THE P-Q THEORY FOR ACTIVE FILTER CONTROL: SOME PROBLEMS AND SOLUTIONS

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ABSTRACT

The instantaneous active and reactive power theory or the *p-q Theory* is widely used to design controllers for active filters. This paper deals with some problems due to the misinterpretations of this theory. A historical background of the theory is presented and the problem caused by distorted voltages at the point of common connection (PCC) is analyzed. In addition, the appearance of source current harmonic component not present in the load current (hidden current) caused by different filtering characteristics for the calculation of the oscillating real and imaginary power components is discussed. The problem caused by the voltage distortion can be solved using a phase locked loop (PLL) circuit. For the hidden current, filters with similar characteristics can avoid them. These analysis and solutions are presented to clarify some aspects of the *p-q Theory* not clear in the original approach of the theory.

KEYWORDS: Instantaneous active and reactive power theory, active power filters.

RESUMO

A teoria de potência ativa e reativa instantâneas ou *Teoria p-q* é largamente utilizada no projeto de controladores de filtros ativos. Este trabalho trata de alguns problemas devido a má interpretação desta teoria. É feito um levantamento da história desta teoria e o problema causado pela tensão distorcida no ponto conexão comum (PCC) é analisado. Além disto, o aparecimento na fonte de componentes de harmônicos de corrente não presentes na corrente da carga (corrente escondida), causada pelas diferentes formas de filtragem das componentes oscilantes da potência real e imaginária são discutidas. O problema causado pela distorção na tensão pode ser resolvido com o uso de circuito *phase locked loop* (PLL) . O problema da "corrente escondida" pode ser resolvido com o uso de filtros de características similares. Estas análises são apresentadas para esclarecer alguns aspectos da *Teoria p-q* que não estavam explícitos na proposta original da teoria.

PALAVRAS-CHAVE: Teoria da potência ativa e reativa instantânea, filtros ativos de potência.

Artigo submetido em 15/12/02
1a. Revisão em 12/02/03
Aceito sob recomendação do Ed. Assoc. Prof. José A. Pomilio

1 INTRODUCTION

The paper presented by Akagi, Kanazawa and Nabae (1983) is the first publication of the p-q Theory in English. However, it only became known worldwide after their second publication (Akagi, Kanazawa and Nabae (1984)). The development of this theory was based on various previously published papers dealing with reactive power compensation. In the end of the 1960's Erlicki and Emanuel-Eigeles (1968) and in the beginning of the 1970's Sasaki and T. Machida (1971) as well as Fukao, Iida and Miyairi (1972) published papers related to what can be considered the basic principle of controlled reactive power compensation. Erlicki and Emanuel-Eigeles (1968) presented some basic ideas like *"... compensation of distortive power are unknown to date... "*. They also assure that *"... a nonlinear resistance behaves like a reactive power generator while having no energy-storing elements... "*, and presented a very first approach to actively correct power factor. Fukao, Iida and Miyairi (1972) says that *"... by connecting a converter based reactive power source in parallel with the load and by controlling it in such a way as to supply reactive power to the load, the power supply will only generate active power and an ideal power transmission would be possible."*

Gyugyi and Pelly (1976) presented the idea that reactive power could be compensated by a naturally commutated cycloconverter without energy storage elements. This idea was explained from the physical point of view, but no specific mathematical proof was presented. Harashima, Inaba and Tsuboi (1976) presented probably by the first time the term "instantaneous reactive power" for a single-phase circuit. In this same year, Gyugyi and Strycula (1976) used the name "active ac power filters" for the first time. Takahashi and Nabae (1980) and Takahashi, Fujiwara and Nabae (1981) gave a hint to the derivation of the *p-q Theory*. The formulation they reached, is in fact a sub-set of the *p-q Theory*, but the physical meanings of the variables were not explained.

Using this background, Akagi, Kanazawa and Nabae (1983 and 1984) presented the *p-q Theory* that is valid for any voltage or current waveform. Later, Watanabe, Stephan and Aredes (1993) presented a paper analyzing in detail this theory, including the interpretation of the powers when the system is unbalanced or contain neutral conductor (three-phase four-wire systems).

This theory has been widely used as a basis for the control algorithm of active filters. However, some intrinsic characteristics of the control algorithm, as presented by Akagi, Kanazawa and Nabae (1984), are not clearly discussed in the literature. For instance, if the voltages used in the calculation of the compensation currents are not sinusoidal and balanced the current waveforms in the source are not purely sinusoidal. In fact, this is not a problem of the theory, but this result may go against the expectation of some engineers.

Another interesting point to discuss appears when the filters in the active filter controller that separate the oscillating components of the real and imaginary powers have different cut-off frequencies. The use of filters with different cut-off frequencies allows the synthesis of active filters that present characteristics not possible to reach with passive filters or even others control algorithm. For example, Akagi, Nabae and Ato (1986) have shown that it is possible to design a filter that compensates only the current components that produce imaginary power leaving the currents that are responsible for the oscillating real power. Filtering the oscillating real power and leaving the imaginary power is also possible. Consequently, it is possible to say that this theory is highly flexible.

The objective of this paper is to analyze some of these situations, which in some cases is advantageous (Akagi, H. (1994)) and in others may be a problem, and propose or formalize solutions to overcome them.

2 THE P-Q THEORY

The *p-q Theory* is based on the $\alpha\beta 0$ transformation, also known as the *Clarke Transformation* [Clarke (1943)], which consists in a real matrix to transform three-phase voltages and currents into the $\alpha\beta 0$ stationary reference frame, given by:

$$\begin{bmatrix} v \\ v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}. \quad (1)$$

The inverse transformation is given by:

$$\begin{bmatrix} v \\ v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & 1 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}. \quad (2)$$

Similarly, generic instantaneous three-phase line currents (i_a, i_b, i_c) can be transformed into $\alpha\beta\mathfrak{w}$ axis.

One advantage of applying the $\alpha\beta\mathfrak{w}$ transformation is the separation of zero-sequence components into the zero-sequence axis. Naturally, the α- and β-axis do not have any contribution from zero-sequence components. If the three-phase system has three wires (no neutral conductor), no zero-sequence current components are present andi_0 can be eliminated in the above equations, simplifying them. The present analysis will be focused on three-wire systems. Therefore, zero-sequence voltage or current is not present. In this situation the real and imaginary powers are given by:

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} v_\alpha & v_\beta \\ v_\beta & -v_\alpha \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}. \quad (3)$$

where, p is the real power and represents the total energy flow per time unity in the three-wire three-phase system ($p_{3\phi}$), in terms of $\alpha\beta$components; q is the imaginary power and has a non-traditional physical meaning and gives the measure of the quantity of current or power that flows in each phase without transporting energy at any instant.

It is worth to note that in the above equation, q is equal to that defined in Akagi, Kanazawa and Nabae (1984) and Watanabe, Stephan and Aredes (1993), however, with a minus signal. With this change of signal, for a balanced positive sequence voltage source and balanced capacitive or inductive load, the new reactive (imaginary) power defined in (3) will have the same magnitude and signal of that calculated using conventional power theory ($Q = 3VI\sin\phi$).

3 SHUNT ACTIVE POWER FILTER

The basic structure of a shunt active power filter is shown in Fig. 1. This filter generates currents for the compensation of the non-desirable current components in the load current.

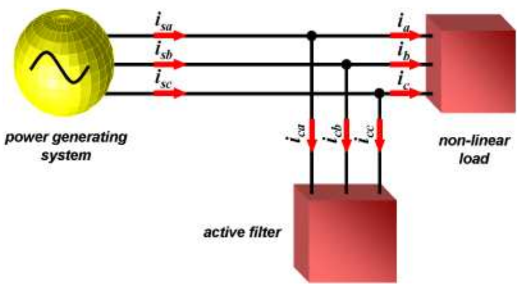
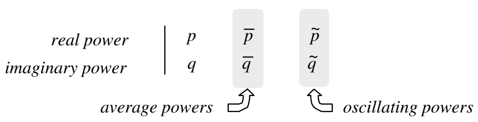


Figure 1: Basic structure of shunt active power filter.

Fig. 2 shows the basic algorithm commonly used for the calculation of the compensating currents. In this figure, p_c and q_c are the compensation reference powers.

In general, when the load is nonlinear the real and imaginary powers can be divided in average and oscillating components, as shown below.



For balanced voltage sources, the oscillating powers $\tilde{p}$ and $\tilde{q}$ represent the undesirable powers due to harmonic components in the load current. In some situation $\bar{q}$ is an undesirable power as well. From these oscillating powers, it is possible to calculate the compensating currents in the $\alpha\beta$ reference frame. Then, by using the Clarke inverse transformation, it is possible to calculate the currents to be injected by the active filter to compensate for these harmonic components in the load. This technique has proven to be very efficient and practical. However, the compensated currents are not sinusoidal if the voltage used in the control algorithm is not balanced and purely sinusoidal. This problem may happen if the voltage at the point of common connection (PCC) is distorted or unbalanced and used in the control algorithm.

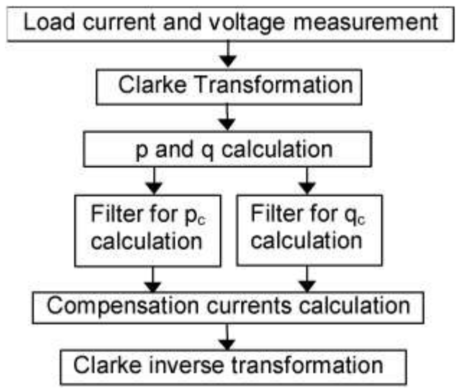


Figure 2: Basic control algorithm for shunt active power filter based on *p-q Theory*.

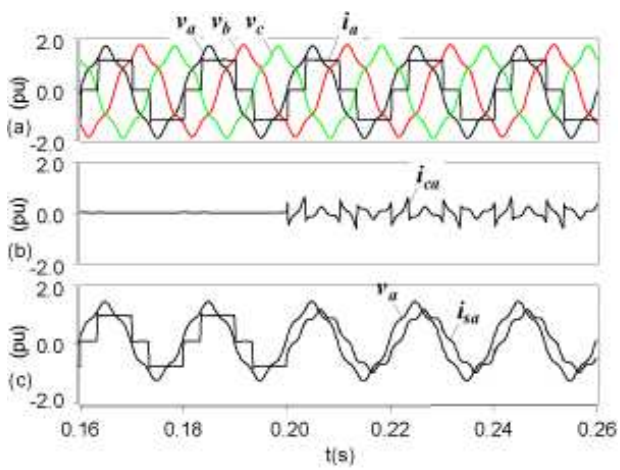


Figure 3: Shunt current compensation under distorted voltages.

Fig. 3(a) shows an example of nonlinear load, connected to a PCC with distorted voltages. The nonlinear load is a rectifier and it is assumed that the currents are not affected by the distorted voltages. Fig. 3(b) shows the compensation current of an active power filter, which was started at t = 0.20 s. The control algorithm is the conventional one, as shown in Fig. 2 and voltages used in the calculations are the voltages at the PCC. In this case, the harmonic components in the voltages at the PCC produce a non-sinusoidal current drawn from the network. In fact, the compensation is correct if we consider that the objective is to have constant real and imaginary powers. When we use the *p-q Theory* with the above algorithm to filter the harmonics, it is supposed that the voltages are

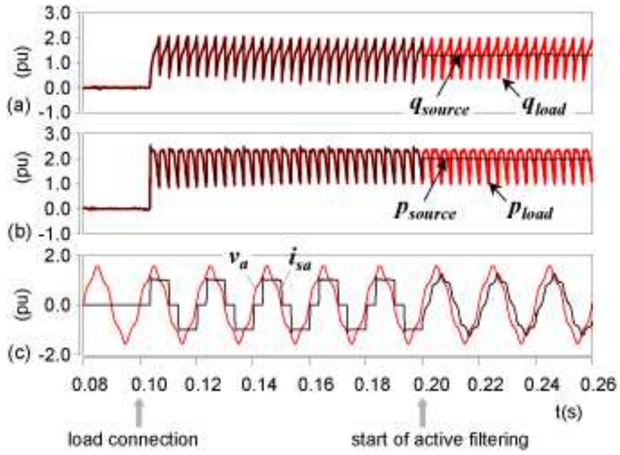


Figure 4: Instantaneous real and imaginary powers of the load and source.

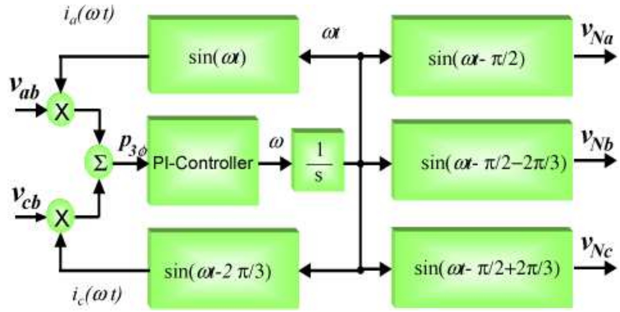


Figure 5: The PLL circuit.

sinusoidal and balanced. If they are not, this algorithm generates a current with harmonic components, which produces constant real and imaginary powers for the distorted voltage, as can be seen in Fig. 3(c), for t > 0.20 s. This explains why the currents are not purely sinusoidal in this figure. Fig. 4 (a) and (b) show the imaginary and real powers at the load and in the source. This figure shows that these powers become constant (without ripple) in the source when the active filter is turned on. However, as shown in Fig. 4 (c) the compensated current is not sinusoidal.

The concept of having real power without ripple (and imaginary power without ripple, as well), may be an interesting solution for motor drives with non-sinusoidal counter electromotive voltages, but not desirable for conventional sinusoidal generation system. In the case of motor drives where the counter electromotive forces are not sinusoidal, a non-sinusoidal current would produce a constant real power. Therefore, no torque ripple will be present in the machine shaft.

If the generation system is based on sinusoidal voltages, which is true in almost all cases, the non-sinusoidal current is a problem and the solution can be found in two ways.

The first solution is to use a filter to eliminate the harmonics components in the voltages at the PCC before using it in the control algorithm. This technique works well if the harmonics components are at high frequencies and the filter do not change the voltage angular phases. In addition, it will only work if no fundamental negative-sequence component is present. Therefore, it is a limited technique.

The second method is based on the use of a phase-locked-loop circuit (PLL circuit), which is used to detect the fundamental positive-sequence component of the voltage at the PCC. This technique is the best option to guarantee the decoupling of the current and the distorted voltages at the PCC. This means that the PLL circuit eliminates the influence of other loads on the shunt filter performance.

Using a PLL circuit, as shown in Fig. 5, Aredes, Häfner and Heumann (1997) proposed a control strategy that guarantees sinusoidal and balanced compensated currents drawn from the network. This is true even when the voltages at the PCC are unbalanced and/or distorted. Simulation results using this control strategy, for the same conditions as in Fig. 3 and 4, are shown in Fig. 6. Note that for this control technique, the real and imaginary powers produced by the sinusoidal currents (compensated currents) are no longer constant. One important conclusion here is that when the voltages are distorted at the PCC, it is impossible to have sinusoidal currents and constant real and imaginary powers at the same time.

The previous analysis was centered in the case where the voltage at the PCC contains harmonic components. However, in some cases, the voltage at PCC is sinusoidal, but with different magnitudes (unbalanced due to fundamental negative-sequence component). In this case, the compensated current using the control algorithm without PLL will contain second order harmonic components. Therefore, the use of the PLL circuit is again very important.

4 THE HIDDEN CURRENT

As shown above, the $p-q$ *Theory* is very precise for the calculation of the compensating currents in shunt active filters if the voltages are balanced and sinusoidal. In addition, the compensation may be perfect if the inverter used to synthesize the compensation current has a high frequency response. However, one interesting phenomenon happens when the gain of the filter to separate the oscillating real power p_c used for the compensation has a different gain from the filter to separate the oscillating imaginary power q_c.

This phenomenon is clearer in the case when only $\tilde{p}$ or only $\tilde{q}$ (or q) is used for the compensation. In practical applications

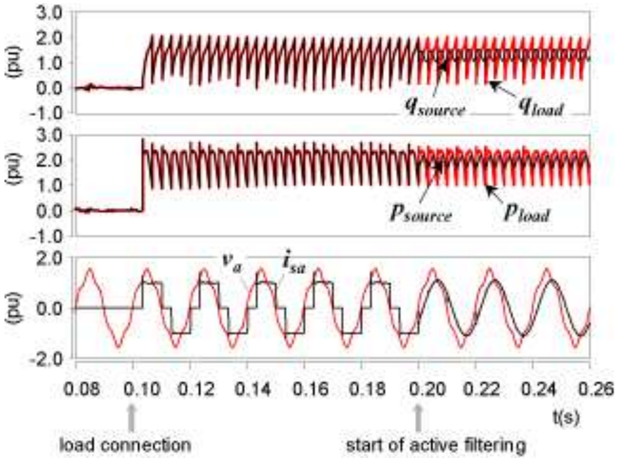


Figure 6: Simulation results from the sinusoidal source current control strategy.

it may not be very common to filter only $\tilde{q}$, but it may be common to have applications were q, including $\tilde{q}$, may be filtered without filtering $\tilde{p}$. In fact, when such compensations are done the filtering algorithm introduces some harmonics that are not present in the original current (load current). Depenbrok et al. criticized the *pq Theory* because of this; however, as it will be shown in this section, this phenomenon is not exactly a problem and depends on what one wants to have as a result of the compensation. To simplify the analysis, first a circuit with fifth order harmonic component of the negative sequence type will be analyzed, then the seventh harmonic component of the positive sequence type. Of course, this problem is not limited to these two harmonic components.

The Fifth Harmonic Component

Let us consider that the current of a given load has only the fifth order harmonic component. The phase angle difference between phases is such that this current is of the negative-sequence type and given by,

$$\begin{cases} i_{a5}(t) = \sqrt{2} I_5 \sin\left(5\omega t + \phi_5\right) \\ i_{b5}(t) = \sqrt{2} I_5 \sin\left(5\omega t + \phi_5 + \frac{2\pi}{3}\right) \\ i_{c5}(t) = \sqrt{2} I_5 \sin\left(5\omega t + \phi_5 - \frac{2\pi}{3}\right) \end{cases} \quad (4)$$

The Clarke Transformation of these currents is given by:

$$\begin{cases} i_{\alpha 5} = \sqrt{3} I_5 \sin(5\omega t + \phi_5) \\ i_{\beta 5} = \sqrt{3} I_5 \cos(5\omega t + \phi_5) \end{cases} \quad (5)$$

The $\alpha - \beta$ transformation of the voltage is given by:

$$\begin{cases} v_{\alpha} = \sqrt{3} V \sin(\omega t) \\ v_{\beta} = -\sqrt{3} V \cos(\omega t) \end{cases} \quad (6)$$

Therefore, the real and imaginary powers are given by:

$$\begin{aligned} p_5 &= -3VI_5 \cos(6\omega t + \phi_5) \\ q_5 &= -3VI_5 \sin(6\omega t + \phi_5) \end{aligned} \quad (7)$$

These powers have only oscillating component at the six times the line frequency. If we decide to compensate for these oscillating powers it is possible to calculate the α and β compensation currents using

$$i_{\alpha p5} = \frac{v_\alpha}{v_\alpha^2 + v_\beta^2} p_5 \quad \text{and} \quad i_{\alpha q5} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2} q_5 \quad (8)$$

$$i_{\beta p5} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2} p_5 \quad \text{and} \quad i_{\beta q5} = \frac{-v_\alpha}{v_\alpha^2 + v_\beta^2} q_5 \quad (9)$$

Substituting (7) in (8) and (9) the following currents can be calculated:

$$\begin{aligned} i_{\alpha p5} &= \frac{\sqrt{3}}{2} I_5 \sin(5\omega t + \phi_5) - \frac{\sqrt{3}}{2} I_5 \sin(7\omega t + \phi_5) \\ i_{\alpha q5} &= \frac{\sqrt{3}}{2} I_5 \sin(5\omega t + \phi_5) + \frac{\sqrt{3}}{2} I_5 \sin(7\omega t + \phi_5) \\ i_{\beta p5} &= \frac{\sqrt{3}}{2} I_5 \cos(5\omega t + \phi_5) + \frac{\sqrt{3}}{2} I_5 \cos(7\omega t + \phi_5) \\ i_{\beta q5} &= \frac{\sqrt{3}}{2} I_5 \cos(5\omega t + \phi_5) - \frac{\sqrt{3}}{2} I_5 \cos(7\omega t + \phi_5) \end{aligned}$$

where,

$$i_{\alpha 5} = i_{\alpha p5} + i_{\alpha q5} \text{ and } i_{\beta 5} = i_{\beta p5} + i_{\beta q5}. \quad (10)$$

Observing the above equations it is possible to note two interesting facts:

- $i_{\alpha p5}$and $i_{\alpha q5}$ as well as $i_{\beta p5}$ and $i_{\beta q5}$ contain seventh order harmonic components and this harmonic component was not present in the original current;
- The seventh order harmonic components in α-axis, $i_{\alpha p5}$, is equal to the seventh order component of the $i_{\alpha q5}$, except that the signal is the inverse, so normally they sum zero and do not appear in the circuit. The same is valid for the β-axis current $i_{\beta p5}$ and $i_{\beta q5}$.

If the idea is to compensate for the currents that is dependent on $\tilde{p}$ or $\tilde{q}$, using the $p - q$ Theory it is possible to define compensation currents using the gain k_p and k_q as given in the following equation:

$$i_{\alpha 5c} = k_p i_{\alpha p5} + k_q i_{\alpha q5} \text{ and } i_{\beta 5c} = k_p i_{\beta p5} + k_q i_{\beta q5} \quad (11)$$

In this case the source current will be

$$i_{\alpha s} = i_\alpha + i_{\alpha 5c} \text{ and } i_{\beta s} = i_\beta + i_{\beta 5c}. \quad (12)$$

If the gains k_p and k_q have the same value, the seventh order harmonic component is totally eliminated in the source current. However, if $k_p \neq k_q$ the seventh order harmonic components in $i_{\alpha p5}$does not cancel the seventh order harmonic component in $i_{\alpha q5}$, therefore an harmonic component that was not present in the original current is introduced in the source current. For this reason, these seventh order harmonic component current and all the current of this type will be called the "hidden current".

Half of the fifth order harmonic current component produces oscillating real power $\tilde{p}$ and therefore is responsible for the active oscillating energy flowing in a three-phase circuit. The other half of the fifth order current harmonic component transports no energy at all, because it produces the oscillating imaginary power $\tilde{q}$.

In the filtering process, the worst situation happens when the *p-q Theory* is used to compensate for the oscillating imaginary power $\tilde{q}$only or $\tilde{p}$ only. In these cases, the seventh order harmonic components (the hidden current) in $i_{\alpha p5}$ and $i_{\beta p5}$ will not be cancelled by the seventh order harmonic component in $i_{\alpha q5}$ and $i_{\beta q5}$, respectively. Therefore, the hidden current will appear with maximum magnitude. It is important to note that in these cases as the fifth order harmonic component is of the negative-sequence type the hidden current component is at a higher frequency.

When only q is used to filter the current, the source current after compensation will contain the hidden current component. In principle, it is not possible to say that this is bad or good thing. What can be said truly is that the imaginary power in the source is zero. In other words, only current that contributes to the energy transport is present.

When only $\tilde{p}$ is used to filter the current, the source current after compensation will also contain the hidden current, as in the previous case. In this case, all the oscillating real power in the source is eliminated. Therefore, if the objective is to eliminate the oscillating energy in the circuit this is the solution. This compensation technique may be more interesting when dealing with drives or specific generation system than with conventional generation. In fact, this procedure is important when torque ripple in the motor or generator has to be eliminated.

On the other hand, if the objective of the filter is to eliminate partially or all the fifth order harmonic component without introducing the hidden current, the oscillating real power and the oscillating imaginary power must be compensated with $k_p = k_q$. This is the case when a passive filter is used, which has no capability to filter separately the "real" harmonic, that portion that produces oscillating real power or to filter the "imaginary" harmonic, that portion that produces the imaginary power.

It is clear from the analysis that the use of *p-q Theory* for the current compensation allows much more flexibility for the filter design than the passive one.

The Seventh Harmonic Component

Next, the seventh harmonic current component of the positive sequence type is analyzed. These currents are given by:

$$\begin{cases} i_{a7}(t) = sqrt2 I_7 \sin\left(7\omega t + \phi_7\right) \\ i_{b7}(t) = \sqrt{2} I_7 \sin\left(7\omega t + \phi_7 - \frac{2\pi}{3}\right) \\ i_{c7}(t) = \sqrt{2} I_7 \sin\left(7\omega t + \phi_7 + \frac{2\pi}{3}\right) \end{cases} \tag{13}$$

The Clarke Transformation of these currents are given by:

$$\begin{cases} i_{\alpha 7} = \sqrt{3} I_7 \sin(7\omega t + \phi_7) \\ i_{\beta 7} = \sqrt{3} I_7 \cos(7\omega t + \phi_7) \end{cases} . \tag{14}$$

The real and imaginary powers generated by these seventh order harmonic currents have only oscillating components at six times the line frequency, like in the case of the fifth order harmonic and are given by:

$$\begin{aligned} p &= 3VI_7 \cos(6\omega t + \phi_7) \\ q &= 3VI_7 \sin(6\omega t + \phi_7) \end{aligned} \tag{15}$$

Here, like in the case for the fifth order harmonic analysis, it is possible to calculate the currents as a function of p and q powers in α and β axis as:

$$\begin{aligned} i_{\alpha p7} &= -\frac{\sqrt{3}}{2} I_7 \cos(5\omega t + \phi_7) + \frac{\sqrt{3}}{2} I_7 \cos(7\omega t + \phi_7) \\ i_{\alpha q7} &= \frac{\sqrt{3}}{2} I_7 \cos(5\omega t + \phi_7) + \frac{\sqrt{3}}{2} I_7 \cos(7\omega t + \phi_7) \\ i_{\beta p7} &= -\frac{\sqrt{3}}{2} I_7 \sin(5\omega t + \phi_7) + \frac{\sqrt{3}}{2} I_7 \sin(7\omega t + \phi_7) \\ i_{\beta q7} &= \frac{\sqrt{3}}{2} I_7 \sin(5\omega t + \phi_7) + \frac{\sqrt{3}}{2} I_7 \sin(7\omega t + \phi_7) \end{aligned}$$

These equations show that the hidden currents have exactly the same properties like in the case of the fifth order harmonic. However, its frequency is lower than the original frequency and is at five times the line frequency. All the conclusions made for the case of the fifth order harmonic is valid for this case.

The active power filter has high level of flexibility and can be designed to filter a current component that produces imaginary power or real power. However, as shown above, this filter may create the hidden current. In many cases these hidden currents are in a high frequency range and do not change the current characteristics. However, for some filter operating at low frequencies the design should put some attention on this problem to avoid undesirable harmonic frequency in the circuit. Naturally, in most cases the flexibility of the *p-q Theory* for the design of active filter control is a positive point as shown in Akagi, Nabae, and Ato (1986) and Akagi (1994).

5 CONCLUSIONS

This paper analyzed two problems of the *p-q Theory*: the influence of the distorted voltage waveforms at the PCC and the hidden current. The solution for the first problem is the use of a phase locked loop (PLL) circuit so as to used only the fundamental positive-sequence (or negative sequence) for the *p-q Theory* algorithm. The hidden current may not be a problem if it is at a high frequency, but it can be avoided if filters with similar gains are used both for $\tilde{p}$ and $\tilde{q}$ calculation.

ACKNOWLEDGEMENT

This work was partially supported by PRONEX/CNPq.

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