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P.C.J. Sarmiento
https://orcid.org/0000-0002-9711-1561
R. Takata
https://orcid.org/0000-0003-3385-6895
T.M. Freitas
https://orcid.org/0000-0002-4329-7401
M.M.B. Pereira
https://orcid.org/0000-0001-6780-2073
L.S. Freire
https://orcid.org/0000-0003-1442-5539
V. Lugert
https://orcid.org/0000-0001-7173-080X
C. Sarturi
https://orcid.org/0000-0001-5600-4834
M.M. Pereira
https://orcid.org/0000-0002-1898-2722

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Nonlinear regression analysis of length growth in cultured rainbow trout

[*Análise não linear do crescimento em comprimento de truta-arco-íris de aquicultura*]

P.C. *Janampa-Sarmiento*[1], R. *Takata*[2], T.M. *Freitas*[2], M.M.B. *Pereira*[2], L. Sá-*Freire*[2],
V. *Lugert*[4,5], C. *Sarturi*[1], M.M. *Pereira*[1,3]

[1]Aluno de pós-graduação - Instituto de Zootecnia - Universidade Federal Rural do Rio de Janeiro - Seropédica, RJ
[2]Fundação Instituto da Pesca do Estado do Rio de Janeiro - Escritório Regional Serrana - Nova Friburgo, RJ
[3]Fundação Instituto da Pesca do Estado do Rio de Janeiro - Centro de Treinamento
de Aquicultura - Rio das Flores, RJ
[4]Alfred Wegener Institute - Helmholtz Centre for Polar and Marine Research - Bremerhaven, Germany
[5]Thünen Institute of Fisheries Ecology - Herwigstraße - Bremerhaven, Germany

ABSTRACT

Length growth as a function of time has a non-linear relationship, so nonlinear equations are recommended to represent this kind of curve. We used six nonlinear models to calculate the length gain of rainbow trout (*Oncorhynchus mykiss*) during the final grow-out phase of 98 days under three different feed types in triplicate groups. We fitted the von Bertalanffy, Gompertz, Logistic, Brody, Power Function, and Exponential equations to individual length-at-age data of 900 fish. Equations were fitted to the data based on the least square method using the Marquardt iterative algorithm. Accuracy of the fitted models was evaluated using a model performance metrics combining mean squared residuals (MSR), mean absolute error (MAE) and Akaike's Information Criterion corrected for small sample sizes (AICc). All models converged in all cases tested. Evaluation criteria for the Logistic model indicated the best overall fit (0.67 of combined metric MSR, MAE and AICc) under all different feeding types, followed by the Exponential model (0.185), and the von Bertalanffy and Brody model (0.074, respectively). Additionally, ΔAICc results identify the Logistic and Gompertz models as being substantially supported by the data in 100% of cases. The logistic model can be suggested for length growth prediction in aquaculture of rainbow trout.

Keywords: aquaculture, logistic model, *Oncorhynchus mykiss*, non-linear equations

RESUMO

O crescimento em comprimento em função do tempo tem uma relação não linear; por isso, funções não lineares são recomendáveis para descrever essa relação. Seis modelos não lineares foram usados para calcular o ganho em comprimento de truta-arco-íris (Oncorhynchus mykiss) *durante 98 dias, na fase final da engorda, submetidas a três dietas diferentes em grupos triplicados. Foram ajustadas as equações de von Bertalanffy, Gompertz, logístico, Brody, função potencial e exponencial a dados individuais de comprimento-idade de 900 peixes. O ajuste foi feito pelo método dos mínimos quadrados, usando-se o algoritmo iterativo de Marquardt. A precisão do ajuste foi avaliada pelo uso de critérios combinados de ajuste: quadrado médio do resíduo (QMR), erro médio absoluto (EMA) e o critério de informação de Akaike corrigido para tamanhos amostrais pequenos (AICc). Todos os modelos atingiram a convergência para cada caso avaliado. Os critérios de avaliação do modelo logístico indicaram o melhor ajuste geral (0,67 vez dos critérios combinados MSR, MAE e AICc) para cada grupo de peixe avaliado, seguido pelo modelo exponencial (0,185) e os modelos von Bertalanffy e Brody, com 0,074, respectivamente. Similarmente, os resultados de ΔAICc identificaram-se ao modelo logístico e ao de Gompertz, com grande suporte das informações em 100% dos casos. Por fim, o modelo logístico pode ser sugerido na predição do crescimento em comprimento de truta-arco-íris cultivada.*

Palavras-chave: aquicultura, modelo logístico, Oncorhynchus mykiss, *equações não lineares*

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E-mail: peterjs_0126@hotmail.com

INTRODUCTION

Mathematical modeling is defined as the use of equations to describe or simulate processes in a system, such as animal growth (Santos, 2008). Mostly, growth is described as an increase in body dimension (mass, volume or length) as a function of time, and when this relationship is plotted, it results in a growth curve.

Length increase in fish has been studied for a long time in population and fisheries research. In contrast, aquaculture studies mostly refer to body weight. Because both measurement (weight and length) are understood to be largely caused by the same genes (Gunnes and Gjedrem, 1981) weight and length are closely linked by mathematical relationships. Accordingly, length, just as weight, can be affected by the environmental factors present in aquaculture facilities, i.e. tank design (Ross *et al*., 1995; Üstündağ and Rad, 2014) water quality and stocking density (Person *et al*., 2008).

Therefore, length growth and other size related treatment studies in cultured fish have received increasing attention since they can be used in the management of aquaculture production (Furuya *et al*., 2014; Silva *et al*., 2015; Lugert *et al*., 2017). In fact, during the last decade the interest to record these growth measurements *in situ* has grown, in order to improve the automatization of rearing practices in commercial fish aquaculture with the goal to advance in terms of productivity and profitability (Miranda *et al*., 2017; Saberioon and Císař, 2018).

Predicting growth in aquaculture facilities with high accuracy is possible using statistically based models, i.e. nonlinear equations, since statistical processing software are capable of handling complex mathematical algorithms in order to achieve analytical solutions (Lugert *et al*., 2016; Powel *et al*., 2019). Thus, the aim of this work was to fit six nonlinear models to length growth data of cultured rainbow trout by nonlinear regression and evaluate which model or models have the highest accuracy to display the growth curve.

MATERIAL AND METHODS

The data were collected at a commercial rainbow trout farm. The farm is located in the municipality of Nova Friburgo, a mountain region of the state of Rio de Janeiro, Brazil (22 ° 23'36 "S, 42 ° 29'12" W, 1.032 m altitude). This research was approved by the Ethics Committee on Animal Use (CEUA) of the Rio de Janeiro State Fisheries Foundation-FIPERJ with document number 007/2017007/2017.

The fish, without sex distinction, were acquired from the farms' own breeding program. Nine hundred fish with an age of 273 days post-hatch (dph), and length (fork length) mean of 22.42 ± 0.71cm, were selected. Fish were distributed randomly into nine masonry tanks with a volume of 40 m^3 each. Fish were fed with three different types of extruded pellets (two commercials diets, A and B, and one experimental diet, C) in triplicates [(A/1, A/2, A/3) (B/1, B/2, B/3) (C/1, C/2, C/3)]. Rations were offered twice a day until apparent saturation. The experimental period was 98 days. Length measures at the beginning and the end of the trial for each feed type are shown in Table 1.

The six nonlinear equations chosen were von Bertalanffy, Brody, Gompertz, Logistic, Exponential, and Power Function; the mathematical expression of each function is presented in Table 2. Models were fitted using the Levenverg-Marquardt algorithm through the nlsLM computational process in the statistical software R (Elzhov *et al*., 2015). This process uses the nonlinear least squares (nls) method. The default convergence conditions were used with the exception of the maximum number of iterations being increased to 1000.

Table 1. Average length (AL) of cultured rainbow trout in centimeters with Standard Deviation (SD) at the beginning and the end of grow-out phase

Diet/Repetition	AL (cm) ± SD
Beginning	
All repetitions	22,42 ± 0,71cm
Final	
A/1	32.31 ± 1.62
A/2	29.80 ± 1.88
A/3	32.06 ± 1.45
B/1	33.09 ± 1.41
B/2	33.00 ± 1.66
B/3	33.42 ± 1.69
C/1	33.07 ± 1.78
C/2	33.45 ± 1.72
C/3	33.01 ± 1.84

Table 2. Mathematical expression of the seven equations fitted to length growth data of cultured rainbow trout

Models	Equation	References
Bertalanffy	$Y = A * (1 - exp\,(-B * (t - T_0))$	Bertalanffy, 1934
Brody	$Y = A * (1 - C * exp\,(-B * t))$	Brody, 1945
Logistic	$Y = A * (1 + exp\,(-B * (t - T)))^{-1}$	Pearl, 1930
Gompertz	$Y = A * exp\,(-exp\,(-B * (t - T)))$	Tjorve and TJorve, 2017
Exponential	$Y = L_0 * exp(t * k)$	Santos *et al.*, 2008
Power Function	$Y = L_0 * (t^{k})$	Huxley, 1932

Y = dependent variable; t = independent variable; A = asymptote; B = exponential rate of approximation to the asymptote; T = the location of the point of inflection (POI); C = an integration constant without biological interpretation; T_0 = is the intercept on the x-axis; Y_1 and Y_2 = first and the last recorded length data, respectively; T_1 and T_2 = age of fish at the beginning and the final of period experiment, respectively; L_0 = is the intercept on the y-axis; and k = exponential rate to infinity.

The accuracy of the fitted models was evaluated using a model performance metrics. The performance criteria to evaluate the goodness of fit are: The mean squared residuals ($MSR = RSS * [n - p]^{-1}$); where RSS is the residual sum of squares, n is the number of observations, p is the number of parameters of the model (Rawlings *et al*., 1998). The Akaike Information Criterion (AIC) corrected for small sample sizes (AICc). $AIC = 2k - 2\ln(\hat{L})$; where k is the number of estimated parameters in the model and $\hat{L}$ is the maximum value of the likelihood function for the model, and *ln* is the natural logarithm (Akaike, 1973). $AICc = \mathrm{AIC} + \frac{2\mathrm{k}^2+2k}{\mathrm{n-k-1}}$; where n is the sample size and k is the number of parameters.

We calculated the difference in AICc (ΔAICc) values to test the support of inferior models by the data. ΔAICc is calculated as: AICc ($AICc_i - AICc_{min}$) (Katsanevakis and Maravelias, 2008). Models with ΔAICc >10 have no support from the data, while models with ΔAICc < 2 have substantial support (Burnham and Anderson, 2002). Models with ΔAICc between 4-7 are somewhat supported by the data and might be taken into consideration.

The Mean Absolute Error (MAE) is the average absolute difference between observed and predicted outcomes and is calculated as: $MAE = mean(|observed - predicted|)$. The MSR, AICc, and MAE were calculated using SAS (Statistical…, 2013). Finally, the results from MSR, AICc, and MAE were analyzed using a scoring system in which each best fit accounted for one score. The model that had the best fit in most tested cases achieved the highest score. In addition, we interpreted the estimated regression parameters of each model in regard to the biological attributes of the species whenever possible.

RESULTS

All models met convergence in all (9 of 9 evaluations) tested cases through Levenverg-Marquardt´s iterative method. All models needed a comparably low number of iterations, and convergence was generally met within 100 iterations. The estimated parameters for each model are shown in Table 3.

Table 3. Estimated parameters of Bertalanffy, Brody, Logistic, Gompertz, Exponential and Power function

Diet/Repetition	Bertalanffy	Brody	Logistic	Gompertz	Exponential	Power Function
A/1					---	---
A	680.637	350.691	353.047	10740.9	---	---
B	0.0001	0.0003	0.0040	0.0006	---	---
T_0	59.533	---	---	---	---	---
T	---	---	930.653	3156.32	---	---
C	---	1.0205	---	---	---	---
L_0	---	---	---	---	7.9986	0.02517
K	---	---	---	---	0.0038	1.2097
A/2						
A	630.258	422.841	925.313	30423	---	---
B	0.0001	0.0002	0.0030	0.0004	---	---
T_0	-14.539	---	---	---	---	---
T	---		1491.86	5010.23	---	---
C	---	0.9979	---	---	---	---
L_0	---	---	---	---	9.8555	0.1116
K	---	---	---	---	0.0029	0.9421
A/3						
A	127.134	127.133	48.1825	59.0688	---	---
B	0.001	0.001	0.0086	0.0048	---	---
T_0	81.7453	---	---	---	---	---
T	---	---	290.004	267.306	---	---
C	---	1.086	---	---	---	---
L_0	---	---	---	---	8.1859	0.0279
K	---	---	---	---	0.0037	1.1919
B/1						
A	47.6917	47.6918	39.627	42.2835	---	---
B	0.0058	0.0058	0.0144	0.0101	---	---
T_0	164.443	---	---		---	---
T	---	---	255.23	228.48	---	---
C	---	2.6016	---	---	---	---
L_0	---	---	---	---	7.633	0.0159
K	---	---	---	---	0.004	1.2946
B/2						
A	88.5135	88.5139	46.3962	54.5543	---	---
B	0.0018	0.0018	0.0101	0.0059	---	---
T_0	113.95	---	---	---	---	---
T	---	---	280.435	254.149	---	---
C	---	1.23147	---	---	---	---
L_0	---	---	---	---	7.59	0.0165
K	---	---	---	---	0.004	1.2868
B/3						
A	60.645	60.6451	43.0691	47.8483	---	---
B	0.0036	0.0036	0.0124	0.0079	---	---
T0	145.668	---	---	---	---	---
T	---		266.573	238.481	---	---
C	---	1.694	---	---	---	---
L0	---	---	---	---	7.3661	0.0127
K	---	---	---	---	0.0041	1.3341

Parameter *A*, values range between 39.63 and 10740.9. Within each group, the lowest value was mostly obtained by the Logistic and Gompertz models, while the highest value was usually estimated by the Bertalanffy and Brody models. In contrast, values in parameter *B* range between 0.00144 and 0.0001 with the lowest values being obtained by the Brody and Bertalanffy models, and the highest values by the Logistic and Gompertz models. Parameter *T* ranges between 5010.23 and 228.48 in the Gompertz model, and between 1491.86 and 255.23 in the Logistic model. Parameter T_0 for the von Bertalanffy model has values between -14.539 to 164.443.

Table 3 (continue). Estimated parameters of Bertalanffy, Brody, Logistic, Gompertz, Exponential, and Power Function models

Diet/Repetition	Bertalanffy	Brody	Logistic	Gompertz	Exponential	Power Function
C/1						
A	75.084	75.084	45.5924	52.3174	---	---
B	0.0023	0.0023	0.0103	0.0063	---	---
T_0	119.958	---	---	---	---	---
T	---	---	275.762	246.341	---	---
C	---	1.3212	---	---	---	---
L_0	---	---	---	---	7.7727	0.0186
K	---	---	---	---	0.0039	1.2663
C/2						
A	309.924	244.385	52.7605	68.3329	---	---
B	0.0004	0.0005	0.0088	0.0046	---	---
T_0	89.1451	---	---	---	---	---
T	---	---	308.098	297.346	---	---
C	---	1.05	---	---	---	---
L_0	---	---	---	---	7.3111	0.0131
K	---	---	---	---	0.0041	1.3263
C/3						
A	80.0456	80.0459	45.4537	52.7584	---	---
B	0.0021	0.0021	0.0105	0.0063	---	---
T_0	120.185	---	---	---	---	---
T	---	---	276.587	249.388	---	---
C	---	1.2927	---	---	---	---
L_0	---	---	---	---	7.5515	0.0159
K	---	---	---	---	0.004	1.2929

L_0 values are between 7.31 and 9.85 for the Exponential model, and between 0.0127 and 0.0279 for the Power Function Model. Similarly, *k* values are between 0.0029 and 0.0041 for the Exponential model, and between 1.3341 and 1.3341 for the Power Function Model. *k* values are between 0.004 to 0.041 for the Exponential model, and between 0.9421 to 1.3341 for the Power Function model. Graphic growth simulations for dph (days post hatch) 100 to 600 by each equation are shown in Figure 1 for three-parameter functions (Logistic, von Bertalanffy, Gompertz and Brody models) and, two-parameter functions (Power Function and Exponential models).

The model performance metrics for each model are presented in Table 4. Lowest MSR-values are produced by the Logistic model in 0.67 of tested cases, followed by the Von Bertalanffy (0.11), Brody (0.11), Exponential (0.11) and Power Function (0.11) models. The Gompertz model did not perform the lowest MSR in any case. MAE is lowest in the Logistic model in 5 out of 9 tested groups, 0.55 respectively. The Gompertz, Exponential and Power function models produced lowest MAE once (0.11) (Table 4).

The AICc values of each model and all tested cases are listed in Table 4. Lowest AICc values are most often obtained by the Logistic model (6 of 9 cases). The Exponential model produced lowest AICc in 2 out of 9 cases, and the von Bertalanffy and Brody model both achieved lowest AICc values in 1 of 9 cases. The Gompertz and Power Function models never achieve lowest AICc.

The overall score obtained by the models are presented at the bottom of Table 4. Undisputedly, the Logistic model achieved the best overall-scoring with 18 of 27 best fits (0.67). The Exponential model achieved best overall fit in 5 of 27 cases. The von Bertalanffy, and the Brody models scored only 2 out of 27 (0.07), and the Gompertz, and Power Function models achieved best fit just in one tested case and criteria (0.04). ΔAICc values range between 0.026 as the lowest and 44.61 as the highest. The Logistic and Gompertz models had substantial support by the data in all cases (Table 4). The von Bertalanffy and Brody models in 6 cases and, the Exponential and Power Function models in 2 cases each.

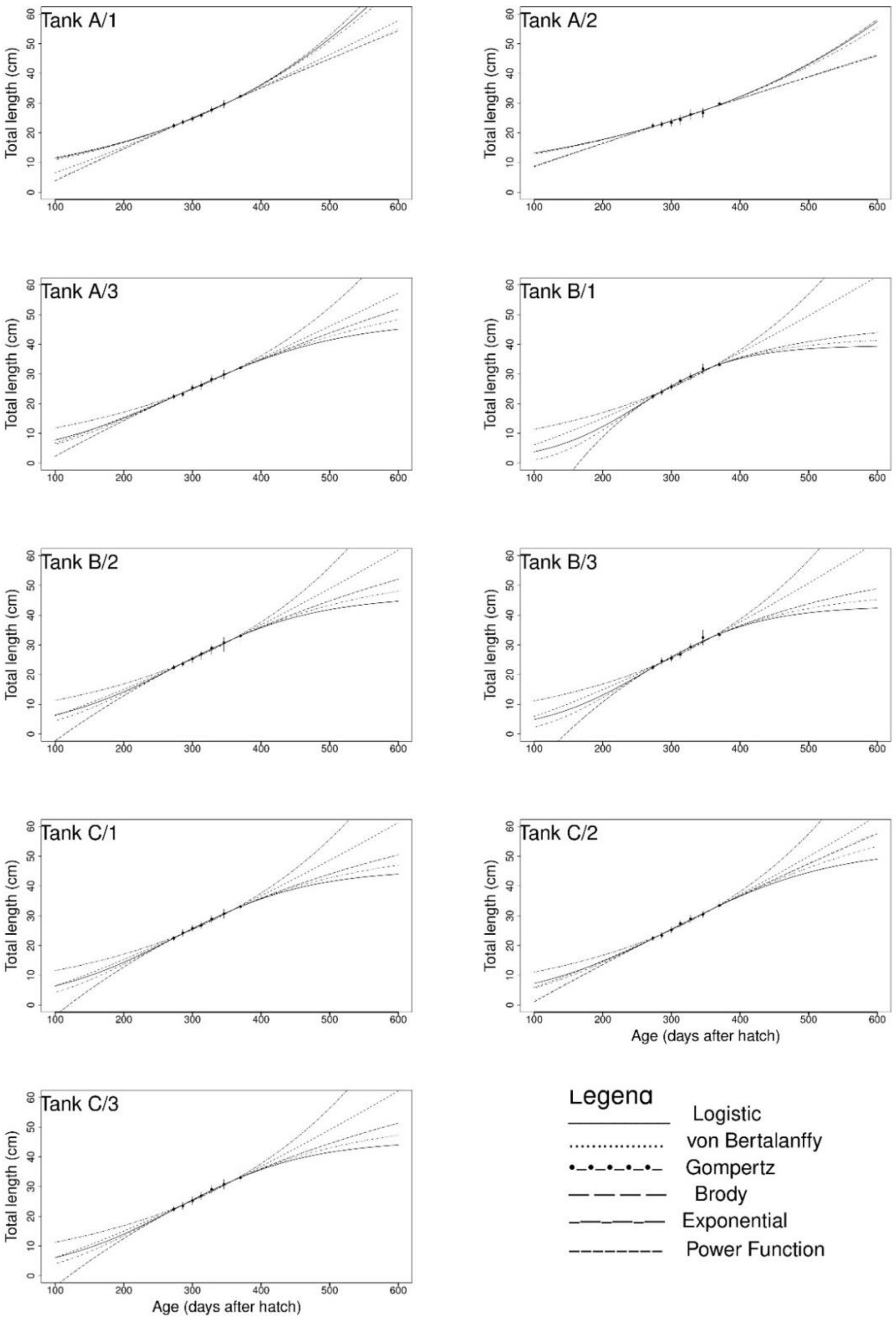

Figure 1. Growth simulations of rainbow trout from 100 until 600 age-days obtained by Logistic (solid line), von Bertalanffy (dotted), Gompertz (dot dash), Brody (long dash), Exponential (two dash) and Power function (dashed) models. Average length in cm (○) ± Standard Deviation.

Table 4**.** Goodness of fit criteria of the von Bertalanffy, Brody, Logistic, Gompterz, Exponential and Power Function equations fitted to length gain data of rainbow trout. Mean Square Residual (MSR), Mean Absolute Error (MAE) and Akaike Information Criterion corrected for small sample sizes (AICc). ΔAICc values indicate support of the model by the data. *best value on same criteria. **number of times that the best score for each model is met

Diet Criteria	Bertalanffy	Brody	Logistic	Gompertz	Exponential	Power Function
A/1						
MSR	1.2147166	1.217811	1.1796001	1.1801987	1.1737527*	1.194842
MAE	0.8236849	0.824574	0.8183127	0.8180777*	0.8188233	0.820356
AICc	598.4779364	598.979191	592.6988709	592.7988288	592.6727535*	596.180959
ΔAICc	5,805	6,306	0,026	0,126	0,000	3,508
A/2						
MSR	2.12683	2.128863	2.046860	2.055098	2.034366*	2.117822
MAE	1.11847	1.119290	1.094625	1.097862	1.093890*	1.118859
AICc	701.63786	701.824204	694.164381	694.947650	693.922849*	701.762634
ΔAICc	7,715	7,901	0,242	1,025	0,000	7,840
A/3						
MSR	1.4333163	1.4333163	1.4308570*	1.432026	1.4826628	1.4324416
MAE	0.9358926	0.9358926	0.9349358*	0.935386	0.9506103	0.9365388
AICc	631.0775059	631.0775059	630.7392007*	630.900026	638.6985844	631.9101009
ΔAICc	0,338	0,338	0,000	0,161	7,959	1,171
B/1						
MSR	1.2999818	1.2999818	1.2862617*	1.2928879	1.6090242	1.4409779
MAE	0.9030997	0.9030996	0.8998888*	0.9015261	0.9885733	0.9384269
AICc	605.6428412	605.6428412	603.5738641*	604.5758229	648.1843622	626.6747177
ΔAICc	2,069	2,069	0,000	1,002	44,610	23,101
B/2						
MSR	1.3745884	1.3745884	1.3689902*	1.3716829	1.4727694	1.3902388
MAE	0.8820453	0.8820452	0.8791945*	0.8804776	0.9140912	0.8842226
AICc	619.6801683	619.6801683	618.8803047*	619.2654429	634.1548813	622.8517529
ΔAICc	0,800	0,800	0,000	0,385	15,275	3,971
B/3						
MSR	2.144757	2.144757	2.127780*	2.136353	2.350096	2.210897
MAE	1.022760	1.022760	1.021003*	1.021942	1.084951	1.038324
AICc	703.274695	703.274695	701.724975*	702.509058	722.055856	710.149607
ΔAICc	1,550	1,550	0,000	0,784	20,331	8,425
C/1						
MSR	1.5411509*	1.5411509*	1.5423322	1.5416740	1.6465393	1.5620830
MAE	0.9567037	0.9567037	0.9565924*	0.9566168	0.9970732	0.9699101
AICc	642.0976960*	642.0976960*	642.2478726	642.1642159	656.0149855	645.6945052
ΔAICc	0,000	0,000	0,150	0,067	13,917	3,597
C/2						
MSR	1.3967752	1.3968260	1.3949278*	1.3957112	1.4559129	1.395989
MAE	0.9172188	0.9175387	0.9158973	0.9165441	0.9292423	0.915059*
AICc	622.8184881	622.8256100	622.5590881*	622.6691286	631.8986434	623.660770
ΔAICc	0,259	0,267	0,000	0,110	9,340	1,102
C/3						
MSR	1.807639	1.807639	1.799842*	1.803624	1.918966	1.827667
MAE	1.012463	1.012463	1.009982*	1.011283	1.032893	1.013757
AICc	676.787560	676.787560	675.935976*	676.349501	689.514099	679.911158
ΔAICc	0,852	0,852	0,000	0,414	13,578	3,975
Score**	2	2	18	1	5	1

DISCUSSION

Convergence is met when the iterative process successfully estimates parameters for the function within the given maximum number of iterations set in the fitting algorithm. In this study, all models met convergence in all tested cases using the Marquardt algorithm. This algorithm is described to be more robust than others offered on statistical software (Elzhov, *et al.*, 2015; Lugert *et al.*, 2017). This is especially important, as non-convergence situations of models for aquaculture data are described by several authors (Costa *et al.*, 2009; Mansano *et al.*, 2012; Allaman *et al.*, 2013; Sousa *et al.*, 2014).

Parameter *A* for three-parameter models (von Bertalanffy, Brody, Logistic and Gompertz) describe the infinite size of an organism and can be interpreted as the possibility of the model to reflect the biological properties of the species. *O. mykiss* is known to exceed 120cm in length (Eaton *et al*., 1995). Accordingly, Logistic and Gompertz models, estimated *A* within the biological range of the species in 7 of 9 cases, and for von Bertalanffy and Brody models in 5 of 9 cases.

Parameter *B* for three-parameter models denotes the precocity index. This means the larger the numeric value, the quicker the fish will reach the asymptotic or infinite size (Malhado *et al*., 2009). Estimated *B* values for Logistic and Gompertz models (0.0001 and 0.0144) in this study have the tendency to be greater than those being obtained from wild rainbow trout (0.002 to 0.049) in 7 and 6 of 9 cases, respectively (Blair *et al*., 2013; Sloat and Reeves, 2014; Cilbiz and Yalim, 2017). Similarly, Lugert *et al*. (2016) found similar differences in parameter *B* between cultured and wild *Scophthalmus maximus*, referring these differences to the positive effect of controlled environmental conditions in recirculating aquaculture systems (RAS). There are no differences in parameter *B* observed between von Bertalanffy and Brody models, having values between 0.001 and 0.058. The Logistic model had the highest values of *B* followed by the Gompertz model. Von Bertalanffy and Brody models generally have the lowest values. Contrary to our findings, Gomiero *et al*. (2009) showed larger *B* estimates for Brody and von Bertalanffy models on length growth of cultured *Brycon orbignyanus*. Our results agree with results by Santos *et al*. (2013) on length growth modeling of *Oreochromis niloticus.*

The point of inflection (POI) (parameter *T*) of the growth curve is only parameterized in the Gompertz and the Logistic model. At the POI, the rate of grow this largest, before diminishing asymptotically against zero. In this study, *T* values obtained by the Gompertz and Logistic models are generally lower than those estimated by Sloat and Reeves (2014) in weight data of wild rainbow trout using the Gompertz model. Furthermore, in aquaculture operations, the parameter *T* can be useful in the empiric adjustment of management strategies, as it is proven to correlate with other husbandry information. For instance, *T* parameter has significant meaning on cultured *Carassius auratus gibelio* because it positively correlates with dietary protein levels (Yun *et al*., 2015). Likewise, *Oreochromis niloticus* shows significant influence of water temperature on weight gain and at the age at the inflexion point (Santos *et al*., 2013).

Parameter T_0 for the von Bertalanffy model defines the hatching day of rainbow trout. In this study, this parameter does not have congruence with biological features since it is not possible to have negative or positive hatching age (up to 54 days). Similarly, parameter L_0 for Exponential and Power Function models which define the hatching length differ between both models. In addition, L_0 of the Power function model shows values (0.0127 to 0.1116cm) smaller than the biologic features of rainbow trout (1.2 to 2cm) as described by Lavens and Sorgeloos (1996).

*P*arameter *k* represents the constant growth rate of rainbow trout trough all growth-curve for Exponential and Power Function model. *k* values of Exponential model are lower than those obtained by the Power Function. These values must be taken with care since both models display exponential shape and are not intended for longer growth periods or extrapolation of data. However, because of their simplicity they are frequently used in aquaculture studies (Santos *et al*., 2008; Costa *et al*., 2009).

In model selection, the goodness of fit should generally not be based on a single criterion. Correspondingly, it has become common practice to evaluate the most suitable model based on an evaluation metrics of mostly three statistical parameters of different properties (e.g. Yun *et al*., 2015; Lugert *et al*., 2017; Powell *et al*., 2019). One parameter should be based on the residuals from fitting the model. The second parameter is often based on information theory either AIC, AICc or BIC. A third parameter is mostly somehow based on the deviation between estimated and sampled data. For these three categories of evaluation parameters, several different statistical parameters are available. In each scenario, the author needs to decide individually, which parameter is most suitable for the current study.

In our study, we used Mean Squared Residual (MSR), Akaike Information Criterion for small sample sizes (AICc) and Mean Absolute Error

(MAE). The non-linear least squares method aims to achieve non-linear equation parameter by minimizing the Residual Sum of Squares (RSS). The smaller RSS, the smaller the MSR and the better the fit (Rawlings *et al*., 1998). In this study, the Logistic model most often achieved the smallest MSR values. Similar results were obtained by Costa *et al*. (2009) in growth studies of *Orechormis niloticus,* but are in contrast to Mansano *et al*. (2012) on *Lithobates catesbeianus*, with both species being reared under aquaculture conditions.

We used ΔAICc to identify whether our datasets were supported by more than one model. This was necessary, as the outcome from the analysis revealed very close numeric results between different models within tested groups. ΔAICc < 2 indicates substantial support of a model by the data (Burnham and Anderson, 2002). Indeed, in all 9 analysis, 2 out of 6 tested models were supported by the data, namely Logistic and Gompertz. This might be due to the specific pattern of our recorded data (grow-out phase), which are distributed around the POI of the growth curve. Accordingly, several models of sigmoidal behavior can equally well reflect this segment of the curve.

Primarily, we observed that the different non-linear models adjusted their fit individually to the various growth trajectories expressed by rainbow trout caused by different diet treatments. Araneda *et al*. (2013) observed similar results when fitting models on various growth data of *Penaeus vannamei*. This specific application has huge potential in predicting the effects of new feed formulations, harvest size and production period in all aquaculture species. However, it is necessary to verify and validate this potential through studies with rigorous control of diet quality and quantity as recorded in carp (Yun *et al*., 2015).

CONCLUSION

All six models (von Bertalanffy, Brody, Logistic, Gompertz, Exponential and Power Function) have shown the capacity to fit the length-at-age data of cultured rainbow trout during the grow-out phase. However, in the current study, the Logistic model achieved the highest accuracy in fit. Despite the growth-length curve of cultured rainbow trout not clearly follows a sigmoidal shape, the diminishing-return shaped von Bertalanffy and Brody models, as well as the exponential shaped Power Function and Exponential models do not meet the mathematical attributes needed to reflect length-at-age data. This is also verified by ΔAICc values, which indicate the Logistic and Gompertz model, as the only models having substantial support by the data in all cases. Furthermore, we showed that it is possible to model the impact of varies feeding strategies to predict long-term influences on growth and harvest size and production period.

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COMMITTEE ON ETHICS AND BIOSAFETY

The procedures adopted were approved by the Ethics Committee on Animal Use (CEUA) of the Rio de Janeiro State Fisheries Foundation-FIPERJ, document number 007/2017, been in accordance with the ethical principles in animal experimentation elaborated by the Brazilian College of Animal Experimentation (COBEA).

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