Villa (2004) deserve particular mention. The former propose DEA CCR type models based on limited inputs/outputs in which the allocation of resources/production can be influenced either by the inputs or the outputs. The construction of these models was based on the geometrical profile of three-dimensional CCR frontier that is replaced by a hyperbolic or spherical frontier depending on the nature of the variable to be allocated. Attention is drawn to the fact that these models postulate a type of function for the frontier as opposed to the practice in classic DEA. Guedes (2007) names this type of model as "parametric" DEA". On the other hand, Lozano & Villa's (2004) model, called *Constant Sum of Outputs* (CSO), refers to resource allocation decision making being centralized. The authors propose a DEA BCC model in two phases in which the efficiency maximization for each individual DMU occurs simultaneously with the minimization of total resources or maximization of total production.

3. Sequential Algorithm to Allocate Resources in DEA

The authors of this paper are not aware of any models to be found in the literature that reallocate resources already existing and/or do not take into account the possibility of the resources being discreet. As proposed in Soares de Mello *et al.* (2006), the sequential algorithm to allocate resources in DEA takes into account the sharing of discreet resources that will be allocated at a later stage. The authors describe a sequential algorithm to allocate resources based on the efficiency measurements of the DMU's. It is a dynamic algorithm that computes new efficiencies at each step.

The first approach to allocate new resources on the basis of DEA efficiency measurements, as showed in Soares de Mello *et al.* (2006) would be the allocation of available resources to efficient DMU's only. However this rather simplistic technique may present difficulties. The first one has to do with the fact that the number of resources may not be a multiple of the number of efficient DMU's, which would lead to some sort of ranking for the 100% efficient DMU's. Another difficulty is that this method of allocation does not take into account those inefficient DMU's that are, however, very close to efficiency. In this case, it may not be right to allocate all resources to the efficient DMU's and nothing to one that is 99% efficient.

In the algorithm proposed by Soares de Mello *et al.* (2006), the efficiencies of all DMU's are computed and only one single resource is allocated to each of the 100% efficient DMU's. Once this is done, it is assumed that the resources have been effectively allocated and new efficiencies are computed. This procedure is repeated until all resources are allocated, the aim of this step being to find out what would be the behaviour of the DMU's if they maintained the same output but could count on the resources allocated at this stage. This is then a sequential algorithm to allocate discreet resources based on efficiencies calculated by the classical DEA models.

Whenever the resources to be allocated are fewer than the number of efficient DMU's, the authors propose that the resources be allocated, firstly, to those efficient DMU's that have not received any in the previous steps and, next, to those efficient DMU's with the lesser number of the original resource. The first rule is justified by a more equitable allocation. The second follows the principle that the same resource is better used by a DMU with fewer resources than by a DMU with more resources.

4. Proposed Hybrid Model

As previously mentioned, DEA-ZSG models as they exist now are adequate for the allocation of continuous resources. To allocate discreet resources, rounding off the allocated value may be an option, but it may lead to values the sum of which is different from the established total.

As opposed to this, the sequential algorithm for DEA allocation is valid for discreet resources. However good the allocation it promotes, this algorithm may become very slow, should the number of resources be large and the number of efficient DMU's be small. The cause for becoming slow is the small number of resources that are reallocated at any iteration.

To combine the advantages and potentialities of both models, we propose herein their joint use. To do so, the allocation of resources is done in two steps. Firstly, resources are allocated in accordance with the proportional strategy DEA-ZSG model. As resources are allocated assuming that the variables are continuous, which they are not, only the integer part is allocated as the first parcel for each DMU. In the second allocation step, the difference between what should have been allocated and what really was, the left-over from the first step is allocated using the sequential algorithm previously described.

5. Numerical Example

To give an example of the hybrid approach herein proposed, we have adapted the Korhonen & Syrjänen (2004) case study. Data were obtained from the real case of 25 Finnish supermarkets belonging to the same chain. The authors have used two output variables, namely sales and profits (10^6 Finnish Mark, the previous Finnish currency), whereas man-hours (10^2 h) and sales (10^2 m^2) were used as inputs.

To apply the model it has been assumed that the variable "sales area" is a non-discretionary input. Indeed, it is not possible, at short notice, to change the area of any supermarket and it may not even be in the management's interest to do so.

Korhonen & Syrjänen (2004) original model measured labour in man-hours because this is a continuous variable, the only type accepted by the model. As the model herein proposed can only deal with discreet variables the variable number of workers (1862) has been used instead. The data of the original paper were used rounded off to the closest integer. Table 1 shows the data for this paper.

Table 1 – Numeric Example Data.

Units	Nº of Workers	Area	Sales	Profits	DEA CCR Efficiency
S1	79	4,99	115,3	1,71	0,490
S2	60	3,3	75,2	1,81	0,427
S3	128	8,12	225,5	10,39	0,877
S4	154	6,7	185,6	10,42	0,722
S5	66	4,74	84,5	2,36	0,557
S6	77	4,08	103,3	4,35	0,568
S7	50	2,53	78,8	0,16	0,654
S8	45	2,47	59,3	1,30	0,426
S9	18	2,32	65,7	1,49	1,000
S10	90	4,91	163,2	6,26	1,000
S11	57	2,24	70,7	2,80	0,656
S12	113	5,42	142,6	2,75	0,372
S13	107	6,28	127,8	2,70	0,455
S14	55	3,14	62,4	1,42	0,443
S15	49	4,43	55,2	1,38	0,701
S16	59	3,98	95,9	0,74	0,523
S17	75	5,32	121,6	3,06	0,550
S18	95	3,69	107,0	2,98	0,389
S19	47	3,00	65,4	0,62	0,495
S20	44	3,87	71,0	0,01	0,682
S21	90	3,31	81,2	5,12	0,518
S22	95	4,25	128,3	3,89	0,542
S23	80	3,79	135,0	4,73	1,000
S24	67	2,99	98,9	1,86	0,734
S25	62	3,10	66,7	7,41	1,000
Total	1862				

It is assumed that the chain management's aim is to improve shop efficiency by reallocating workers. The hybrid approach herein proposed is used within the terms of an input oriented CCR model. The efficiencies generated by the original workers allocation is shown on Table 1, and were calculated with the SIAD software (Angulo-Meza *et al.*, 2005).

It should be pointed out that the objective proposed here is not the same as Korhonen & Syrjänen's (2004). For those authors, the chain management's interest was to maximise production through the reallocation of available resources. To do so, they proposed an interactive multi-objective DEA model, which suggests the best allocation plan. This model assumes that units are able to change their output within the set of production possibilities following a certain number of rules. That model does not constrain resources to be constant. The results obtained by those authors show alterations both at the output and work force level.

The first phase of the approach herein proposed is to compute the targets using the DEA-ZSG model. The reallocation results are shown on Table 2. It can be noticed that the reallocated value is a fraction, a characteristic of the DEA-ZSG model. So, only the integer part of this value is reallocated to each DMU and the sequential algorithm, as previously explained, reallocates the remainder. According to the CCR model, the remainder are 15 workers to be reallocated in the second phase.

Table 2 – Input oriented DEA-ZSG CCR model results.

Units	Original nº of Workers	Nº of workers after DEA-ZSG CCR model reallocation	Efficiency after reallocation	Workers allocation for the beginning of the second phase
S1	79	61,79	0,999	61
S2	60	40,89	0,999	40
S3	128	179,17	1,000	179
S4	154	177,47	1,000	177
S5	66	58,68	1,000	58
S6	77	69,81	1,000	69
S7	50	52,19	1,000	52
S8	45	30,60	0,999	30
S9	18	28,73	1,000	28
S10	90	143,65	1,000	143
S11	57	59,68	0,999	59
S12	113	67,09	1,000	67
S13	107	77,71	1,000	77
S14	55	38,89	0,999	38
S15	49	54,82	1,000	54
S16	59	49,25	1,000	49
S17	75	65,84	1,000	65
S18	95	58,98	1,000	58
S19	47	37,13	1,000	37
S20	44	47,90	1,000	47
S21	90	74,41	1,000	74
S22	95	82,18	1,000	82
S23	80	127,69	1,000	127
S24	67	78,49	1,000	78
S25	62	98,96	1,000	98
Total	1862	1862		1847

The second phase follows the steps of the sequential algorithm and its results are shown on Table 3. At this stage, the initial step consists of computing the DEA efficiency measurements with the resource allocation shown in the last column of Table 2, one resource unit being allocated to each DEA CCR model efficient DMU. This iteration allocates resources to DMU's S9, S10, S11, S23 and S25, 10 workers being left to reallocate. The CCR model is run again with the new resource, i.e., DMU's S9, S10, S11, S23 and S25 have respectively 29, 144, 60, 128 and 99 workers. In this second iteration 14 units are CCR efficient with this resource arrangement: S2, S4, S6, S7, S8, S9, S10, S14, S17, S18, S21, S23, S24 and S25.

In the second iteration, the number of resources to be allocated is less than the number of efficient DMU's. According to the sequential algorithm, resources should be allocated first to those efficient DMU's that were not contemplated in the previous steps and, after, to those that originally had fewer of this resource. Applying the first tie-breaking criterion, 4 of 14 DMU's (S9, S10, S23 and S25) did receive resources in the previous phase and, thus, they are ineligible to receive any. So, in this iteration, the third, the remaining 10 workers can be reallocated without having recourse to the second tie-breaking criterion.

Table 3 – Second phase results: use of the sequential algorithm.

Units	Iteration 1		Iteration 2		Iteration 3	
	Nº of workers	DEA CCR Efficiency	Nº of workers	DEA CCR Efficiency	Nº of workers	DEA CCR Efficiency
S1	61	0,987	61	0,992	61	1,000
S2	40	0,996	40	1,000	41	0,994
S3	179	0,988	179	0,994	179	1,000
S4	177	0,991	177	1,000	178	0,997
S5	58	0,986	58	0,990	58	1,000
S6	69	0,994	69	1,000	70	1,000
S7	52	0,991	52	1,000	53	0,990
S8	30	0,994	30	1,000	31	0,984
S9	28	1,000	29	1,000	29	1,000
S10	143	1,000	144	1,000	144	1,000
S11	59	1,000	60	0,994	60	0,995
S12	67	0,976	67	0,998	67	1,000
S13	77	0,984	77	0,987	77	0,998
S14	38	0,997	38	1,000	39	0,985
S15	54	0,990	54	0,993	54	1,000
S16	49	0,980	49	0,987	49	0,996
S17	65	0,988	65	1,000	66	0,998
S18	58	0,998	58	1,000	59	1,000
S19	37	0,979	37	0,982	37	0,989
S20	47	0,994	47	0,996	47	1,000
S21	74	0,992	74	1,000	75	0,995
S22	82	0,988	82	0,999	82	1,000
S23	127	1,000	128	1,000	128	1,000
S24	78	0,998	78	1,000	79	0,998
S25	98	1,000	99	1,000	99	1,000
Total	1847		1852		1862	
Slack	15		10		0	

The proposed hybrid approach reached its initial objective of allocating the supermarket chain workers in the "fairest possible way" as the table shows. "Fair way" (the result of the DEA-ZSG model imposed paradigm) means the increase of the average chain efficiency, which grows from 63,1% (Table 1) to 99,7% (Table 3). Furthermore, the reallocation of resources took only three iterations.

6. Conclusions

A hybrid approach has been proposed in this paper for the (re) allocation of discreet resources. This approach uses together the DEA-ZSG models and the DEA sequential algorithm to allocate discreet resources. DEA-ZSG models are used as a first step to allocate resources in a continuous manner. The sequential allocation algorithm is used thereafter to allocate discreet resources.

Only three iterations were needed to reallocate all the resources in this case. Moreover, the average shop efficiency was nearly 100%. So, this new approach was very advantageous both to reallocate resources and to increase the average efficiency of the DMU's at the end of the process. This is of particular interest to units subject to a central authority that wishes its productive centres to be as efficient as they can.

DEA-ZSG models and the sequential allocation algorithm can be used together for other purposes. One example is to allocate a large quantity of new resources beside those already existing. As already mentioned, the use of the sequential algorithm on its own might lead to numberless iterations to reach the complete allocation of a particular resource. This could render the process too slow for practical purposes. Two alternatives can be used to start off the hybrid model. Either the new resources are added to the previous lot, the total being then uniformly allocated to the units, or the initial system is unchanged and only the new resources are uniformly allocated to the units. In the first case, the total value serves as starting point for the model. In the second, the sum of the two values becomes the variable to be allocated in the DEA-ZSG model to be followed by the sequential algorithm.

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