Pesquisa Operacional (2013) 33(3): 343-359

Printed version ISSN 0101-7438 / Online version ISSN 1678-5142
www.scielo.br/pope

DECOMPOSITION APPROACH FOR GENERATION AND TRANSMISSION EXPANSION PLANNING WITH IMPLICIT MULTIPLIERS EVALUATION

Fernanda S. Thomé[1], Silvio Binato[1], Mario V.F. Pereira[1],
Nora Campodónico[1], Marcia H.C. Fampa[2*] and Luiz Carlos da Costa Jr.[1]

Received June 11, 2012 / Accepted June 22, 2013

ABSTRACT. In an electric power systems planning framework, decomposition techniques are usually applied to separate investment and operation subproblems to take benefits from the use of independent solution algorithms. Real power systems planning problems can be rather complex and their detailed representation often leads to greater effort to solve the operation subproblems. Traditionally, the algorithms used in the solution of transmission constrained operation problems take great computational advantage with compact representation of the model, which means the elimination of some variables and constraints that don't affect the problem's optimal solution. This work presents a new methodology for solving generation and transmission expansion planning problems based on Benders decomposition where the incorporation of the traditional operation models require an additional procedure for evaluating the Lagrange's multipliers associated to the constraints which are not explicitly represented yet are used in the construction of the Benders cuts during the iterative process. The objective of this work is to seek for efficiency and consistency in the solution of expansion planning problems by allowing specialized algorithms to be applied in the operation model. It is shown that this methodology is particularly interesting when applied to stochastic hydrothermal problems which usually require a large number of problems to be solved. The results of this methodology are illustrated by a Colombian system case study.

Keywords: expansion planning, Benders decomposition, implicit Lagrange multipliers.

1 INTRODUCTION

The expansion planning process of generation-transmission systems is, in most of the cases, originated from the necessary changes in the electrical system in face of the growing energy demand, requiring new generators and transmission lines to be constructed in order to supply

*Corresponding author
[1]Power Systems Research, Praia de Botafogo, 228/1701-A, 22250-145 Rio de Janeiro, RJ, Brazil.
E-mails: {fernanda,silvio,mario,nora}@psr-inc.com/ luizcarlos@psr-inc.com
[2]Federal University of Rio de Janeiro, COPPE, C.P. 68530, 21945-970 Rio de Janeiro, RJ, Brazil.
E-mail: fampa@cos.ufrj.br

those needs. The definition of a consistent expansion planning policy is an essential requirement in order not to compromise the countries development.

In many countries the investments made in generation plants are often much greater than the investments in transmission equipments due to resources location and availability. In these countries an hierarchical procedure is usually adopted for the system expansion planning process because economically feasible results can be obtained from a more simplified solution method. This procedure consists in a two-step decision process: in the first step a decision is taken for the construction of new generators [20, 1] and in the second step transmission reinforcements are decided, given the generation decision in the first step [19, 9, 5].

In recent years, renewable sources, such as hydro, wind, solar, etc., have drawn the attention of energy policy makers because of their competitiveness and environmental characteristics. However, these resources are usually located far away from the load centres requiring more significant investments in new transmission equipments. In these systems, the hierarchical procedure becomes less attractive, since it disregards the trade off relationship between the generation and transmission expansion options. For this reason, economical aspects motivate the development of integrated generation-transmission expansion planning methodologies [7, 6, 13].

If we take into account that the system's operation problem is essentially stochastic, due to, for example, the generation unpredictability of hydro plants and renewable sources, and time coupled since the water reservoirs work as limited energy stocks and link the decision in the current stage with the future consequences of this decision, then the characteristics of the operation problem lead the expansion decision process to a complex large-scale problem to be solved.

In order to deal with the problem's complexity, mathematical decomposition techniques have been successfully adopted [4, 11]. Usually, these techniques divide the expansion problem into two separate problems: (1) determining the optimal investment in new plants and transmission lines, (2) determining the system's optimal operation scheduling associated to the construction of the new capacities.

The Benders decomposition technique [2] is particularly interesting when applied to the expansion planning problem because it allows the use of specialized algorithms to the solution of the complex operation subproblem, for example the Stochastic Dual Dynamic Programming (SDDP) methodology [12]. This is all thanks to the fact that investment and operation problems can be modeled independently, and coupled by investment decisions and associated operation total cost and cost sensibilities, the last one given by Lagrange multipliers of the operation problem.

A significant issue about using a specific model for solving the operation problem is that such sensibilities are not always explicitly available in the model. For example, in a purely operative problem one does not represent the candidate generators that do not have been decided for construction along the study horizon. Most importantly, by incorporating the transmission network in the model, efficient techniques are commonly combined for model compactness and flow limit constraints relaxation. This means that while the use of specialized models ensure greater efficiency and consistency for solution of expansion problems, the Lagrange multipliers associated to constraints which, however, are not explicitly represented, have to be inferred from the dual variables of the model [17]. This information is used in the construction of the Benders cuts which are incorporated into the investment model to obtain new expansion plans within the iterative solution procedure.

This paper presents a new integrated generation-transmission expansion planning methodology based on Benders Decomposition approach, whose innovation relies on the use of implicit Lagrange multipliers, allowing the use of customized and efficient algorithms. In the next sections the methodology is described first for a generation expansion problem and then extended to the integrated generation-transmission expansion problem. A case study with the Colombian system is presented for a comparison between the hierarchical and the integrated planning approaches, using the methodology proposed.

Without loss of generality, we assume that all generators and transmission circuits are candidates to expansion as the existent elements can be represented as candidate projects with fixed investment decision and zero investment cost. The main mathematical symbols used throughout this paper are listed (in alphabetical order) in Table 1.

2 GENERATION EXPANSION PLANNING

A linear model for a multistage generation expansion planning problem can be formulated as:

$$\text{Min} \quad \sum_{t\in T}\left[\sum_{i\in I}\left(b_g^i \cdot y_g^{t,i} + c^i \cdot g^{t,i}\right) + \sum_{j\in J} b_h^j \cdot y_h^{t,j}\right] \tag{1a}$$

$$\text{s/t} \quad \sum_{i\in I} g^{t,i} + \sum_{j\in J} \rho^j \cdot u^{t,j} = \sum_{n\in N} d^{t,n} \qquad \forall t \in T \tag{1b}$$

$$v^{t,j} - v^{t-1,j} + u^{t,j} + s^{t,j} = a^{t,j} \qquad \forall t \in T,\ j \in J \tag{1c}$$

$$u^{t,j} - \bar{u}^j \cdot x_h^{t,j} \leq 0 \qquad \forall t \in T,\ j \in J \tag{1d}$$

$$v^{t,j} - \bar{v}^j \cdot x_h^{t,j} \leq 0 \qquad \forall t \in T,\ j \in J \tag{1e}$$

$$g^{t,i} - \bar{g}^i \cdot x_g^{t,i} \leq 0 \qquad \forall t \in T, i \in I \tag{1f}$$

$$x_h^{t,j} = \sum_{\tau\leq t} y_h^{\tau,j} \qquad \forall t \in T,\ j \in J \tag{1g}$$

$$x_g^{t,i} = \sum_{\tau\leq t} y_g^{\tau,i} \qquad \forall t \in T, i \in I \tag{1h}$$

$$x_h^{t,j}, y_h^{t,j} \in \{0, 1\} \qquad \forall t \in T,\ j \in J \tag{1i}$$

$$x_g^{t,i}, y_g^{t,i} \in \{0, 1\} \qquad \forall t \in T, i \in I \tag{1j}$$

$$v^{t,j}, u^{t,j}, s^{t,j} \geq 0 \qquad \forall t \in T,\ j \in J \tag{1k}$$

$$g^{t,i} \geq 0 \qquad \forall t \in T, i \in I, \tag{1l}$$

Table 1 – Nomenclature.

b_f	Investment cost vector of circuits.
b_g	Investment cost vector of thermal plants.
b_h	Investment cost vector of hydro plants.
c	Production cost vector of thermal plants.
d	Demand vector of system buses.
a	Water inflow vector of hydro plants.
ρ	Production factor vector of hydro plants.
e	Unitary vector.
f	Flow vector of circuits.
$\bar{f}$	Flow capacity vector of circuits.
g	Production vector of thermal plants.
$\bar{g}$	Production capacity vector of thermal plants.
v	Stored volume vector of hydro plants.
$\bar{v}$	Storage capacity vector of hydro plants.
u	Turbined outflow vector of hydro plants.
$\bar{u}$	Turbining capacity vector of hydro plants.
s	Spilled outflow vector of hydro plants.
I	Set of thermal plants.
J	Set of hydro plants.
N	Set of power system buses.
K	Set of transmission circuits.
T	Set of time periods.
M	Disjunctive constant vector of circuits.
S	Bus-circuit incidence matrix.
y_f	Investment decision vector of circuits.
y_g	Investment decision vector of thermal plants.
y_h	Investment decision vector of hydro plants.
x_f	Accumulated investment decision vector of circuits.
x_g	Accumulated investment decision vector of thermal plants.
x_h	Accumulated investment decision vector of hydro plants.
w	Objective function value of operation problem.
α	Operation cost function.
β	Sensitivity matrix.
γ	Susceptance vector of circuits.
π	Lagrange multiplier vector.
θ	voltage angle vector of system buses.
$[\phi]$	Diagonal matrix of elements of generic vetor ϕ.

where the decision variables of the problem are the thermal generator's production g^t, hydro plant's stored volume v^t, turbining u^t and spillage s^t, and investment decisions y_g^t and y_h^t, the last ones consisting of binary variables such that for $y_g^{t,i} = 1$ the generator i is constructed in stage t, otherwise, for $y_g^{t,i} = 0$ the plant is not constructed in that stage. The auxiliary investment variables x_g^t and x_h^t represent the accumulated decision until stage t. The objective function minimizes the sum of the generators investment and operative costs, the first one given by construction cost of new capacities and the second one given essentially by thermal fuel cost and generation deficit cost.

Constraints (1b) represent the system's load supply while constraints (1c) represent hydro plant's water balance and constraints (1d)-(1f) correspond to the plants operative limits. Note that these capacity constraints reflect the investment decision in new generators once their production amount g^t or turbined outflow u^t must be null in stage t if their accumulated investment decision is not to be constructed until that stage $x_g^{t,i} = 0$ or $x_h^{t,j} = 0$.

The structure of expansion planning problems allows a two-stage decision process to be employed in the solution methodology. By applying the Benders decomposition scheme to solve problem (1), at each iteration the relaxed generation expansion problem produces a new investment plan which is fixed in the operation subproblem whose solution is used in the construction of a Benders cut to be added in the investment problem.

The operation problem, in particular, can in principle be solved by linear programming algorithms, however, the actual scheduling problem involves several hydro plants whose inflow uncertainties representation over time leads to an exponential increase of the problems dimension, resulting in a stochastic optimization problem that quickly becomes computationally intractable. This has motivated the development of solution approaches, such as the SDDP algorithm which is similar to a multi stage Benders decomposition approach that solves a set of single stage and single inflow scenario problems. For this reason, and for notation simplicity, we can use an one-stage deterministic problem formulation in order to illustrate the proposed methodology.

Let x_g^* and x_h^* be the expansion solution obtained from the relaxed investment problem, the following operation subproblem must be solved:

$$\text{Min} \quad w = c^{\mathsf{T}} g \tag{2a}$$

$$\text{s/t} \quad e^{\mathsf{T}} g + \rho^{\mathsf{T}} u = e^{\mathsf{T}} d \tag{2b}$$

$$v - u - s = v_0 + a \tag{2c}$$

$$u \leq [\bar{u}]\, x_h^* \tag{2d}$$

$$v \leq [\bar{v}]\, x_h^* \tag{2e}$$

$$g \leq [\bar{g}]\, x_g^* \tag{2f}$$

$$u, s, v, g \geq 0. \tag{2g}$$

By solving problem (2), the optimal value of the objective function w^* and the Lagrange multiplier vectors associated to the capacity constraints (2d), (2e) and (2f) are obtained and used in the evaluation of a Benders cut which is added to the investment problem. The Benders cut is obtained by the following linear constraint:

$$\alpha \geq w^* - \left(\pi_u^{*\mathsf{T}} [\bar{u}] + \pi_v^{*\mathsf{T}} [\bar{v}]\right) \left(x_h - x_h^*\right) - \pi_g^{*\mathsf{T}} [\bar{g}] \left(x_g - x_g^*\right),$$

where π_u^*, π_v^* and π_g^* are the vectors of Lagrange multipliers associated to constraints (2d), (2e) and (2f), respectively.

Although the formulation of problem (2) represents all generators independently of their associated investment decision, the variables g^i for all thermal plant i such that $x_g^{*i} = 0$ and the variables u^j, s^j and v^j for all hydro plant j such that $x_h^{*j} = 0$ are not actually represented in a purely operative problem. It means that specialized operative models don't represent these variables and their associated constraints in the formulation since it has no effect on the problem's optimal solution. However, in a expansion planning framework, the Lagrange multipliers associated to the removed constraints are not explicit known and need to be calculated in order to build the Benders cuts for the investment problem.

In the case of thermal generators, the Lagrange multipliers associated to constraints (2f) are trivially calculated according to the procedure shown as follows. In the case of the hydro generators, however, due to the hydro balance constraints which establishes decisions temporal coupling, it is easier to keep the hydro balance constraints for the non constructed plants and represent them as run-of-the-river plants with null turbining capacity, and the multipliers are directly obtained from the problem's solution. Although the evaluation of the implicit multipliers for the hydro plants could be done with some extra mathematical effort, this work wants to emphasize that the implicit multipliers evaluation procedure receives most significant recognition when transmission network is incorporated in the expansion problem, which will be seen in the next section.

From linear programming theory, in the optimal solution, the values of the Lagrange multipliers of a primal problem are equal to the values of the variables of the dual problem. The dual formulation of problem (2) is:

$$\text{Max} \quad w' = d^{\mathsf{T}} e \pi_\lambda + x_h^{*\mathsf{T}} [\bar{u}] \pi_u + x_h^{*\mathsf{T}} [\bar{v}] \pi_v + x_g^{*\mathsf{T}} [\bar{g}] \pi_g + (v_0 + a)^{\mathsf{T}} \pi_h \tag{3a}$$

$$\text{s/t} \quad \pi_\lambda e + \pi_g \leq c \tag{3b}$$

$$\pi_\lambda \rho - \pi_h + \pi_u \leq 0 \tag{3c}$$

$$\pi_h + \pi_v \leq 0 \tag{3d}$$

$$\pi_\lambda \text{ free}, \quad \pi_h \geq 0 \tag{3e}$$

$$\pi_u, \pi_v, \pi_g \leq 0. \tag{3f}$$

According to constraint (3b), since it's a maximization problem and $\pi_g \leq 0$, in the optimal solution we have:

$$\pi_g^i = \min\left\{0, c^i - \pi_\lambda\right\}, \quad \forall i \in I.$$

From this result, the implicit Lagrange multipliers $\{\pi_g^i, \forall i \in I | x_g^i = 0\}$ are obtained so that a Benders cut can be calculated and added to the investment problem to be solved in the following iteration of the solution method.

3 GENERATION-TRANSMISSION EXPANSION PLANNING

Now considering the transmission network system, the expansion planning problem is traditionally formulated according to a linearized power flow model [14, 15], governed by Kirchhoff's first and second laws. The problem shown next is the integrated generation-transmission expansion problem which is the same used in the second step of the hierarchical planning approach for transmission reinforcement expansion, fixing all generation investment decision accordingly to the solution of the generation expansion problem.

$$\text{Min} \quad \sum_{t\in T}\left[\sum_{i\in I}\left(b_g^i \cdot y_g^{t,i} + c^i \cdot g^{t,i}\right) + \sum_{j\in J} b_h^j \cdot y_h^{t,j} + \sum_{k\in K} b_f^k \cdot y_f^{t,k}\right] \tag{4a}$$

$$\text{s/t} \quad \sum_{k\in K_n} f^{t,k} + \sum_{i\in I_n} g^{t,i} + \sum_{j\in J_n} \rho^j \cdot u^{t,j} = d^{t,n} \qquad \forall t\in T, n\in N \tag{4b}$$

$$f^{t,k} - \gamma^k\left(\theta^{t,from(k)} - \theta^{t,to(k)}\right) \le M^k\left(1 - x_f^{t,k}\right) \qquad \forall t\in T, k\in K \tag{4c}$$

$$f^{t,k} - \gamma^k\left(\theta^{t,from(k)} - \theta^{t,to(k)}\right) \ge -M^k\left(1 - x_f^{t,k}\right) \qquad \forall t\in T, k\in K \tag{4d}$$

$$v^{t,j} - v^{t-1,j} + u^{t,j} + s^{t,j} = a^{t,j} \qquad \forall t\in T, j\in J \tag{4e}$$

$$u^{t,j} - \bar{u}^j \cdot x_h^{t,j} \le 0 \qquad \forall t\in T, j\in J \tag{4f}$$

$$v^{t,j} - \bar{v}^j \cdot x_h^{t,j} \le 0 \qquad \forall t\in T, j\in J \tag{4g}$$

$$g^{t,i} - \bar{g}^i \cdot x_g^{t,i} \le 0 \qquad \forall t\in T, i\in I \tag{4h}$$

$$f^{t,k} - \bar{f}^k \cdot x_f^{t,k} \le 0 \qquad \forall t\in T, k\in K \tag{4i}$$

$$f^{t,k} + \bar{f}^k \cdot x_f^{t,k} \ge 0 \qquad \forall t\in T, k\in K \tag{4j}$$

$$x_h^{t,j} = \sum_{\tau\le t} y_h^{\tau,j} \qquad \forall t\in T, j\in J \tag{4k}$$

$$x_g^{t,i} = \sum_{\tau\le t} y_g^{\tau,i} \qquad \forall t\in T, i\in I \tag{4l}$$

$$x_f^{t,k} = \sum_{\tau\le t} y_f^{\tau,i} \qquad \forall t\in T, k\in K \tag{4m}$$

$$x_h^{t,j}, y_h^{t,j} \in \{0,1\} \qquad \forall t\in T, j\in J \tag{4n}$$

$$x_g^{t,i}, y_g^{t,i} \in \{0,1\} \qquad \forall t\in T, i\in I \tag{4o}$$

$$x_f^{t,k}, y_f^{t,k} \in \{0,1\} \qquad \forall t\in T, k\in K \tag{4p}$$

$$v^{t,j}, u^{t,j}, s^{t,j} \ge 0 \qquad \forall t\in T, j\in J \tag{4q}$$

$$g^{t,i} \ge 0 \qquad \forall t\in T, i\in I, \tag{4r}$$

where $f^{t,k}$, $\theta^{t,n}$ and $y_f^{t,k}$ are the additional decision variables of the problem, and auxiliary variable $x_f^{t,k}$ is the accumulated investment decision over circuits until stage t. The objective function minimizes the sum of generation and transmission investment costs and thermal operational cost. Constraints (4b) represent the load balance for each bus (Kirchhoff's first law) and constraints (4i)-(4j) correspond to the circuits operative capacity limit.

Disjunctive constraints (4c)-(4d) were proposed by [8, 18] and later by [10] and represents the Kirchhoff's second law equations. The vector M of disjunctive constants allows the relaxation of the these constraints when $x_f^k = 0$, that means circuit k is not constructed, and imposes the Kirchhoff's second law when the circuit k is constructed or $x_f^k = 1$. The problem is that using high values for the disjunctive constants M can lead to an ill-conditioned problem which results in bad convergence process when solving the problem. In order to avoid that poor numerical conditioning in the problem, the evaluation of the minimum values of the disjunctive constants M was applied [3].

By applying Benders decomposition scheme to this problem, and once again reducing the problem to an one-stage deterministic formulation, the following operation subproblem must be solved for an expansion solution $\{x_g^*, x_h^*, x_f^*\}$ obtained from a relaxed investment problem:

$$\text{Min} \quad w = c^{\mathsf{T}} g \tag{5a}$$

$$\text{s/t} \quad Sf + e^{\mathsf{T}} [\rho] u + g = d \tag{5b}$$

$$f - [\gamma] S^{\mathsf{T}} \theta \le [M] \left(e - x_f^*\right) \tag{5c}$$

$$f - [\gamma] S^{\mathsf{T}} \theta \ge -[M] \left(e - x_f^*\right) \tag{5d}$$

$$v - u - s = v_0 + a \tag{5e}$$

$$u \le [\bar{u}] x_h^* \tag{5f}$$

$$v \le [\bar{v}] x_h^* \tag{5g}$$

$$g \le [\bar{g}] x_g^* \tag{5h}$$

$$f \le [\bar{f}] x_f^* \tag{5i}$$

$$f \ge -[\bar{f}] x_f^* \tag{5j}$$

$$u, s, v, g \ge 0. \tag{5k}$$

Considering the optimal solution of problem (5), let $\pi_{\gamma+}^*$, $\pi_{\gamma-}^*$, π_{f+}^* and π_{f-}^* be the values of the vectors of Lagrange multipliers associated to constraints (5c)-(5d) and (5i)-(5j), respectively, and define $\pi_\gamma^* = \pi_{\gamma+}^* + \pi_{\gamma-}^*$ and $\pi_f^* = \pi_{f+}^* + \pi_{f-}^*$. Since, for each circuit, at most one of the multipliers $\pi_{\gamma+}^*$ and $\pi_{\gamma-}^*$ will be nonzero, as well as multipliers π_{f+}^* and π_{f-}^*, then the Benders cut can expressed by:

$$\alpha \ge w^* - \left(\pi_u^{*\mathsf{T}} [\bar{u}] + \pi_v^{*\mathsf{T}} [\bar{v}]\right) (x_h - x_h^*) - \pi_g^{*\mathsf{T}} [\bar{g}] \left(x_g - x_g^*\right)$$
$$- \left(-\pi_\gamma^{*\mathsf{T}} [M] + \pi_f^{*\mathsf{T}} \left[\bar{f}\right]\right) \left(x_f - x_f^*\right).$$

Although this disjunctive expansion planning problem can be solved by a standard solution algorithm, such as Branch-and-Bound, this formulation uses variables f and θ for the power flow model, according to constraints (5b)-(5d). For existing circuits, we have:

$$Sf + e^{\top}[\rho]u + g = d \tag{6}$$

$$f = [\gamma]S^{\top}\theta. \tag{7}$$

Replacing variable f from (7) in equation (6), we obtain:

$$B\theta + e^{\top}[\rho]u + g = d,$$

where $B = S\gamma S^{\top}$ is the susceptance matrix. The solution of this linear system on variables θ allows the circuits power flow to be expressed as:

$$f = \beta(d - g - e^{\top}[\rho]u),$$

where $\beta = \gamma S^{\top}B^{-1}$ is the sensibility matrix and, since B is not a full rank matrix, one bus is selected as an angle reference bus by fixing null value to its corresponding row/column in matrix B^{-1}. The generation of the reference bus is then obtained from the total load balance:

$$e^{\top}g + \rho^{\top}u = e^{\top}d.$$

This way, the formulation shown next is equivalent to problem (5) and is derived from this algebraic handling of the problem equations, resulting in compact formulation of the problem, where circuits flows are expressed as function of variables g and u.

$$\text{Min} \quad w = c^{\top}g \tag{8a}$$

$$\text{s/t} \quad e^{\top}g + \rho^{\top}u = e^{\top}d \tag{8b}$$

$$-\bar{f} \le \beta\left(d - g - e^{\top}[\rho]u\right) \le \bar{f} \tag{8c}$$

$$v - u - s = v_0 + a \tag{8d}$$

$$u \le [\bar{u}]x_h^* \tag{8e}$$

$$v \le [\bar{v}]x_h^* \tag{8f}$$

$$g \le [\bar{g}]x_g^* \tag{8g}$$

$$u, s, v, g \ge 0. \tag{8h}$$

where the circuits capacity constraints (8c) can be represented only for the circuits k whose investment decision is to be constructed, $x_f^{*k} = 1$.

This formulation has a compact transmission network representation which allows the use of a special algorithm for solving the operation subproblem, involving a relaxation scheme of the circuits capacity constraints [16]. First the operation problem is solved without any transmission constraints (8c), then a simple power flow evaluation indicates the overloaded circuits and for each one of them a limit constraint is then added to the operation problem for a new solution. The iterative process stops when no overloaded circuits are found in the system which, in most cases, occur in a few iterations with a few circuits being added to the relaxed problem. This scheme leads to computational effort decrease and less stored data volume during the solution process, which becomes especially interesting when a great number of problems must be solved such as in the case of the SDDP algorithm applied to the stochastic operation problem.

Nevertheless, besides allowing the elimination of generation variables associated to investment decision $x_g^{*i} = 0$, the compact formulation of the problem doesn't explicitly represent the Kirchhoff's second law constraints and compulsorily eliminates the circuit capacity constraints associated to investment decision $x_f^{*k} = 0$.

For this reason, for the Benders cut coefficients evaluation, the Lagrange multipliers associated to the removed constraints $\{\pi_g^i, \forall i \in N | x_g^i = 0\}$, $\{\pi_\gamma^k, \forall k \in K\}$ and $\{\pi_f^k, \forall k \in K | x_f^k = 0\}$ are not explicit known and need to be calculated as follows.

The dual formulation of problem (5) is:

$$
\begin{aligned}
\text{Max} \quad & w' = d^\top \pi_d + x_h^{*\top} [\bar{u}] \pi_u + x_h^{*\top} [\bar{v}] \pi_v + x_g^{*\top} [\bar{g}] \pi_g \\
& \qquad + \left(e - x_f^*\right)^\top [M] \pi_\gamma + x_f^{*\top} [\bar{f}] \pi_f + (v_0 + a)^\top \pi_h && \text{(9a)} \\
\text{s/t} \quad & S^\top \pi_d + \pi_{\gamma_+} - \pi_{\gamma_-} + \pi_{f_+} - \pi_{f_-} = 0 && \text{(9b)} \\
& \pi_d + \pi_g \le c && \text{(9c)} \\
& [\rho] \pi_d - \pi_h + \pi_u \le 0 && \text{(9d)} \\
& \pi_h + \pi_v \le 0 && \text{(9e)} \\
& \pi_d \text{ free}, \quad \pi_h \ge 0 && \text{(9f)} \\
& \pi_u, \pi_v, \pi_g, \pi_{\gamma_+}, \pi_{\gamma_-}, \pi_{f_+}, \pi_{f_-} \le 0, && \text{(9g)}
\end{aligned}
$$

and the dual formulation of problem (8) is:

$$
\begin{aligned}
\text{Max} \quad & w' = e^\top d \pi_\lambda + x_h^{*\top} [\bar{u}] \pi_u + x_h^{*\top} [\bar{v}] \pi_v + x_g^{*\top} [\bar{g}] \pi_g \\
& \qquad + x_f^{*\top} \left[\bar{f}\right] \pi'_f - d^\top \beta^\top \pi'_{f_+} + d^\top \beta^\top \pi'_{f_-} + (v_0 + a)^\top \pi_h && \text{(10a)} \\
\text{s/t} \quad & \pi_\lambda e + \pi_g - \beta^\top \pi'_{f_+} + \beta^\top \pi'_{f_-} \le c && \text{(10b)} \\
& \pi_\lambda \rho - [\rho]^\top e \beta^\top \pi'_{f_+} + [\rho]^\top e \beta^\top \pi'_{f_-} - \pi_h + \pi_u \le 0 && \text{(10c)} \\
& \pi_h + \pi_v \le 0 && \text{(10d)} \\
& \pi_\lambda \text{ free}, \quad \pi_h \ge 0 && \text{(10e)} \\
& \pi_u, \pi_v, \pi_g, \pi'_{f_+}, \pi'_{f_-} \le 0. && \text{(10f)}
\end{aligned}
$$

Since the constraints (8c) are only available for circuits k whose investment decision is $x_f^k = 1$ then their associated Lagrange multiplier vectors are $\pi'_{f_{+,-}} = \{\pi_{f_{+,-}}^k, \forall k \in K | x_f^k = 1\}$.

According to constraints (9c) and (10b), since the problems (9) and (10) maximize the objective function and $\pi_g \leq 0$ then, in the optimum solution we have:

$$\pi_g = \min\{0, c - \pi_d\} \tag{11a}$$

$$\pi_g = \min\{0, c - \pi_\lambda e + \beta^\top \pi'_f\}. \tag{11b}$$

From equations (11a) and (11b):

$$c - \pi_d = c - \pi_\lambda e + \beta^\top \pi'_f,$$

re-arranging the terms of this equation:

$$\pi_d = \pi_\lambda e - \beta^\top \pi'_f,$$

therefore:

$$\pi_d^i = \pi_\lambda - \sum_{k \in K | x_f^k = 1} \beta^{k,i} \pi_f'^k, \quad \forall i \in N.$$

From constraint (9b):

$$\pi_f = -S^\top \pi_d - \pi_\gamma.$$

Let from(k) and to(k) be the terminal buses of circuit k, since $\pi_\gamma^k = 0, \forall k \in K | x_f^k = 0$ because the associated constraints (5c)-(5d) are not active in the optimum solution, then:

$$\pi_f^k = \pi_d^{\text{from}(k)} - \pi_d^{\text{to}(k)}, \quad \forall k \in K | x_f^k = 0.$$

Finally, from constraint (9b) we have:

$$\pi_\gamma = -S^\top \pi_d - \pi_f,$$

therefore:

$$\begin{aligned} \pi_\gamma^k &= 0, && \forall k \in K | x_f^k = 0 \\ \pi_\gamma^k &= \pi_d^{\text{from}(k)} - \pi_d^{\text{to}(k)} - \pi_f'^k, && \forall k \in K | x_f^k = 1 \end{aligned}$$

The equations obtained for the evaluation of the implicit Lagrange multipliers $\{\pi_g^i, \forall i \in N | x_g^i = 0\}$, $\{\pi_\gamma^k, \forall k \in K\}$ and $\{\pi_f^k, \forall k \in K | x_f^k = 0\}$ are used in the calculation of the coefficients of a Benders cut which is incorporated in the integrated expansion problem according to the solution algorithm illustrated in Figure 1.

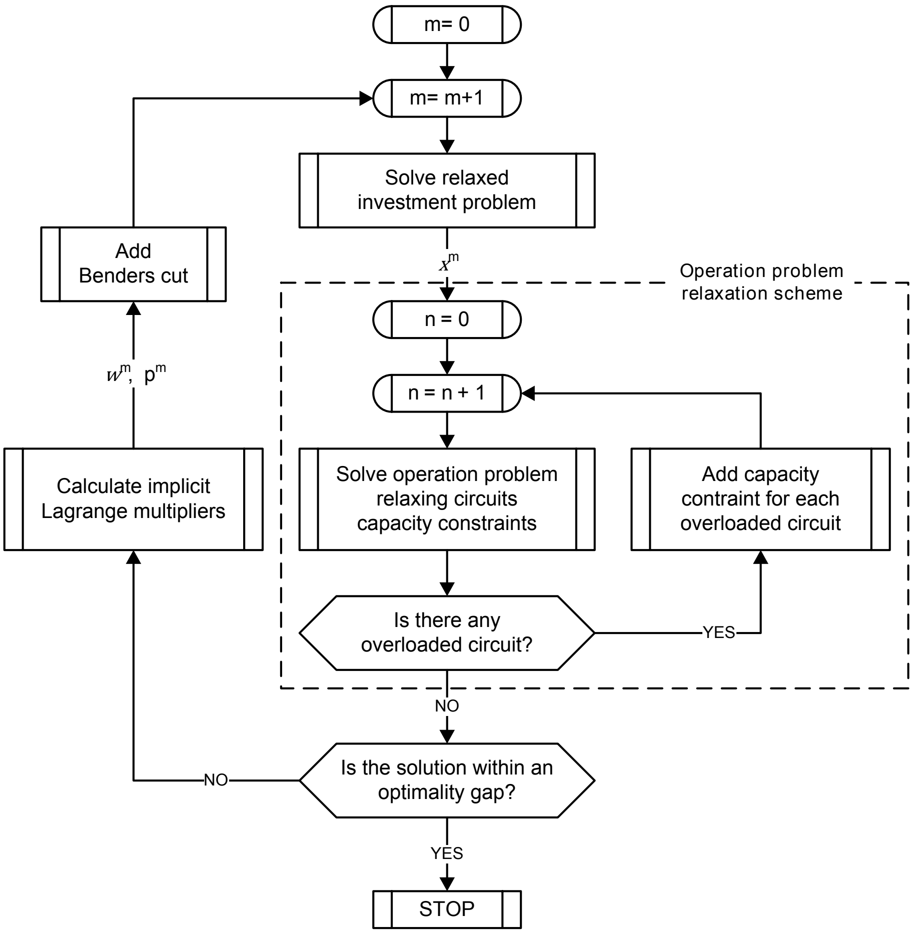

Figure 1 – Expansion problem solution algorithm.

4 Case Study

The proposed methodology was applied to the Colombian power system, which, in January 2010, consisted of 58 hydro plants and 56 thermal plants. The total thermal and hydro installed capacity was 4902 MW and 9263 MW, respectively. A six-year horizon 2010-2015 was considered for the expansion and four additional years 2016-2019 were represented in order to diminish the end-of-horizon effect. Maximum total load considered for the study period is shown in Figure 2.

This study represented the transmission network of 115-500 kV, containing 107 system buses and 219 circuits. The generation and transmission candidates considered for the system expansion were composed by 22 thermal plants with total installed capacity of 3837 MW, 5 hydro plants with 2512 MW and 25 transmission circuits.

The algorithm described in the last section was used in the analysis of two cases, one for the hierarchical and the second one for the integrated planning procedure. Both approaches were

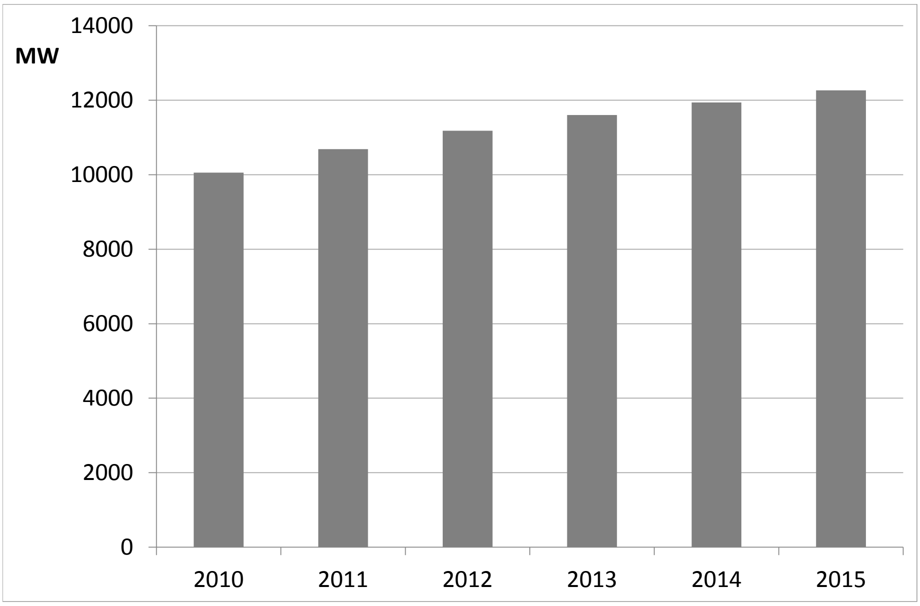

Figure 2 – Colombian system peak load.

solved by Xpress optimization package version 19.00.17 from FICO and under the same conditions, using an Intel Quad Core 2.4GHz processor 8Gb of RAM in Windows 7 operating system. Also, both solutions were obtained for a convergence gap lower than 3%.

In the first case, the methodology was applied in two steps, first to the generation expansion problem and then applied to transmission reinforcement expansion problem. Figure 3 illustrates the expansion amount in generation capacity.

Now considering an integrated procedure, the proposed methodology took into account the generation and transmission candidates in the same expansion problem and the generation capacity expansion is shown in Figure 4.

Figures 3 and 4 show that the hierarchical planning procedure leads to a higher hydro expansion, what was expected once the generation expansion decisions were taken without accounting for the plants physical location and, therefore, also leading to higher transmission reinforcement costs. This solution shows that the integrated planning procedure may completely change the expansion planning profile by simultaneously optimizing the generation and transmission resources. In Table 2, we observe that the total cost associated to the integrated planning solution is 180M$ lower, approximately 5%. Therefore, by considering the trade-off relationship between investment and operation costs, the integrated expansion planning methodology reaches the most economic solution among the possible system expansion alternatives.

Tables 3 and 4 contains the convergence results of each planning approach, where we find a highly relevant comparison between computational time spent with the compact model versus the disjunctive model traditionally used in the transmission expansion planning studies. The incorporation of the compact representation of the transmission network in an expansion problem,

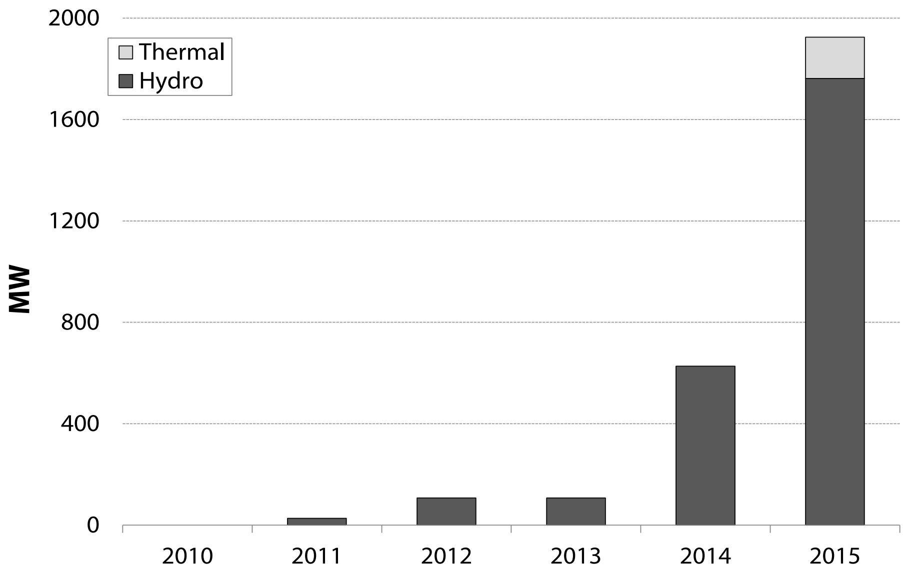

Figure 3 – Hierarchical planning – generation additional capacity.

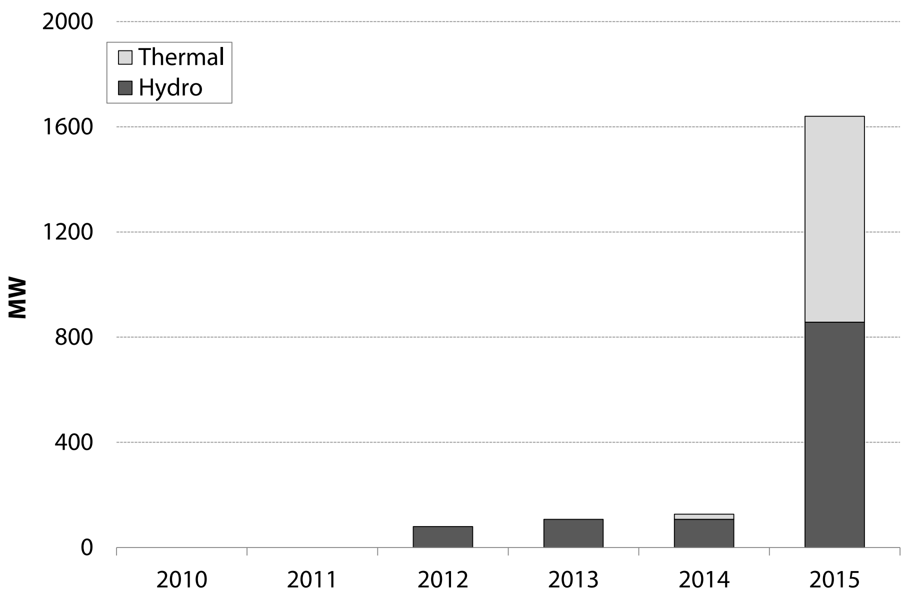

Figure 4 – Integrated planning – generation additional capacity.

made possible by the implicit multipliers evaluation methodology, has shown gains of approximately 8 times in speed-up time, which means a reduction of near 87% in CPU time for both hierarchical and integrated planning approaches.

Table 2 – Planning costs.

Costs (M$)	Hierarchical	Integrated
Investment	**889**	**588**
Generation	812	523
Transmission	77	65
Operation	**2881**	**3002**
Total	**3770**	**3590**

Table 3 – Convergence results.

Planning	# Iterations
Hierarchical	
Generation	9
Transmission	6
Integrated	
Gen.+Trans.	212

Table 4 – Execution time results.

Planning	CPU Time (min)
Hierarchical	
Generation	2
Trans.(compact)	61
Trans.(disjunct.)	478
Integrated	
Gen.+Trans.(compact)	1840
Gen.+Trans.(disjunct.)	14318

5 CONCLUSIONS

The decomposition techniques applied to the electrical systems expansion planning problem into an operation and an investment subproblems allow the use of specialized algorithms to solve each problem. In this sense, the adoption of the most traditional and well-consolidated operation models is intuitively adequate in order to ensure coherency and efficiency in the solution of expansion planning problems, otherwise the results of an expansion planning study could be very distorted. In this context, it's significantly important to remind that, in a purely operative problem, the representation of the non-constructed elements of the system is essentially unnecessary, more over, the use of compact model for transmission network representation make crucial the evaluation of the Lagrange multipliers which are associated to the constraints that are not explicit represented in order to obtain the Benders cut coefficients that must be added to the investment problem.

In addition, when dealing with hydrothermal operation, the stochastic dual dynamic programming (SDDP) algorithm is known as the state-of-the art in solving this type of problems. Even though this iterative algorithm can handle large scale systems by efficiently reducing the number of problems to be solved, this number can still be very large. To be more illustrative, assuming an expansion planning study for mid to long term horizon considering hydrological uncertainties, it would mean that, in each iteration of the expansion model, the number of optimization problems to be solved by the model could reach tens of millions, therefore, it's worth mentioning how significant can be the speed-up obtained by the proposed methodology in the solution of real hydrothermal problems.

From another aspect, as it was seen, the hierarchical planning procedure consists in a decision process whose transmission reinforcement decisions are taken in order to accommodate the generation investment decisions which were made without considering transmission network congestion. For this reason, by the Colombian case study results, we can conclude that this procedure leads to a higher total cost if compared to the integrated planning procedure because it does not take into account the jointly optimization of generation and transmission expansion alternatives. So finally, the proposed methodology, based on the Benders decomposition approach allied to the compact network model, aims to make coherent and computationally feasible and efficient the solution of large-scale integrated generation-transmission expansion planning problems.

ACKNOWLEDGEMENTS

This work was partially funded by CNPq – Conselho Nacional de Desenvolvimento Científico e Tecnológico.

REFERENCES

[1] ALBUQUERQUE LL, ALMEIDA AT & CAVALCANTE CAV. 2009. Aplicabilidade da programação matemática multiobjetivo no planejamento da expansão de longo prazo da geração no Brasil. *Pesquisa Operacional*, **29**(1): 153–177.

[2] BENDERS JF. 1962. Partitioning methods for solving mixed variables programming problems. *Numerische Mathematik*, **4**: 238–252.

[3] BINATO S, PEREIRA MVF & GRANVILLE S. 2001. A new Benders decomposition approach to solve power transmission network design problems. *IEEE Transactions on Power Systems*, **16**(2): 235–240.

[4] BLOOM J. 1983. Solving an Electricity Generating Capacity Expansion Planning Problem by Generalized Benders Decomposition. *Operations Research*, **31**(1): 84–100.

[5] BUYGI MO, SHANECHI HM, BALZER G, SHAHIDEHPOUR M & PARIZ N. 2006. Network Planning in Unbundled Power Systems. *IEEE Transactions on Power Apparatus and Systems*, **21**(3): 1379–1387.

[6] ENAMORADO JC, GOMEZ T & RAMOS A. 1999. Multi-Area Regional Interconnection Planning Under Uncertainty. *Proceedings of the 13th Power System Computing Conference*, Trondhein, Jun-Jul 1999.

[7] GORENSTIN BG, CAMPODÓNICO NM, DA COSTA JP & PEREIRA MVF. 1993. Power System Expansion Planning Under Uncertainty. *IEEE Transactions on Power Apparatus and Systems*, **8**(1): 129–136.

[8] GRANVILLE S & PEREIRA MVF. 1985. Analisys of the linearized power flow model in Benders decomposition. System Optimization Lab, Dept. of Operations Research, Stanford University, SOL 85-04.

[9] LATORRE G, CRUZ RD & ALEIZA JM. 2003. Transmission Expansion Planning: A Classification of Publications and Models. *IEEE Transactions on Power Apparatus and Systems*, **18**(1): 938–946.

[10] OLIVEIRA G, BAHIENSE L & PEREIRA MVF. 2003. Modelo disjuntivo de expansão ótima de redes de transmissão em sistemas hidrotérmicos. *Pesquisa Operacional*, **23**(1): 129–140.

[11] PEREIRA MVF, CAMPODÓNICO NM, GORENSTIN BG & DA COSTA JP. 1995. Application of Stochastic Optimization to Power System Planning and Operation. *IEEE Stockholm Power Tech*, IEEE, Stockholm, Sweden, Jun (1995).

[12] PEREIRA MVF & PINTO LMVG. 1991. Multi-Stage Stochastic Optimization Applied to Energy Planning. *Mathematical Programming*, **52**(1-3): 359–375.

[13] ROH JH, SHAHIDEHPOUR M & FU Y. 2007. Market-Based Coordination of Transmission and Generation Capacity Planning. *IEEE Transactions on Power Apparatus and Systems*, **22**(4): 1406–1419.

[14] STOTT BAO. 1974. Fast Decoupled Load Flow. *IEEE Transactions on Power Apparatus and Systems*, **93**: 859–869.

[15] STOTT BAO. 1974. Review of Load Flow Calculation Methods. *Proceedings of the IEEE*, **62**(7): 916–929.

[16] STOTT B & MARINHO JL. 1979. Linear programming for power system network security applications. *IEEE Transactions on Power Apparatus and Systems*, PAS-99.

[17] THOMÉ FS. 2008. Application of Decomposition Technique with Evaluation of Implicit Multipliers in Electrical Systems Generation and Network Expansion Planning. Dissertação de Mestrado, COPPE/UFRJ, Rio de Janeiro, Brazil (in portuguese).

[18] VILLASANA R. 1984. Transmission network planning using linear and linear mixed integer programming. Ressenlaer Polytechnic Institute, Ph.D. Thesis.

[19] YEHIA M, CHEDID R, ILIC M, ZOBIAN A, TABORS R & LACALLE-MELERO J. 1995. A Global Planning Methodology for Uncertain Environments: Application to the Lebanese Power System. *IEEE Transactions onPower Apparatus and Systems*, **10**(1): 332–338.

[20] ZAMBON KL, CARNEIRO AM, SILVA ANR & NEGRI JC. 2005. Análise de decisão multicritério na localização de usinas termoelétricas utilizando SIG. *Pesquisa Operacional*, **25**(2): 183–199.