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# TRANSMISSION LOSS ALLOCATION UNDER COMBINED POOL AND BILATERAL OPERATION

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## ABSTRACT

The allocation of transmission losses in an open system operating under both pool and bilateral contracts is analyzed using the method of differential loss allocation. This approach allows to consider loss supply as marketable product since each bilateral contract can chose its more appropriate loss supplier. The relative time-dependence of the various contracts including those with the pool are shown. It is also studied the relative influence of implementing bilateral contracts on the pool dispatch performance. The theory is presented together with a number of numerical examples.

**KEYWORDS:** Open access, transmission loss allocation, combined pool and bilateral transactions.

## RESUMO

A alocação das perdas de transmissão de sistemas de potência abertos operando com mercado de contratos bilaterais e mercado 'spot' é analisada utilizando um método de alocação de perdas incremental. Esta abordagem permite considerar a compensação de perdas como um produto a ser comercializado dado que cada contrato bilateral pode escolher seu mais apropriado fornecedor. A dependência temporal relativa entre os vários contratos incluindo os contratos feitos com o mercado

'spot' é mostrada. É estudado o impacto relativo da implementação de contratos bilaterais no desempenho do despacho no mercado 'spot'. Os aspectos teóricos são apresentados junto com vários exemplos numéricos.

**PALAVRAS-CHAVE:** Acesso aberto, alocação de perdas de transmissão, transações conjuntas de mercado 'spot' e bilateral.

## 1 INTRODUCTION

Under competition, power systems can generally operate on the basis of both bilateral contracts and pool dispatch. Bilateral contracts are typically agreed to ahead of time and are of relatively longer duration. Such contracts take place between a load (buyer) and a generator (seller) who independently negotiate a mutually profitable price driven by market forces (Wu and Varaiya, 1995 and Tutorial, 1997). On the other hand, Pool or spot market dispatch takes care of supplying the load components not met by the bilateral contracts. The Pool accomplishes this by minimizing the total cost paid to those generators that bid to supply power to the Pool (Schweppe *et alii.*, 1998). It is important to recognize that the Pool generation dispatch and the system incremental cost are strongly influenced by the pre-defined bilateral contracts since such contracts move the operating points of the generators. It is assumed that the bilateral contracts have received approval by the ISO (Independent System Operator) and meet the system security conditions.

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As discussed in (Galiana, 1998), transmission losses form a significant component of the amount of power that has to be generated in order to meet the power demand. This paper deals with the issue of transmission loss calculation and allocation under combined pool and bilateral operation.

In an earlier study (Phelan, 1998 and Galiana and Phelan, 1998), the authors developed and tested a theory for the allocation of system transmission losses to bilateral contracts. This result differs from existing methods that are primarily heuristic and allocate losses on a pro-rata basis, or from others that work in the context not of bilateral contracts with individual slack buses but of bus loads (Bialeck J. , 1997), (Kirschen D. *et alii* ,1997), or with a single slack bus. The differential loss allocation strategy presented in (Phelan, 1998 and Galiana and Phelan, 1998), recognizes that for an infinitesimal change in any given bilateral contract, it is possible to associate a unique increment in system losses corresponding to the contract increment. These incremental relations are then shown to yield a set of algebraic-differential equations governing the loss allocation for contracts of any magnitude. The theory developed accounts for the relative locations in the network of the buyer and seller, the amount of the bilateral contract, the interaction of all simultaneous bilateral contracts, as well as the choice of location of the loss supplier for each individual contract. The latter point is important as it recognizes that losses need not necessarily be supplied by one slack generator only but rather that the loss supplier could vary from contract to contract. One significant result of this theory of loss allocation is that, arguably quite correctly, some bilateral contracts may be allocated negative losses. Such a condition arises if a given contract has the net tendency to send power through the network along paths opposite to the net flow from generators to loads. Our earlier work on this subject also examined ways to simplify the calculation of bilateral loss allocation components without excessively compromising the accuracy of the results.

In this paper, we extend the differential loss allocation scheme to the likely situation where both bilateral and pool operation co-exist. The difficulty in formulating and solving the loss allocation problem with combined pool/bilateral operation is that each mode of operation influences the other in a complex non-linear manner. We resolve this difficulty as in the purely bilateral case by incrementing the bilateral and pool contracts either sequentially or simultaneously in small amounts thus making it possible to calculate both the incremental losses associated with each bilateral contract increment as well as the increments in the optimum pool gener-

ation components. The variations thus calculated are progressively summed until the final bilateral and pool load components are fully satisfied. Because of the pool operation, at each iteration step the method solves an optimum dispatch over the pool variables. Similarly, at each step, the full load flow and all operational limits are enforced.

## 2 THEORY OF POOL/BILATERAL LOSS ALLOCATION

If the real demand at bus  $j$  is denoted by,  $P_{dj}$  of which one part,  $P_{dj}^p$ , is supplied by the pool while the remaining component,  $P_{dj}^b$ , is satisfied by bilateral contracts, then,

$$P_{dj} = P_{dj}^p + P_{dj}^b \quad (1)$$

The bilateral load components,  $P_{dj}^b$ , are defined by the sum of the bilateral contracts with the supplying generators, that is,

$$P_{dj}^b = \sum_{i=1}^n GD_{ij} \quad (2)$$

where, as shown in reference (Galiana and Ilic, 1998 and Galiana, 1998), the notation  $GD_{ij}$  is used to characterize a bilateral contract transferring this amount of real power between a generator at bus  $i$  and a load at bus  $j$ .

As with the loads, the generation at each bus,  $P_{gi}$ , has two components: (i)  $P_{gi}^p$ , dedicated to supplying the pool demand and associated losses; and (ii)  $P_{gi}^b$ , dedicated to satisfying the bilateral contracts and corresponding losses. Clearly,

$$P_{gi} = P_{gi}^p + P_{gi}^b \quad (3)$$

The combined problem of loss allocation consists of determining the optimum levels of the pool components,  $P_{gi}^p$ , in the sense that they minimize the total pool payments to the bidding generators, that is,

$$\underset{P_g^p}{\text{Minimize}} \quad \sum_{i=1}^n C_i(P_{gi}^p + P_{gi}^b)$$

subject to the load flow equations,

$$(P_g^p + P_g^b) - (P_d^p + P_d^b) = P(\delta) \quad (4)$$

to any generation and transmission constraints and to the simultaneous presence of the bilateral contract generation levels. These satisfy,

$$P_g^b = \sum_{j=1}^n GD_{ij} + \sum_{r,s=1}^n L_{rs/i} \quad (5)$$

where  $L_{rs/i}$  represents the loss allocated to an arbitrary bilateral contract  $GD_{rs}$  obtaining its loss supply from bus  $i$ . The last term in equation (5) represents the sum of all the loss allocation components for those bilateral contracts whose losses are supplied by generator  $i$ .

It is considered that there are enough reactive power flows to maintain bus voltage magnitude near to 1.0 p.u..

### 3 SOLUTION STRATEGY

The incremental loss allocation approach followed in (Phelan, 1998 and Galiana and Phelan, 1998), is applied here to determine the loss terms  $L_{rs/i}$ . This approach states that, because of the trilateral nature of a bilateral transaction, an infinitesimal change in the contract 'rs',  $dGD_{rs}$ , produces an infinitesimal increment in three bus injections *only*, namely, at the selling bus  $r$ , where the incremental injection is  $dP_r = dGD_{rs}$ , at the buying bus  $s$ , where the incremental injection is  $dP_s = -dGD_{rs}$ , and finally in the bus supplying the corresponding incremental system losses, that is, bus  $i$ , where  $dP_{loss} = L_{rs/i}$  is injected. One can think of bus  $i$  as the slack bus for bilateral contract  $rs$ . Now, in general, for an arbitrary set of injections  $dP_j$  with  $i$  as the slack, one has,

$$dP_{loss} = \sum_{j=1}^n \left[ \frac{\partial P_{loss}}{\partial P_j} \right]_i dP_j \quad (6)$$

where the partial derivatives are the so-called incremental transmission loss (ITL) coefficients with bus  $i$  as the slack. It follows thus, that for an increment in contract  $rs$  as indicated above,

$$dP_{loss} = dL_{rs/i}$$

$$dP_{loss} = \left[ \frac{\partial P_{loss}}{\partial P_r} \right]_i dGD_{rs} + \left[ \frac{\partial P_{loss}}{\partial P_s} \right]_i (-dGD_{rs}) \quad (7)$$

The essence of the incremental loss allocation approach is to increase each contract gradually in steps of  $dGD_{rs}$ , use the above equation (7) to estimate the bilateral loss allocation increments,  $dL_{rs/i}$  use equation (5) to obtain the bilateral generation levels including losses, and then solve the load flow equations (4) for the phase angles,  $\delta$ , the system losses,  $P_{loss}$ , and, the incremental transmission loss coefficients. This process is repeated until the final level of the bilateral contracts is reached. As discussed and analyzed in (Phelan, 1998 and Galiana and Phelan, 1998), the loss allocation process is path dependent, the final results depending on the order in which the contracts are implemented.

In order to avoid re-computing ITLs relative to different slack buses (different bilateral contract loss providers),

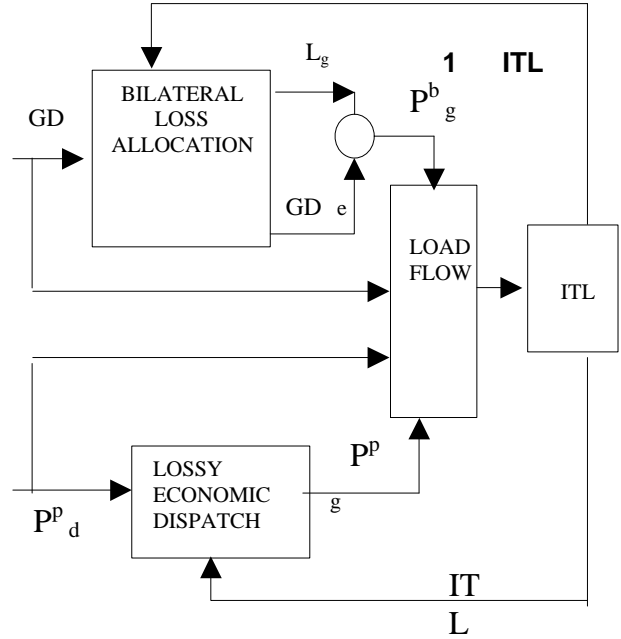


Figure 1: Combined pool and bilateral incremental loss allocation.

the following useful identity (Galiana, 1998) is recalled,

$$\frac{1 - \left[ \frac{\partial P_{loss}}{\partial P_i} \right]_k}{1 - \left[ \frac{\partial P_{loss}}{\partial P_s} \right]_k} = 1 - \left[ \frac{\partial P_{loss}}{\partial P_i} \right]_s$$

(ID1) In summary, the essence of the loss allocation calculations is via the following steps also illustrated in Figure 1. Begin with an initial guess of phase angles. Solve for the ITLs. Assume that the pool,  $P^p_d$ , and bilateral demand components,  $GD$ , are nil.

Increase the pool bus demands by  $\Delta P^p_d$  and solve the economic dispatch problem for the pool generation components keeping the bilateral generation constant.

Solve the load flow and ITL problems.

Keeping pool load constant, increase the bilateral load contracts by  $\Delta GD$  and re-compute the bilateral generation using equation (5).

Repeat step (C) and return to step (B) until the bilateral and pool load contracts are met.

The above procedure is the standard one, basically solving the pool dispatch and meeting the bilateral contracts simultaneously in small increments. A variation of this approach, called the sequential approach, increases the pool demands incrementally while solving the pool dispatch at each increment until the complete pool load is

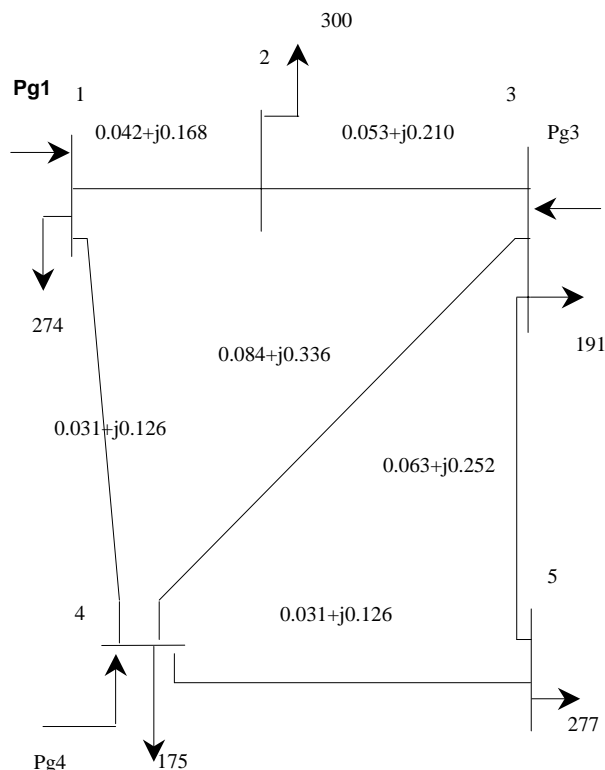


Figure 2: Line impedances on 100 MVA basis. Bus loads in MW. Total load=1153 MW.

met. Bilateral contracts are gradually dispatched next. An alternative sequential approach, meets the bilateral contracts first, followed by a gradual introduction of the pool contracts.

## 4 SIMULATION RESULTS

The theory presented is tested on the network described in Figure 2. The system load is 1153 MW distributed among the five buses as indicated in the Figure 2. The bus voltage magnitudes are taken to be 1.0 pu. The price and capacity data of the three available generators are described in Table 1.

Table 1: Generator Data.  $C(P_g) = C_0 + aP_g + 0.5bP_g^2$

Gen	Pmax MW	$c_0$ \$/h	a \$/MWh	b \$/MW <sup>2</sup> h
1	500	928	21.8	0.04
3	500	553	18.1	0.03
4	600	664	21.8	0.03

The loads shown in Figure 2 purchase some of their power demand in the form of bilateral contracts given by the 5 by 5 matrix GD denoted by,

$$GD = \begin{bmatrix} 7 & 102 & 102 & 33 & 140 \\ 0 & 0 & 0 & 0 & 0 \\ 101 & 58 & 1 & 79 & 10 \\ 103 & 140 & 88 & 63 & 127 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (8)$$

Note that the second and fifth rows are zero since there are no generators available at those buses. In addition, to each bilateral contract there must exist an associated loss contract to supply the corresponding losses. For the above GD matrix, these loss contracts are defined as follows: All bilateral demand contracts with generators 1,3 and 4 also have their losses respectively supplied by generators 1, 3 and 4 with one exception, namely, contract GD<sub>42</sub> which has its losses supplied by generator 3.

No transmission constraints are assumed in these simulations. Three cases are presented here, all of them with the same total and individual bus demands:

Case A: Only bilateral contracts. No pool contracts.

Case B : 100 MW of load 2 supplied by pool. Reduction of contract GD<sub>12</sub> from 102 MW to 2 MW.

Case C: 100 MW of load 2 and 50 MW of load 5 supplied by pool. Reduction of contract GD<sub>12</sub> from 102 MW to 2 MW plus reduction of contract GD<sub>15</sub> from 140 MW to 90 MW.

Case D: In cases A, B, and C the allocation process is simultaneous in the sense described earlier. Case D presents a sequential dispatch where the pool is dispatched first and the bilateral contracts after. Case D is otherwise identical to case B.

The results of the loss allocation process for cases A through D are summarized in Tables 2 to 5. All quantities shown in these tables are in MW with the exception of the pool incremental price,  $\lambda$ , which is in \$/Mwh.

## 5 DISCUSSION OF RESULTS

From Table 2, Case A, one can observe from the loss allocation matrix that some bilateral contracts are allocated a proportionately higher loss component while others receive a negative allocation. For example, bus 3 injects a relatively low amount into the network compared to buses 1 and 4 which are large net generating

Table 2: Purely Bilateral Operation. Case A ( $P_d^p = 0$ ).

Loos Allocation Matrix					$P_g^b$	$P_g^p$
0	11	5.2	-0.4	10.8	410.5	0
0	0	0	0	0	0	0
-4.8	3.1	0	-4.6	0.2	258.3	0
1.2	15.6	5.5	0	11.3	538.5	0
$P_{log} = 54.2,$					$P_{loss}^p = 0,$	$P_{loss}^b = 0, \lambda = 0$

Table 3: Pool and Bilateral Operation. Case B ( $P_{d2}^p = 100, AGD_{12} = -100$ ).

Loos Allocation Matrix					$P_g^b$	$P_g^p$
0	0.16	-0.3	-1	5.9	288.6	0
0	0	0	0	0	0	0
0.3	4.9	0	-2.3	0.5	267.9	108.5
3.4	15.8	2.7	0	9.8	536.4	0
0	0	0	0	0	0	0
$P_{log} = 48.2,$					$P_{loss}^p = 8.5,$	$P_{loss}^b = 39.8, \lambda = 29.7$

buses. Contracts 3 to 1 and 3 to 4 are therefore rewarded with negative allocations. Alternatively, bilateral contracts between net generating bus 4 and buses 2 and 5, which are net consumers, are allocated relatively high loss components.

Considering Table 3, Case B, we note that with the introduction of a pool contract in bus 2, and a corresponding reduction in its bilateral contract with generator 1, the system losses decrease significantly from 54.2 to 48.3 MW. This can be explained by the fact that load 2 does not now receive most of its power from the heavily loaded generators 1 and 4, spreading its supply out to generator 3 as well. Note also that the losses attributed to the bilateral contracts have decreased proportionately more than the decrease in bilateral demand. From the loss allocation matrix one sees for example that contract 15 has reduced its loss allocation from 10.8 to 5.9 MW for the same transfer of 140 MW. More generally, relative to their respective load contracts, generators 1 and 4 decrease the proportion of loss allocation quite significantly. For example, generator 1 goes from  $26.5/384=6.9\%$  in case A to  $4.6/284=1.6\%$  in case B. On the other hand, generator 3 supplies a higher proportion of the losses.

Again, in case B, we note that optimum pool dispatch has selected only generator 3 to supply the pool demand of 100 MW since this generator is the cheapest and since it is not heavily loaded by bilateral contracts. Note also, that the pool generator supplies 8.5 MW of losses, or 8.5% of its contract.

Table 4: Pool and Bilateral Operation. Case C ( $P_{d2}^p = 100, P_{d5}^p = 50, AGD_{12} = -100, AGD_{15} = -50$ ).

Loos Allocation Matrix					$P_g^b$	$P_g^p$
0	0.1	2.6	-1.3	2.5	232.6	20
0	0	0	0	0	0	0
2.6	5.7	0	-1.3	0.6	272.4	141.8
4.4	16	1.4	0	9.2	535.5	0
0	0	0	0	0	0	0
$P_{log} = 49.2,$					$P_{loss}^p = 11.8,$	$P_{loss}^b = 37.4, \lambda = 31.9$

Table 5: Sequential Dispatch, pool, then Bilateral Operation. Case D ( $P_{d2}^p = 100, AGD_{12} = -100$ ).

Loos Allocation Matrix					$P_g^b$	$P_g^p$
0	0.2	-2.8	-1.4	3.8	283.6	5.8
0	0	0	0	0	0	0
2.9	7.4	0	-1.1	0.6	278.4	97.9
4.5	20	1.3	0	9.1	535.4	0
0	0	0	0	0	0	0
$P_{log} = 48.2,$					$P_{loss}^p = 3.8,$	$P_{loss}^b = 44.4, \lambda = 22$

As the pool demand increases to 150 MW (Case C, Table 4), the total losses increase slightly and are covered by the pool generation. The bilateral losses decrease since generator 3, once again, picks up most of the increase in pool generation.

As expected, the pool incremental price,  $\lambda$ , increases with pool demand as seen by comparing cases B and C. Note that the incremental price is relatively high because the bilateral contracts push the committed generators to higher levels of production. This phenomenon is particularly pronounced in the pool dispatch of generator 4 which although “cheaper” than generator 1 (see Table 1), is not dispatched even in case C since generator 4 must generate 521 MW of bilateral contracts plus losses and is therefore operating at a very high marginal rate.

Comparing the sequential and simultaneous cases (D versus B respectively), the principal difference appears in the loss allocation between pool and bilateral, seeing that in both cases the system losses are identical. Pool incremental price is also affected by bilateral contracts in case B compared with case D. In case D, pool dispatch increases generation at bus 1 (near six MW) rather than increasing generation at buses 3 and 4 (apparently cheaper, see Table 1) since generator at bus 1 is electrically close to the pool load at bus 2. In other words, supplying this level of load from generator at bus 1, becomes economically attractive considering losses.

## 6 CONCLUSIONS

The difficult question of how to separate the losses attributed to the bilateral contracts from the losses attributed to the pool contracts in a fair and systematic manner has been studied. Since the relationship between these two loss components is very non-linear, any separation procedure will be subject to interpretation and, possibly, controversy. We recognize that the proposed scheme is, in spite of its many capabilities, still an operating strategy, a loss allocation tool, to which the network participants must agree. Such agreement will come only if the participants see an eventual benefit. This benefit, we believe, will arise from the fact that the proposed scheme is more fair and reflective of reality when compared to typical approaches. These, in our opinion, do and cannot explicitly reflect real considerations, such as the relative location and size of the bilateral contracts, the simultaneous role of the pool contracts, the incremental loss behaviour to individual contracts, the possible choice of loss supplier for each contract, and the relative time-dependence of the various contracts including those with the pool.

A number of questions remain to be investigated, namely how best to implement such a scheme in an ISO, whether its application should be in real-time or off-line, after-the-fact. Agreements must be arrived at on the application of sequential or simultaneous allocation schemes. Additional work is needed to render the algorithms more efficient, possibly at the expense of some accuracy. The impact of a non-flat voltage profile is also a subject for further study. In this case, we expect an increase in some loss allocation contracts as well as the total losses.

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