versão impressa ISSN 0101-7438 / versão online ISSN 1678-5142

UNIFORMIZATION OF FRONTIERS IN NON-RADIAL ZSG-DEA MODELS: AN APPLICATION TO AIRPORT REVENUES

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Recebido em 10/2008; aceito em 07/2009 após 1 revisão
Received October 2008; accepted July 2009 after one revision

Abstract

We propose in this paper an extension to the Zero Sum Gains Data Envelopment Analysis model (ZSG-DEA). The proposed approach takes into account, simultaneously, non-radial projections and cone-ratio weights restrictions. We developed an iterative approximate algorithm to solve this model, as in the case study it is oriented only to the constant sum output. The theoretical approach is applied to the concession of discounts and surcharges problem, in terms of airport fees.

Keywords: DEA; zero sum gains; non-radial projections; airport fees.

Resumo

É proposta neste artigo uma extensão do modelo de Análise de Envoltória de Dados com Ganhos de Soma Zero (DEA-GSZ). Esta extensão considera simultaneamente projeções não radiais e restrições do tipo *cone-ratio*. Mostra-se necessário para a resolução do modelo proposto, o uso de um algoritmo iterativo aproximado. Isto se deve ao fato de o modelo ser orientado a somente um dos *outputs*, o de soma constante. O modelo teórico desenvolvido é exemplificado a um problema de concessão de descontos e sobretaxas em tarifas aeroportuárias.

Palavras-chave: DEA; ganhos de soma zero; projeções não radiais; tarifas aeroportuárias.

1. Introduction

One of the most studied applications for measurements obtained using DEA is the distribution or redistribution of resources. Research in this area has been conducted, for example, by Yan *et al.* (2002), Beasley (2003), Korhonen & Syrjänen (2004), Lozano & Villa (2004, 2005), Soares de Mello *et al.* (2006), Gomes *et al.* (2007), Fang & Zhang (2008), Haidi-Vencheh *et al.* (2008) and Asmild *et al.* (2009).

In the past few years, DEA models have been developed following the concept that resources are limited and, therefore, in order to attribute more resources to a DMU one needs to remove them from another. These models seek to obtain a frontier with the maximum number of efficient DMUs (ideally all should be efficient).

One of these models parameterizes a final frontier that is obtained when all DMUs are efficient. Avellar *et al.* (2005, 2007) and Guedes *et al.* (2009) arbitrated a spherical form for this frontier. The Spherical Frontier DEA Model, as it is called, deals with the distribution of both new and existing resources.

Another class of models solve problems of the same type without the need to choose a function for the frontier. This type of model was primarily applied in cases where the production of the DMUs was not independent. It was first developed to establish Olympic medal goals at the Sydney Olympic Games (Lins *et al.*, 2003) and is named Zero Sum Gains DEA model (ZSG-DEA). Initially, the condition for zero sum gains was only applied to the DEA BCC model, proposed by Banker *et al.* (1984).

Gomes *et al.* (2003) have adapted the ZSG-DEA model to the uniformization of the frontier with a redistribution of outputs. For this they have used a modelling of iso-efficiency layers (Barr *et al.*, 2000; Gomes *et al.*, 2009b). Gomes *et al.* (2004) used the ZSG-DEA model for a redistribution of outputs and inputs with the uniformization of the frontier in a different way: instead of reducing the production and preserving the resources of initially efficient DMUs, they forced them to diminish outputs as well as reduce inputs. In order to solve the mathematical problem of determining one equation for each facet of the frontier, they used the smoothed frontier developed by Soares de Mello *et al.* (2002, 2004) and Nacif *et al.* (2009).

Gomes *et al.* (2005) extended the use of ZGS-DEA models to constant returns to scale (Charnes *et al.*, 1978), still with the redistribution of inputs. Gomes & Lins (2008), apply the uniformization of frontier techniques in the ZGS-DEA model to the distribution of an undesirable output, modelled as an input, in a case of environmental interest.

In radial projection cases, the uniformization does not represent major problems given that theorems already exist, which guarantee the methods in use – as stated in the articles mentioned hereabove. In case there are any restrictions in relation to weight, the success of the uniformization will depend on the validity of a conjecture formulated by Gomes (2003). Although it has not yet been demonstrated, this conjecture has been valid in previous case studies.

However, if there are any variables in the model that cannot be altered by the manager (non-controlled variables), the situation becomes more complex. This article shows (numerically) that the methods of uniformization previously used may not work in this case. A justification is also outlined as to why it does not work. A method of successive approximations is used in the case study presented, which, apparently, converges towards the final distribution with all the DMUs having unitary efficiency. In this article, the study is only conducted in an empirical form, not concerning itself with the theoretical foundations of the method for the moment.

The result of the approach developed here is applied to a case of establishing fee criteria at airports. The modelling of this problem requires ZSG-DEA models with weights restrictions and non-controlled variables. Given the lack of data, due to the natural state of secrecy of commercial airliners, a simulation was made in order to exemplify the problem. This simulation makes use of fictitious companies and is based on the average values of real companies from a few years back, so as to avoid confusion between our results and the real values of airline companies.

2. Zero Sum Gains DEA Models

2.1 General Concepts and Radial Projections

The classic DEA models, both the CCR model and the BCC model, in all their variants, assume total freedom of production. This means that the production of one DMU does not interfere with the production of the others. However, in some cases this freedom does not exist. In the case of competition, for example, if one considers as output the final result or an index that aggregates its results (Lins *et al.*, 2003; Villa & Lozano, 2004), a gain of position for any competitor implies a loss of position for one or more of its adversaries.

To deal with this type of situation, the so-called Zero Sum Gains DEA models (ZSG-DEA) were proposed. These represent a situation similar to that of a zero sum game, in which all that is gained by one player is lost by the other(s) (Gomes, 2003; Gomes *et al.*, 2003, 2004, 2005; Lins *et al.*, 2003; Gomes & Lins, 2008). In other words, the net sum of gains must be zero. In contrast to what occurs in traditional models, the way in which a DMU reaches its target on the frontier can result in the alteration of the shape of the efficient frontier. In Lins *et al.* (2003) the authors propose strategies for the radial search of targets, highlighting the proportional reduction strategy. Applications of the ZSG-DEA model, other than those presented in the previously cited articles, can be seen in Gomes *et al.* (2007, 2008), Gomes & Soares Mello (2009), Hu & Fang (2010).

In the proportional strategy, the DMU that seeks efficiency (seeks the frontier) needs to gain certain units of output (or lose input). So that the sum remains constant, other DMUs must lose (or gain) in proportion to their levels of output (input). In this way, whichever has the lowest level of output (input) loses (gains) less; whichever has the highest level of output (input) loses (gains) more.

It may occur that more than one DMU will seek to maximise efficiency, which can be done in competition or cooperation. The most interesting case in ZSG modelling is that in which inefficient DMUs form a cooperation group. In the ZSG-DEA paradigm, the search for cooperation means that DMUs from this group try to allocate a certain amount of input (or to remove an amount of output) only to the DMUs that do not belong to this group.

In a general case of multiple inefficient DMUs operating in a cooperation regime, the ZSG-DEA model is a Multi-objective Nonlinear Programming Problem (Gomes *et al.*, 2003). Problems of this type frequently lead to the use of metaheuristics. However, for the proportional reduction strategy the model is reduced to a Mono-objective Nonlinear Programming model, according to the Proportional Efficiencies in the Proportional Strategies Theorem (Gomes *et al.*, 2005). Its statement establishes that in the problem of various DMUs in cooperation in the search for targets with proportional strategy, the efficiencies of the DMUs in the ZSG-DEA model are directly proportional to their efficiencies in the classic DEA model.

In a case where all the inefficient DMUs form a single cooperation group and seek efficiency in the classic DEA efficiency frontier, the application of the ZSG-DEA model brings about the redistribution of input or of the constant sum output. After this redistribution, all DMUs will belong to the efficient frontier; in other words, all will be 100% efficient.

This new DEA frontier, referred to here as the uniform DEA frontier or maximum efficiency frontier, is located at lower levels than the those of the classic DEA model, as efficient DMUs gain input units (or lose output units) to compensate the loss (or gain) of inefficient units, so as to maintain a constant sum. This situation of uniform efficiency can be seen as desirable by regulatory agencies, since what will be presented to the decision-maker is a distribution of resources (or products) that makes all units 100% efficient. This is the approach followed by Gomes & Lins (2008).

In order to build a uniform frontier directly, in which inefficient DMUs form one single group of cooperation *W*, Gomes *et al.* (2003) demonstrated the Target Determination Theorem, which was used by Gomes & Lins (2008). This theorem states that the target of the DMU under analysis in the ZSG-DEA model with proportional strategy is equal to the target in the classic case multiplied by the reduction coefficient. This theorem, together with the Proportional Efficiencies in the Proportional Strategies Theorem, allows us to reduce the solution to the Nonlinear Programming Problem to one single nonlinear equation. For the CCR and BCC models, with input orientation, we have equation (1), where h_{Ro} and h_o are, respectively, the measurements of efficiency in the ZSG-DEA and classic DEA models for DMU *o*; *W* is the group of DMUs (*j*) in cooperation; $r_{oj} = h_{o-I} / h_{j-I}$ is the factor of proportionality resulting from employing the proportional strategy, with input orientation. Equation (2) is valid for output oriented models, in which $q_{oj} = h_{o-O} / h_{j-O}$ is the proportionality factor.

$$h_{Ro} = h_o \left(1 + \frac{\sum_{j \in W} \left[x_j \left(1 - r_{oj}\, h_{Ro} \right) \right]}{\sum_{j \notin W} x_j} \right) \tag{1}$$

$$h_{Ro} = h_o \left(1 - \frac{\sum_{j \in W} \left[y_j \left(q_{oj} h_{Ro} - 1 \right) \right]}{\sum_{j \notin W} y_j} \right) \tag{2}$$

2.2 Non-Radial Projections

The ZSG-DEA models presented so far impose the constant sum restriction only to the output or to the single input of the model, or to all of the outputs or inputs of constant sum. These are radial models, namely, they consider the proportional reduction or increase of inputs or of outputs, and do not assume simultaneous alterations.

It is interesting to note that there could be cases where an input (or output) of constant sum coexists with other inputs (outputs) of non-constant sum. In these multidimensional models, the projections are not radial, in other words, the variations of inputs and outputs are not proportional. There follows a description of how to treat ZSG-DEA in such a situation.

Take a ZSG-DEA problem modelled with $i = 1...r$ inputs and $k = 1...s$ outputs, in which only the output *f* is of constant sum. The remaining variables are not restricted, as in the classic DEA models. For an output-oriented model, the zero sum gains 'game' will be valid only for the output *f*. As there will be no redistribution or relocation of the other outputs present in the model, these can be assumed as analogous to the non-controllable variables (Cooper *et al.*, 2000).

Equation (3) presents the non-radial CCR ZSG-DEA model, output oriented, in which the efficiency is measured only by the alteration of one of the outputs (namely, of which a constant sum is imposed). In this model h_{Ro} is the inverse of the efficiency of the DMU *o*; x_{ji} and y_{jk} are the values of inputs *i* and outputs *k* of the DMUs *j*, respectively; λ_j represents the contribution of the DMUs *j* in the composition of the target of DMU *o*; y'_{jf} represents the new values of output *f* after reallocation.

Non-radial BCC ZSG-DEA models are analogous, with one exception: the addition of the convexity constraint $\sum_j \lambda_j = 1$.

$$\begin{aligned}
&\text{Max } h_{Ro} \\
&\text{subject to} \\
&x_i \geq \sum_j \lambda_j x_{ji}, \forall i \\
&h_{Ro} y_{of} \leq \sum_j \lambda_j y'_{jf} \\
&y_k \leq \sum_j \lambda_j y_{jk}, \forall k \neq f \\
&\lambda_j \geq 0, \forall j
\end{aligned} \qquad (3)$$

2.3 Weights Restrictions

The classic DEA models allow for total freedom in the choice of weights that will give the maximum efficiency value to a given DMU. This freedom is important in the identification of inefficient units, in other words, of those DMUs that present a poor performance including with their own set of multipliers.

The flexibility in the choice of weights is one of the advantages appointed to DEA modelling. Nevertheless, the calculated weights can be inconsistent with the knowledge in relation to the relative values of inputs and outputs, what can generate the need to introduce additional constraints.

When there are preferences between inputs and/or outputs among the decision agents, these value judgements are incorporated into the DEA models by means of restrictions to the weights (or multipliers) associated with inputs and/or outputs of the assessed units. Allen *et al.* (1997) and Thanassoulis *et al.* (2004), present a survey on the evolution of value judgements incorporation through weights restrictions. Lins *et al.* (2007) show how impracticalities arise and how to avoid them in the DEA LPPs that incorporate weights restrictions. Joro & Viitala (2004) study the relationship between weights restrictions and specialist opinions. Podinovski (2005) analyses the role of limits to the weights in DEA. Halme *et al.* (1999) present an alternative proposal for incorporating preferences in DEA without the use of weights restrictions.

Angulo Meza & Lins (2002) consider that the use of weights restrictions is one of the techniques that improve discrimination in DEA with decision-makers subjective opinions. Adler *et al.* (2002) also include weights restrictions within discrimination improvement techniques and present various types of restrictions. The main ones, according to Angulo Meza & Lins (2002), are: (a) direct restrictions on multipliers; (b) input-output level adjustments observed for capturing value judgements; (c) restriction to virtual inputs and outputs.

Lins *et al.* (2003) present the radial ZSG-DEA model with weights restrictions, output oriented, with k multiple outputs, all of constant sum. According to Gomes's (2003) conjecture, the Reference DMUs Contribution Equality Theorem and the Target Value Determination Theorem remain valid for the radial ZSG-DEA models with weights restrictions. This means that in the ZSG-DEA model with weights restrictions, as in the case without these restrictions, the values of the reference DMUs contributions are equal to those obtained from classic DEA models with weights restrictions, as long as the proportional reduction strategy is adopted. Consequently, the uniformization of the frontier can be achieved.

Nevertheless, as previously mentioned, there could be situations where only one of the outputs in the model must have constant sum and, furthermore, restrictions must be imposed to the output variables weights. In these cases, the corresponding model must be the non-radial ZSG-DEA with weights restrictions. This model (CCR, output oriented) is presented in (4), in which output f is of constant sum and the remaining variables are not restricted. $A^T\gamma$ represents the coefficients matrix of the outputs weights restrictions, $Au \le 0$. Analogous models can be derived for input orientation and the BCC situation.

$$\begin{aligned}
&\text{Max } h_{Ro} \\
&\text{subject to} \\
&x_i \ge \sum_j \lambda_j x_{ji},\ \forall i \\
&h_{Ro} y_{of} \le \sum_j \lambda_j y'_{jf} - A^T\gamma_i \\
&y_k \le \sum_j \lambda_j y_{jk} - A^T\gamma_i,\ \forall k \ne f \\
&\lambda_j, \gamma_i \ge 0,\ \forall j,i
\end{aligned} \tag{4}$$

Through numeric experimentation, which will be shown in section 3, it is verified that Gomes's conjecture (2003) does not remain valid for non-radial ZSG-DEA models with weights restrictions. This means that the uniformization of the efficient frontier is not reached in one round. Therefore, this conjecture cannot be generalised onto non-radial models with weights restrictions. Through numeric examples with two outputs, where one is of constant sum, it is observed that when the weights restriction imposed favours the constant sum output, the efficiency values were the same as those obtained with the non-radial model without weights restrictions. In this situation, uniformization is promptly obtained. However, when the imposed weights restriction favours the output of non-restricted sum, uniformization is only reached after following redistribution iterations of the constant sum output.

3. Case Study

3.1 Air Transport and Fees

Air transport is responsible for a large part of domestic (in countries of great size like Brazil) and international travel. According to Ashford *et al.* (1991), the airport is an essential part of the tourist transport system, because it is the physical location where a transfer is made: from air transport to land and sea transport. One must also observe that, as stated by Martin-Cejas (2006), "the quality of the journey begins at the airport".

In 2000, the Brazilian Airport Infrastructure Company (Infraero) adopted the vision that airports represent much more than simply the air entrance to cities. They must be seen as essential elements in the economic development of the urban regions where they are installed and in the development of tourism (Palhares, 2001). Non-aeronautical revenue is becoming increasingly important to the revenue of airports, so that these do not become overly dependent on strictly aeronautical fees. A study of non-aeronautical revenues in the efficiency of airports can be seen in Soares de Mello & Gomes (2004).

The importance of airline companies in generating non-aeronautical revenue for airports is clear. The more passengers are transported, the more revenue will be generated in services. This is because the consumption of products that are commercialised inside the airport tends to increase with the larger movement of passengers and their companions. In this way, airline companies contribute directly to greater returns in non-aeronautical revenue.

We propose here a fee discount concession model to airline companies as an incentive to increase the occupation of their airplanes and a stimulus to increase the number of passengers and visitors at airports. As a consequence, there would be an increase in non-aeronautical revenue, with a tendency to bring the airport closer to a self-sustainable operation. Aligned with this proposition, the more passengers an airline company brings to the airports, the larger the discount it will have on airport landing and ground stay fees. However, the discounts given to the 'more efficient' companies in terms of passenger transportation should be compensated with a type of surcharge on the companies with a smaller number of transported passengers. In this way, there is a need for both the discount and the surcharge to be calculated for each flight individually. For this purpose, the non-radial Zero Sum Gains DEA models based on the original proposal of ZSG-DEA, as discussed in Lins *et al.* (2003), Gomes *et al.* (2003, 2004, 2005) and Gomes & Lins (2008) are proposed and presented. For a presentation of the main airport fees see Fonseca *et al.* (2004).

Some work has already been done in the field of air transport in respect of the use of the DEA model to evaluate airports. Without any pretence to an exhaustive list, some of these works are mentioned herebellow.

Gillen & Lall (1997) recommend the use of DEA in these cases due to its non-parametric characteristics and due to the fact that DEA takes into account different factors simultaneously, although not including operational costs and revenues. Pels *et al.* (2001) compare European airports using DEA modelling. Adler & Berechman (2001) use DEA to evaluate the quality of airports from the point of view of airline companies, with the intention of determining which airports companies should choose for hubs. Fernandes & Pacheco (2002) use DEA to measure the efficiency of 35 Brazilian airports and, by doing so, evaluate their financial management and their infrastructures level of use. Pacheco &

Fernandes (2003) use DEA to study the same 35 airports so that, under financial aspects and physical dimensions, they can identify means of improving performance. Pestana & Dieke (2007) use DEA to evaluate Italian Airports.

3.2 Modelling

The new concept of attributing discounts in proportion to the quantity of passengers airline companies transport to each airport can be modelled through ZSG-DEA. In this paper, one airport will be analysed and the occupation rate of each flight at a given time on a weekday will be considered. Initially, according to the model proposed here, the discounts and surcharges, for each flight, focus only on the landing fee, so as to guarantee that the Infraero airport- related income remains constant in the airport under study. For this airport a model is applied where the DMUs are the flights operated in one day.

The model used is the non-radial CCR ZSG-DEA, output oriented. In this model, from the point of view of the airport administrator, the inputs are the maximum take-off weight (MTOW) of the aircraft, contained in its navigation certificate and measured in tons, and the duration of ground stay of the aircraft at the airport. The outputs are the non-airport-related revenue generated by the flight and the total fee paid.

The MTOW input indicates the maximum take-off weight of the aircraft. Generally, the greater this value, the larger the size of the aircraft and the greater the number of seats it offers. This implies that a larger number of passengers can be transported, which contributes to a greater collection of non-aeronautical related revenue by the airport's administration in its dependencies. As the landing and ground stay fee values depend on the MTOW of the aircraft, the greater the MTOW, the greater the amount charged by the airport for an aircraft to land. The amount charged by the hour for the aircraft to remain in the manoeuvre locations is also greater.

The input, length of stay of the aircraft in the manoeuvre location, measures the time spent by the aircraft in the manoeuvre location of the airport between flights. As in the point of view of the airline company 'aircraft on the ground make a loss', this variable measures the capacity of the airline company to prepare the aircraft for a new flight. The boarding and disembarking of passengers, the cleaning of the aircraft, the loading and unloading of luggage, and refuelling are some of the activities carried out by the company's team while the aircraft is on the ground. The shorter this time, the better the possibility that this aircraft will make more flights, and consequently, transport more passengers, bringing them to airports and therefore, increasing the collection of non-airport-related revenue. From the airport point of view this variable measures the unavailability of this space, which could be used by another aircraft. In other words, the shorter this time, the greater the possibility that more aircrafts will use this space and bring more passengers to the airport.

The output, non-aeronautical-related revenue generated by the flight, is virtually impossible to measure directly. Therefore, to substitute it, the number of passengers effectively transported was used, because it maintains a causal relationship with the non-aeronautical income.

Total paid fee corresponds to the landing fee plus the ground stay fee. The ground stay fee is only charged to the airline company in the case of its aircraft exceeding the first three hours of parking after landing. Thus, the greater the number of hours parked, the greater the amount that will be charged to the company.

The model proposed here fits into a total reformulation of fees and has the objective of measuring the efficiency of each flight in generating non-aeronautical revenue for the airports. Based on this efficiency, discounts and surcharges are attributed to the value of the total fee (landing and ground stay), so that the amount collected by the airport administration remains constant.

The constant sum output is, therefore, the total landing fee, in other words, after the execution of the model there will be an increase of output to the least efficient flights and a decrease of output to the most efficient flights, maintaining the total of this variable unchanged. Besides verifying which flights were most efficient in generating revenue for the airport administration, it is necessary to identify how much should really have been paid and, thus obtain the discount or surcharge. For this, a new value was calculated for the total landing fee output, for all DMUs (flights) in such a way that, with this new value, all are efficient.

The DMUs analysed in the model are domestic flights operated at a given airport. As there is difficulty obtaining real data, the data here was simulated. The difficulty in accessing trustworthy data about air transport has already been mentioned, for example, in Soares de Mello *et al.* (2005).

A total of 26 flights were considered spread through three different fictitious companies, C1, C2, C3. We supposed that all companies were of similar size, one of them being slightly larger than the other two. In this way, 13 flights were considered for company C1, 13 for company C2 and 10 flights for company C3.

Some aircraft were chosen, from among the ones that operate within the Brazilian air network (and which, therefore, had easily available data) to make up the fleet of these fictitious companies. This fleet is presented in Table 1, where the MTOW and the number of seats of each aircraft for each airline company appear.

Table 1 – Aircraft, MTOW and seats.

	Aircraft	**MTOW (in tons)**	**Number of seats**
C1	767-300	181	196
	737-800	77	156
	737-700	68	132
	737-500	52	117
	737-300	61	132
C2	A-320	70	162
	A-319	64	132
	Fokker 100	44	108
C3	737-700	62	144
	737-800	62	177

3.3 Data

The data used in the model is simulated, because it was not possible to obtain real data. Firstly, it is necessary to know which airliners operate the flights of which company. For this, one considers the aircraft in Table 2 as each company's fleet. This table shows the aircraft classified by type, quantity of each aircraft in the fleet (N) and the probability of each aircraft operating a flight (Prob). The probability of each aircraft operating a flight is given by the ratio between the number of aircraft of each type and the total number of aircraft from each company.

Table 2 – Probability of each aircraft in the fleet of airline companies.

C1			C2			C3		
Aircrafts	**N**	**Prob**	**Aircraft**	**N**	**Prob**	**Aircraft**	**N**	**Prob**
767-300	8	0,1356	A-320	31	0,4769	737-700	21	0,8750
737-800	2	0,0339	A-319	13	0,2000	737-800	3	0,1250
737-700	3	0,0508	Fokker 100	21	0,3230			
737-500	14	0,2373						
737-300	32	0,5423						

A discreet distribution is carried out with this data, where the discreet variables are the aircraft types. The probability of each aircraft operating a flight is also inserted. This is performed for each airline company, in other words, for company C1 five discreet variables are considered with their respective probabilities and 13 random numbers are generated. For C2 three discreet variables are considered and 13 random numbers are also generated. For C3 two discreet variables are considered and 10 random numbers generated.

To obtain the number of passengers transported it is necessary to generate values for the occupation of flights. These values are obtained by a triangular distribution, where the values for the averages are the ones available on the website of the former Civil Aviation Department. For each fictitious company the average data of each of three real companies was used. The calculation of the effective number of passengers transported is given by the 'occupation of each flight' multiplied by the 'number of seats available in the aircraft that operates the flight'.

As the landing and ground stay fees depend on the MTOW of the aircraft, the values for the total paid fee are obtained at *Portaria* N.º 33/DGAC. According to the *Portaria* N.º 440/SOP, the standard MTOW for aircraft classified in group I, for the charging of use of services supplied by the aeronautical infrastructure, is calculated based on the weighted average of the fleet of each company, by type of aircraft. The individual MTOW of each aircraft is considered for the purposes of charging landing and ground stay fees. This is justified by the fact that airline companies with a larger number of smaller sized aircrafts benefit from the value of these fees. In addition, it is because the standard MTOW does not make sense, as the discounts will be given by flight to the airline company. The values charged by category I airports are considered for the landing and ground stay fees.

The data for the ground stay of each aircraft after landing is also simulated through empirical observation.

The values of each variable for each DMU are found in Table 3. The value of the sum of the total fee referring to all the flights corresponds to R$ 4,457.27. It must be stressed that this value is apparently small. However, 36 flights is a number that only corresponds to a small period of time in the daily operation of a busy airport, like Guarulhos and Congonhas, in São Paulo, and Santos Dumont and Galeão, in Rio de Janeiro. On adding up the values throughout a period of 30 days and considering all the flights of a same company in major airports in the country, the value of the fees becomes quite significant. According to the values of the case study presented here, each airline company fictitiously pays the following values for the flights they operate: C1 = R$ 1,870.29; C2 = R$ 1,469.74; C3 = R$ 1,117.24.

Table 3 – Model data.

Company	Aircraft	DMUs	MTOW	Duration of ground stay (min)	Passengers transported	Total fee (R$)
C1	737-300	DMU 1	61	50	68	101.87
	737-800	DMU 2	77	40	119	128.59
	737-700	DMU 3	68	40	100	113.56
	737-500	DMU 4	52	240	87	155.48
	737-500	DMU 5	52	60	85	86.84
	737-500	DMU 6	52	70	61	86.84
	737-300	DMU 7	61	30	67	101.87
	737-300	DMU 8	61	40	58	101.87
	737-300	DMU 9	61	50	69	101.87
	737-500	DMU 10	52	40	67	86.84
	737-300	DMU 11	61	40	126	101.87
	767-300	DMU 12	181	300	138	600.92
	737-300	DMU 13	61	50	95	101.87
C2	F-100	DMU 14	44	40	68	73.48
	A-320	DMU 15	70	50	88	116.90
	A-319	DMU 16	64	240	105	191.36
	A-320	DMU 17	70	70	74	116.90
	A-320	DMU 18	70	50	125	116.90
	A-320	DMU 19	70	50	110	116.90
	A-320	DMU 20	70	40	131	116.90
	F-100	DMU 21	44	40	66	73.48
	A-319	DMU 22	64	30	101	106.88
	A-319	DMU 23	64	50	74	106.88
	F-100	DMU 24	44	40	70	73.48
	F-100	DMU 25	44	40	58	73.48
	A-320	DMU 26	70	180	96	186.20
C3	737-700	DMU 27	62	40	109	103.54
	737-800	DMU 28	62	240	125	185.38
	737-700	DMU 29	62	70	127	103.54
	737-700	DMU 30	62	40	124	103.54
	737-700	DMU 31	62	50	115	103.54
	737-700	DMU 32	62	30	43	103.54
	737-700	DMU 33	62	30	127	103.54
	737-800	DMU 34	62	50	146	103.54
	737-700	DMU 35	62	40	101	103.54
	737-700	DMU 36	62	30	108	103.54

4. Results and Discussion

The non-radial ZSG-DEA CCR model, oriented to outputs, with weights restrictions is calculated. It is desired that the output number of passengers transported has a greater weight than the output total fee. However, as the two variables are measured in different units, the previous statement, by itself, is meaningless (Allen *et al.*, 1997; Gomes *et al.*, 2009a). For it to make sense, it would be necessary to normalise the data, which would bring problems of interpreting the results of the redistribution of the total fee output. Thus, the option was made to perform a normalisation by the sum, incorporated not to the data, but to the weights restriction of these variables. In this way, the restriction added is $u_1 - 1{,}2991 u_2 \geq 0$. This value corresponds to the ratio between the sum of the values of the total fee of all the DMUs and the sum of the values of passengers transported by all of the DMUs.

The first step is to run the non-radial model with weights restrictions so as to obtain the efficient measurements of each DMU. To obtain the uniformised frontier, the DMUs targets for the variable of constant sum, total fee, are determined. The values of the efficiency measurements and the targets are found in Table 4.

It was confirmed that the DMUs 28, 33 and 34 are efficient by the non-radial model. The DMUs 16, 26 and 28 present high values of efficiency, even though they possess high ground stay times. This is because a large aircraft operates them. They also have a reasonably high rate of occupation and, consequently, they transported a number of passengers greater than the average. They also have an increased total fee value due to the ground stay fee, even with the use of weights restrictions. These factors mean that these flights have relatively high efficiency values.

After that, the proportional reduction strategy is applied to the target output and, in this way, new values for the total fee for each DMU are obtained, according to Table 4.

The next step is to run the model again with the new output values obtained. It was expected, according to the conjecture of Gomes (2003) for radial ZSG-DEA models with weights restrictions, to achieve uniformization, in other words, all the DMUs should be efficient. However, this was not observed. An average efficiency of 90.1% was obtained. As observed previously, in non-radial ZSG-DEA CCR models with weights restrictions, uniformization is not reached at first. It is necessary to perform repeated iterations, in order to calculate various redistributions of the output. In this way, the average efficiency improves at each iteration until uniformization is reached. The average efficiencies of the subsequent iterations were: 95.6%, 98.0%, 99.1%, 99.6% and 99.8%. These average efficiencies, which are increasing, suggest that the method converges. It can be said, therefore, that the uniformization of the frontier is reached according to this approximate method. The result of the fifth (and last) iteration is shown in Table 5. In this table, the values of the final efficiency measurements and the final values of the total fee output for each DMU are presented.

Table 4 – Values of each DMU efficiencies and their targets before and after the redistribution.

Company	DMUs	Non-radial efficiency	Target before redistribution	Target after redistribution
C1	DMU 1	0.5084	200.3737	138.7840
	DMU 2	0.7054	182.2937	126.2614
	DMU 3	0.6550	173.3740	120.0833
	DMU 4	0.8702	178.6716	123.7525
	DMU 5	0.6173	140.6771	97.4366
	DMU 6	0.4969	174.7635	121.0457
	DMU 7	0.5730	177.7836	123.1375
	DMU 8	0.5048	201.8027	139.7738
	DMU 9	0.5117	199.0815	137.8890
	DMU 10	0.5549	156.4967	108.3936
	DMU 11	0.8979	113.4536	78.5809
	DMU 12	0.8335	720.9598	499.3554
	DMU 13	0.6162	165.3197	114.5047
C2	DMU 14	0.6070	121.0544	83.8454
	DMU 15	0.5599	208.7873	144.6115
	DMU 16	0.8684	220.3593	152.6265
	DMU 17	0.4895	238.8151	165.4095
	DMU 18	0.7274	160.7094	111.3115
	DMU 19	0.6487	180.2066	124.8157
	DMU 20	0.8316	140.5724	97.3641
	DMU 21	0.5943	123.6413	85.6371
	DMU 22	0.7783	137.3249	95.1148
	DMU 23	0.5225	204.5550	141.6801
	DMU 24	0.6203	118.4588	82.0476
	DMU 25	0.5482	134.0387	92.8386
	DMU 26	0.7700	241.8182	167.4895
C3	DMU 27	0.7434	139.2790	96.4682
	DMU 28	1.0000	185.3800	128.3990
	DMU 29	0.7728	133.9803	92.7982
	DMU 30	0.8644	119.7825	82.9645
	DMU 31	0.7199	143.8255	99.6173
	DMU 32	0.4868	212.6952	147.3182
	DMU 33	1.0000	103.5400	71.7145
	DMU 34	1.0000	103.5400	71.7145
	DMU 35	0.6918	149.6675	103.6636
	DMU 36	0.8074	128.2388	88.8215
			Total	4,457.2700

Table 5 – Final result: efficiency and total fee.

Company	DMUs	Efficiency	Total fee
C1	DMU 1	0.9937	150.1055
	DMU 2	0.9958	123.1194
	DMU 3	0.9952	120.3897
	DMU 4	0.9980	109.5555
	DMU 5	0.9954	96.7640
	DMU 6	0.9943	127.2013
	DMU 7	0.9943	128.9365
	DMU 8	0.9937	151.5673
	DMU 9	0.9937	148.8821
	DMU 10	0.9941	114.5857
	DMU 11	0.9984	68.3838
	DMU 12	0.9937	539.7584
	DMU 13	0.9948	117.0767
C2	DMU 14	0.9949	85.5148
	DMU 15	0.9942	152.4620
	DMU 16	0.9977	136.5431
	DMU 17	0.9937	178.5468
	DMU 18	0.9961	107.2004
	DMU 19	0.9951	125.5497
	DMU 20	0.9974	88.2394
	DMU 21	0.9947	87.9614
	DMU 22	0.9964	90.2306
	DMU 23	0.9938	152.2733
	DMU 24	0.9950	83.0682
	DMU 25	0.9942	97.7476
	DMU 26	0.9958	163.4978
C3	DMU 27	0.9963	92.0654
	DMU 28	1.0000	104.6054
	DMU 29	0.9975	83.8245
	DMU 30	0.9979	73.7162
	DMU 31	0.9960	96.3470
	DMU 32	0.9935	161.1811
	DMU 33	1.0000	58.4251
	DMU 34	1.0000	58.4251
	DMU 35	0.9956	101.8517
	DMU 36	0.9971	81.6675
		Total fee paid	4,457.2700

Using this final result, a calculation is made of how much each airline company, fictitiously should pay after the redistribution of the total fees values in each flight: C1 = R$ 1,996.36; C2 = R$ 1,548.84; C3 = R$ 912.11.

It was confirmed that the total value to be paid by company C1 for the flights analysed here would be 6.7% greater. Company C2 would pay an amount approximately 5.4% greater and C3 would receive a discount in the total value paid for its flights of approximately 18.4%.

It was also noted that 18 flights would receive a discount and another 18 would have to pay a surcharge. The flights referring to the DMUs 8, 17 and 32 would receive a greater percentage increase in their fees: 49%, 53% and 56%, respectively. It was also observed that they are all flights that transport few passengers. However, the flights referring to the DMUs 28, 33 and 34 would receive a greater percentage discount in their fees: 44%. These are flights that transport a greater number of passengers.

Airline company C3 would receive the most discounts in its flights, 9 flights with discounts; company C2 received discounts in 5 flights, and C1 in 4 flights. It was also observed that there was an average occupation of 0.57 for the flights that are surcharge. For the 18 flights that would obtain discounts, the average occupation is 0.78.

5. Conclusions

This article dealt, solely in an empirical way, with an extension of the ZSG-DEA that considers simultaneous weights restrictions and non-controllable variables. This non-radial model does not permit the uniformization of the frontier by the method of clustering all the inefficient DMUs in a single group of cooperation. This shows that the extension of the Gomes (2003) conjecture is no longer valid for the present case. Nevertheless, previous numerical experiments, both with the conjecture and its extension for a non-radial model without weights restrictions, have been shown to be valid. Although it is still necessary to demonstrate the conjecture (which may, possibly, not be valid in the simple case for which it was stated), it is necessary to understand the reason why, when two simple situations are joined, it is no longer possible to use the target proportionality theorem and a single group of cooperation for the uniformization of the frontier. This study is left as a suggestion for future work.

Regarding the algorithm proposed here (successive iterations) there is also no guarantee that this will work in all cases. In the numerical experiments carried out, there was always apparent convergence, although it was not always fast. The properties of this algorithm, including its theoretical validation, should also be the subject of new research.

As regards the case study, the proposal to attribute discounts on the airline fees according to the number of passengers transported signifies a means by which the airport administrators encourage the airline companies to become more efficient, review their costs and, consequently, the price of the airline tickets. In this way, as well as encouraging competition between the airline companies, the concept presented here can result in a greater movement of passengers (and companions) in the airports which opt to use it. In turn, this greater movement will generate greater non-aeronautical revenue. For this to occur, it is necessary that the administrators of Brazilian airports fully realise the importance of this type of revenue as well as investment in shopping and leisure options, also in such a way as to attract diverse visitors. A small beginning of awareness of the non-aeronautical revenue fundamental importance can be notice by the increasing presence of *AeroShoppings* in the Brazilian Airport.

American and European airports, most of them privatised and with a more commercial philosophy, have noticed the importance of the ever increasing non-aeronautical revenues. Privatisation, in turn, requires regulation of prices. Discussion over single till or dual till divides the opinions of air transport specialists and researchers. See for example, Czerny (2006). The model proposed here presents hybrid characteristics of dual till and single till.

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