Testing a New Strategy to Treat Divergent Amplitudes in QED

A. P. Baêta Scarpelli, O. A. Battistel and M. C. Nemes
Instituto de Física, ICEX
Universidade Federal de Minas Gerais
Caixa Postal 702, CEP: 30161-970, Belo Horizonte, MG, Brazil

Received on 15 October, 1997

We analyse a new strategy to manipulate and calculate divergent amplitudes in the context of Quantum Electrodynamics. We compare our results with results of Dimensional Regularization and (one parameter) Pauli-Villars regularization prescriptions and conclude that the present technique allows for an unambiguous determination of the physical content of the divergent amplitude. The results obtained for QED can be easily extended to nonrenormalizable theories where they should be most especially useful.

I. Introduction

Quantum Field Theory is today accepted as the most adequate tool to study the dynamics of interacting particles. The reason for this is the spectacular success of Quantum Eletrodynamics (QED) in the determination of observables in the context of perturbative calculations. The success of QED was not immediate and is intimately connected to an adequate interpretation of the infinities which appear in a perturbative expansion involving loops. This procedure which eliminates the divergencies of the amplitudes in favor of a reinterpretation of physical parameters, Renormalization Theory, has been crucial for the applicability of QFT in general. The criterium "renormalizability" has always been used as a guide for the construction of fundamental theories. This was the case of the Electroweak Theory and Quantum Chromodynamics. Unfortunately the quantitative success of QED could not be estabished in this context, given its much more complicated structure in what concerns a perturbative analisys of low energy processes. On the other hand, the physics of low energy hadrons is an important research subject. In this context one is led to consider effetive theories with symmetry content as close as possible to the symmetries expected to be important from QCD. Such theories, however, are in general nonrenormalizable. This, in turn, involves limitations in what concerns the treatment of divergent amplitudes, since any regularization prescription adopted can not be removed afterwards [1]. The main difficulty in this domain of Quantum Field Theory is to adopt a prescription for the manipulation and calculation of divergent integrals such that the symmetry content of the underlying model and basic precepts of QFT (such as unitarity) be still contained in the calculated amplitude.

Recently a new technique for this purpose has been developed by O. A. Battistel [2] which is essencially independent of regularization prescriptions. One of the important differences between the method of ref [2] and the convencional ones is that it does not introduce modifications of the integrands in any intermediate step of the calculation.

The purpose of the present contribution is to test this technique in the context of QED to answer the following question: can one extract the physical content of the amplitudes in an automatically and unambiguously way? Obtaining a positive answer to this question would present obvious advantages in what refers the use one parameter 4- dimensional regularizations. Even when Dimensional Regularization [3] is used the final result is of course unique but not free from finite constants, typical of such technique, such as Euler's gama constant. The structure of the ambiguity in the finite part in the case of QED is such that it can be absorbed in the renormalization parameters, with the argument that otherwise such constant should be universal. This ambivalency which is harmless in QED may be fatal for nonrenormalizable theories. Before using the new technique in such context it is important to test it in QED and show it is capable of eliminating the ambiguisties.

This work is organized as follows: in section II we introduce the method. Section III contains the results of the electon self energy, vacuum polarization tensor and vertex correction at one loop level. We identify in a regularization independent context the mathematical conditions which are at the root of the ambiguities in the definition of the finite part contribuction and establish consistency conditions for regularization prescriptions such that physically sound results can be found. Conclusions can be found in section IV.

II. The strategy to manipulate and calculate divergent amplitudes

In this section we present the main physical requirements which led to the construcuion of an alternative prescription to manipulate and calculate divergent integrals [2]. This technique satisfies the following three requirements:

1. The final results should not present unphysical behavior such as complex thresholds associated to regularization parameters.

2. The final results should be independent of intermediate steps (uniqueness of the solution).

3. The physical predictions of the theory most not depend on how the integrals are manipulated.

The studies performed revealed that such requirements could be satisfied by adopting a set of rules, which consist in the procedure we will adopt here:

a) Divergent integrals which depend on the external momenta should be written as sum of divergent momentum independent integrals plus finite integrals. These last ones should not be affected by regularizations.

b) Divergent integrals, external momenta independent, should be reduced to the few divergent objects typical of the theory in question.

c) In the case of nonrenormalizable theories the remaining indefinite objects should be directly specified by phenomenology. In the case of renormalizable theories, they should, as usual, be incorporated in the redefinition of the physical constants of the theory at that level.

The first rule is directly associated to requirement 1, since it is necessary and sufficient for the elimination of unphysical behavior introduced by regularization. The second rule is necessary for the uniqueness of the results, i.e., in order that two equivalent forms of the amplitude do not lead to different results. Again for this rule to be satisfied, the following relations between divergent integrals of the same degree of divergence should be fullfiled:

$$\int_\Lambda \frac{d^4K}{(2\pi)^4}\frac{K_\mu K_\nu}{[K^2-M^2]^3}-\frac{g_{\mu\nu}}{4}\int_\Lambda \frac{d^4K}{(2\pi)^4}\frac{1}{[K^2-M^2]^2}=0 \tag{1}$$

$$q^\alpha q^\beta\int_\Lambda \frac{d^4K}{(2\pi)^4}\frac{K_\mu K_\nu K_\alpha K_\beta}{[K^2-M^2]^4}-\frac{1}{24}(q^2 g_{\mu\nu}+2q_\mu q_\nu)\int_\Lambda \frac{d^4K}{(2\pi)^4}\frac{1}{[K^2-M^2]^2}=0 \tag{2}$$

and

$$\int_\Lambda \frac{d^4K}{(2\pi)^4}\frac{K_\mu K_\nu}{[K^2-M^2]^2}-\frac{g_{\mu\nu}}{2}\int_\Lambda \frac{d^4K}{(2\pi)^4}\frac{1}{[K^2-M^2]}=0 \tag{3}$$

The symbol Λ indicates here the use of a regularization prescription. Note that all we need is the existence of a regularizing function which is even in the momentum and which satisfies the above three relations. For example Dimensional Regularization satisfies the three relations. In four dimensions a gaussian-type regularization also does the job [4]. Recently A. L. Mota has shown that the three listed relations are not but the requirement of translational invariance of free fields [4].

III. Electron self-energy, Vacuum Polarization Tensor and Vertex Correction

Electron self-energy

We start with the evaluation of the electron self energy. It is given by

$$-i\Sigma(p) = -e^2 \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{\gamma_\mu[\gamma^\mu p_\mu - \gamma^\mu K_\mu + M]\gamma^\mu}{[(p-K)^2 - M^2][K^2 - m^2]}, \tag{4}$$

where p stands for the electron external momentum, e its bare charge and M its bare mass. The matrices γ are Dirac's gamma matrices [4]. After taking the trace we can rewrite eq. (4) as

$$-i\Sigma(p) = 2e^2\{(\gamma^\mu p_\mu - 2M)I - \gamma^\mu I_\mu\}, \tag{5}$$

where

$$I = \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{1}{[(p-K)^2 - M^2][K^2 - m^2]}, \tag{6a}$$

$$I_\mu = \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{K_\mu}{[(p-K)^2 - M^2][K^2 - m^2]}. \tag{6b}$$

In order to illustrate the strategy we use we shall proceed to the evaluation of the integrals (6a) and (6b) in detail. According to our prescription, the integral I should be manipulated only by means of mathematical identities *at the level of the integrand* in order to separate the momentum dependent contributions as follows

$$I = \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{1}{[K^2 - m^2]} \left\{ \frac{1}{[K^2 - M^2]} - \frac{p^2 - 2p\cdot K}{[(p-K)^2 - M^2][K^2 - M^2]} \right\}, \tag{7}$$

or, identically,

$$I = \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{1}{[K^2 - M^2]^2} - \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{M^2 - m^2}{[K^2 - M^2]^2[K^2 - m^2]} + \\ - \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{p^2 - 2p\cdot K}{[(K-p)^2 - M^2][K^2 - M^2][K^2 - m^2]}. \tag{8}$$

Next we define the first integral as $I_{\log}(M^2)$ and evaluate the other ones which are finite to get

$$I = I_{\log}(M^2) - \frac{i}{(4\pi)^2} Z_0(m^2, M^2, p^2), \tag{9}$$

where we define

$$Z_K(\lambda_1^2, \lambda_2^2, p^2) \equiv \int_0^1 dx x^K \ln\left[\frac{p^2 x(1-x) + (\lambda_1^2 - \lambda_2^2)x - \lambda_1^2}{(-\lambda_2^2)}\right] \tag{10}$$

Now we evaluate I_μ according to the same prescription. Following the same steps as before we rewrite the integrand as

$$I_\mu = \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{K_\mu}{[K^2 - M^2][K^2 - m^2]} - \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{(p^2 - 2p\cdot K)K_\mu}{[K^2 - M^2]^2[K^2 - m^2]} + \\ + \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{(p^2 - 2p\cdot K)^2 K_\mu}{[(K-p)^2 - M^2][K^2 - M^2]^2[K^2 - m^2]}. \tag{11}$$

The first term vanishes since it is an odd integrand. The second term should again be recast into the form

$$I_\mu = \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{2p^\nu K_\mu K_\nu}{[K^2 - M^2]^3} - \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{2p^\nu K_\mu K_\nu (M^2 - m^2)}{[K^2 - M^2]^3[K^2 - m^2]} + \\ + \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{(p^2 - 2p\cdot K)^2 K_\mu}{[(K-p)^2 - M^2][K^2 - M^2]^2[K^2 - m^2]}. \tag{12}$$

After evaluating the finite part we have

$$I_\mu = 2p^\nu \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{K_\mu K_\nu}{[K^2 - M^2]^3} - \frac{i}{(2\pi)^2} p_\mu Z_1(m^2, M^2, p^2). \tag{13}$$

Now the electron self energy can be writen as

$$\Sigma(p) = \frac{e^2}{8\pi^2}[(\gamma^\mu p_\mu - 2M)Z_0(m^2, M^2, p^2) - \gamma^\mu p_\mu Z_1(m^2, M^2, p^2)] +$$
$$+2e^2 i(\gamma^\mu p_\mu - 2M)\int_\Lambda \frac{d^4K}{(2\pi)^4}\frac{1}{[K^2-M^2]^2} - 4e^2 i\gamma^\mu p_\nu \int_\Lambda \frac{d^4K}{(2\pi)^4}\frac{K_\mu K_\nu}{[K^2-M^2]^3}. \tag{14}$$

Now our consistency condition (1) leads to

$$\Sigma(p) = \frac{e^2}{8\pi^2}[(\gamma^\mu p_\mu - 2M)Z_0(m^2, M^2, p^2) - \gamma^\mu p_\mu Z_1(m^2, M^2, p^2)] +$$
$$+ie^2(\gamma^\mu p_\mu - 4M)I_{\log}(M^2). \tag{15}$$

An important remark at this stage is that the consistency condition Eq. (1) was crucial in order to obtain the above result and, as we will discuss in what follows is at the root of the success (or lack of it) of other procedures.

Dimensional Regularization gives for the same amplitude the following well known result [5]

$$\Sigma(p) = \frac{e^2}{8\pi^2}[(\gamma^\mu p_\mu - 2M)Z_0(m^2, M^2, p^2) - \gamma^\mu p_\mu Z_1(m^2, M^2, p^2)] +$$
$$-\frac{e^2}{16\pi^2}(\gamma^\mu p_\mu - 2M) - \frac{e^2}{16\pi^2}(\gamma^\mu p_\mu - 4M)\left[\frac{1}{\epsilon} + A - \ln(-M^2) - \ln(4\pi)\right]. \tag{16}$$

The first term constitutes the finite part of the amplitude and the rest is the divergent part. It is important to call attention to the fact that the choice of the finite part was based on physical arguments. For physical reasons we expect it to have such form. We therefore had to incorporate some finite terms into the divergent part. This is an well known ambiguity inherent to the regularization prescription which is absent in the former procedure.

If we use a one parameter Pauli-Villars Regularization we get

$$I = \frac{i}{(4\pi)^2}[Z_0(\Lambda^2) - Z_0(m^2)],$$
$$I_\mu = \frac{i}{(4\pi)^2}p_\mu[Z_1(\Lambda^2) - Z_1(m^2)] \tag{17}$$

and

$$\Sigma(p) = \frac{e^2}{8\pi^2}[(\gamma^\mu p_\mu - 2M)Z_0(m^2, M^2, p^2) - \gamma^\mu p_\mu Z_1(m^2, M^2, p^2)] +$$
$$-\frac{e^2}{8\pi^2}[(\gamma^\mu p_\mu - 2M)Z_0(\Lambda^2) - \gamma^\mu p_\mu Z_1(\Lambda^2)]. \tag{18}$$

Again we have two terms. The first one corresponds to the finite part of the amplitude and the second to the divergent one. The identification of finite and divergent contributions is not well defined since it depends on the stage of the calculation where the limits of the regularization parameters are taken. For example if possible cacellations are effected first in $[Z_K(\Lambda^2) - Z_K(m^2)]$ (where only the first argument of the functions are explicitely shown) and then the extraction of the finite part is made we would get a different result. This could also cause symmetry violations.

In the context of our procedure renormalization can be performed without having to invoke any specific regularization, by directly incorporating the divergency $I_{\log}(M^2)$ in the redefinition of the electron mass

$$\delta M = 3Mie^2 I_{\log}(M^2). \tag{19}$$

The vacuum polarization

Again according to Feynman's rules and QED's Lagrangean the vacuum polarization tensor can be written as

$$i\Pi_{\mu\nu}(q) = -e^2 \int \frac{d^4K}{(2\pi)^4} \frac{\mathrm{Tr}\{\gamma_\nu[\gamma^\mu K_\mu - \gamma^\mu q_\mu + M]\gamma_\mu[\gamma^\mu K_\mu + M]\}}{[(K-q)^2 - M^2][K^2 - M^2]} \quad (20)$$

which can be conveniently rewritten in the form

$$i\Pi_{\mu\nu}(q) = -4e^2 \left\{ 2I_{\mu\nu} - 2I_\mu q_\nu - \frac{1}{2}\left[I^{(1)} + I^{(2)} - q^2 I \right] \right\} \quad (21)$$

where

$$I_{\mu\nu} = \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{K_\mu K_\nu}{[(K-q)^2 - M^2][K^2 - M^2]}, \quad (22)$$

$$I_\mu = \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{K_\mu}{[(K-q)^2 - M^2][K^2 - M^2]}, \quad (23)$$

$$I = \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{1}{[(K-q)^2 - M^2][K^2 - M^2]}, \quad (24)$$

$$I^{(1)} = \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{1}{[K^2 - M^2]} \quad (25)$$

and

$$I^{(2)} = \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{1}{[(K-q)^2 - M^2]}. \quad (26)$$

We next present the results obtained for each one of the above integrals when treated according to our prescription:

$$I^{(1)} = I^{(2)} = I_{\mathrm{quad}}(M^2), \quad (27)$$

where

$$I_{\mathrm{quad}}(M^2) \equiv \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{1}{[K^2 - M^2]}, \quad (28)$$

$$I = I_{\log}(M^2) - \frac{i}{(4\pi)^2} Z_0(M^2, M^2, q^2), \quad (29)$$

$$I_\mu = \frac{1}{2} q_\mu I_{\log}(M^2) + \frac{i}{(4\pi)^2}\frac{1}{2} q_\mu Z_0(M^2), \quad (30)$$

$$I_{\mu\nu} = \frac{g_{\mu\nu}}{2} I_{\mathrm{quad}}(M^2) + \frac{1}{12}(4q_\mu q_\nu - q^2 g_{\mu\nu}) I_{\log}(M^2) +$$
$$+ \frac{i}{(4\pi)^2}\frac{1}{3q^2}\left\{ \frac{1}{2}(q^2 - 4M^2) q^2 g_{\mu\nu} + 2(2q^2 + M^2) q_\mu q_\nu \right\} Z_0(M^2, M^2, q^2) + \frac{i}{(4\pi)^2}\frac{1}{18}(q_\mu q_\nu - q^2 g_{\mu\nu}) \quad (31)$$

The final result is given by

$$\Pi_{\mu\nu}(q) = -\frac{4}{3} e^2 \frac{1}{(4\pi)^2} [q_\mu q_\nu - q^2 g_{\mu\nu}] \left[\frac{1}{q^2}(q^2 + 2M^2) Z_0(M^2) + \frac{1}{3} \right] +$$
$$-\frac{4}{3} i e^2 [q_\mu q_\nu - q^2 g_{\mu\nu}] I_{\log}(M^2). \quad (32)$$

Note that the quadratic divergences cancel out. Again, to obtain this important result it has been crucial to make use of the consistency conditions. Otherwise the result would be inconsistent. Note also that there is no ambiguity in the determination of the finite and divergent parts of the amplitude and gauge invariance is respected.

Vertex Correction

We next treat the vertex correction at one loop level:

$$-ie\Lambda_\nu(p,q) = -e^3 \int_\Lambda \frac{d^4K}{(2\pi)^4} \frac{\gamma_\mu[\gamma^\mu p_\mu - \gamma^\mu K_\mu + M]\gamma_\nu[\gamma^\mu q_\mu - \gamma^\mu K_\mu + M]\gamma^\mu}{[(p-K)^2 - M^2][(q-K)^2 - M^2][K^2 - m^2]} \quad (33)$$

or, in term of the integrals,

$$-ie\Lambda_\nu(p,q) = 8e^3MJ_\nu - e^3[4M(q_\nu+p_\nu) - \gamma_\nu(p^2+q^2) - 2\gamma^\mu p_\mu\gamma_\nu\gamma^\alpha q_\alpha]J +$$
$$-e^3\gamma_\nu[I^{(1)}(p)+I^{(1)}(q)] + 4e^3\gamma^\mu J_{\mu\nu} +$$
$$-e^3[\gamma_\nu(p^\mu+q^\mu) + 2(q_\alpha\gamma^\mu\gamma_\nu\gamma^\alpha + p_\alpha\gamma^\alpha\gamma_\nu\gamma^\mu)]I_\mu, \tag{34}$$

where we made the definition

$$J = \int \frac{d^4K}{(2\pi)^4}\frac{1}{[(p-K)^2-M^2][(q-K)^2-M^2][K^2-m^2]} \tag{35}$$

and similar for J_μ and $J_{\mu\nu}$, with K_μ and $K_|muK_\nu$ in the numerator respectivaly. As was defined before

$$I^{(1)}(p) = \int_\Lambda \frac{d^4K}{(2\pi)^4}\frac{1}{[(p-K)^2-M^2][K^2-m^2]}. \tag{36}$$

Note that we have already performed some of the integrals. We have to obtain the results of the integrals with three denominators. The integrals J and J_μ are finite and do not need any technique to be solved. So, the only new divergent integral is $J_{\mu\nu}$. The results of the integrals with three denominators are

$$J = \frac{i}{(4\pi)^2}\xi_{00}(p^2,q^2,m^2,M^2), \tag{37}$$

$$J_\mu = \frac{i}{(4\pi)^2}\{p_\mu\xi_{10}(p^2,q^2,m^2,M^2) + q_\mu\xi_{01}(p^2,q^2,m^2,M^2)\}, \tag{38}$$

$$J_{\mu\nu} = \frac{i}{(4\pi)^2}\{p_\mu p_\nu\xi_{20} + q_\mu q_\nu\xi_{02} + (p_\mu q_\nu + p_\nu q_\mu)\xi_{11}\} +$$
$$\frac{i}{(4\pi)^2}\frac{g_{\mu\nu}}{2}F(p^2,q^2) + \frac{g_{\mu\nu}}{4}I_{\log}(M^2), \tag{39}$$

where we have defined

$$\xi_{nm}(p^2,q^2,m^2,M^2) = \int_0^1\int_0^{1-x} dxdy x^n y^m \frac{1}{H^2(p^2,q^2,m^2,M^2)}, \tag{40}$$

with

$$H^2(p^2,q^2,m^2,M^2) = p^2x(1-x) + q^2y(1-y) + (m^2-M^2)x + (m^2-M^2)y - 2p\cdot qxy - m^2, \tag{41}$$

and

$$F(p^2,q^2,m^2,M^2) = \int_0^1\int_0^{1-x} dxdy \ln[H^2(p^2,q^2,m^2,M^2) - \ln(-M^2)]. \tag{42}$$

For simplicity we will omit some arguments of these functions, since in our case the arguments are always the same. The final result for the amplitude is then

$$-ie\Lambda_\nu(p,q) = -e^3\frac{i}{(4\pi)^2}[4M(q_\nu+p_\nu) - \gamma_\nu(p^2+q^2) - 2\gamma^\mu p_\mu\gamma_\nu\gamma^\alpha q_\alpha]\xi_{00} +$$
$$+e^3\frac{i}{(4\pi)^2}8M[p_\nu\xi_{10} + q_\nu\xi_{01}] +$$
$$-e^3\frac{i}{(4\pi)^2}[\gamma_\nu(p^\mu+q^\mu) + 2(q_\alpha\gamma^\alpha\gamma^\mu\gamma_\nu + p_\alpha\gamma^\alpha\gamma_\nu\gamma^\mu)][p_\mu\xi_{10} + q_\mu\xi_{01}] +$$
$$+e^3\frac{i}{(4\pi)^2}\{2\gamma_\nu F(p^2+q^2) + 4\gamma^\mu[p_\mu p_\nu\xi_{20} + q_\mu q_\nu\xi_{02} + (p_\mu q_\nu + p_\nu q_\mu)\xi_{11}]\} +$$
$$+e^3\frac{i}{(4\pi)^2}[Z_0(m^2,p^2) + Z_0(m^2,q^2)] - \gamma_\nu e^3 I_{\log}(M^2) \tag{43}$$

The above result differs from the one obtained by Dimensional Regularization through details. We quote the D. R. result:

$$-ie\Lambda_\nu(p,q) = -e^3\frac{i}{(4\pi)^2}[4M(q_\nu + p_\nu) - \gamma_\nu(p^2+q^2) - 2\gamma^\mu p_\mu\gamma_\nu\gamma^\alpha q_\alpha]\xi_{00} +$$
$$+e^3\frac{i}{(4\pi)^2}8M[p_\nu\xi_{10} + q_\nu\xi_{01}] +$$
$$-e^3\frac{i}{(4\pi)^2}[\gamma_\nu(p^\mu+q^\mu) + 2(q_\alpha\gamma^\alpha\gamma^\mu\gamma_\nu + p_\alpha\gamma^\alpha\gamma_\nu\gamma^\mu)][p_\mu\xi_{10} + q_\mu\xi_{01}] +$$
$$+e^3\frac{i}{(4\pi)^2}\{2\gamma_\nu F(p^2+q^2) + 4\gamma^\mu[p_\mu p_\nu\xi_{20} + q_\mu q_\nu\xi_{02} + (p_\mu q_\nu + p_\nu q_\mu)\xi_{11}]\} +$$
$$+e^3\frac{i}{(4\pi)^2}[Z_0(m^2,p^2) + Z_0(m^2,q^2)] +$$
$$-e^3\frac{i}{(4\pi)^2}\gamma_\nu\left[\frac{1}{\xi} + A - \ln(-M^2) - \ln(4\pi)\right] + e^3\gamma_\nu\frac{i}{(4\pi)^2} \quad (44)$$

The discussion of these differences with the methods will become clear in the conclusion which follows.

IV. Conclusions

We are now in a position to discuss what are the essential steps which characterize the adequacy of regularization schemes. For concreteness we base the discussion in the electron self energy, since it suffices. At a given point of the calculation with our procedure we have used eq. (1). Due to the use of these relation, the result obtained for the finite part was well defined and unambiguous. The divergent content was completely absorbed in the integral

$$I_{\log}(M^2) = \int_\Lambda \frac{d^4K}{(2\pi)^4}\frac{1}{[K^2-M^2]^2}. \quad (45)$$

When we calculated the same amplitude by means of Dimensional Regularization, finite regularization dependent terms have appeared and had to be absorbed in the divergent part of the amplitude. This could in principle be avoided had we simply effected the traces in 4-dimensions and only then extended the dimensions to become continuous and complex. Let us then discuss what happened with the one parameter Pauli-Villars Regularization. Assume we proceed as suggested here, separated finite from divergent contributions and only then apply the regularization. We would have, just before explicitely introducing the regularization

$$-i\Sigma(p) = -i\tilde{\Sigma}(p) + 2e^2(\gamma^\mu p_\mu - 2M)\int_\Lambda \frac{d^4K}{(2\pi)^4}\frac{1}{[K^2-M^2]^2} +$$
$$-4e^2\gamma^\mu p_\nu \int_\Lambda \frac{d^4K}{(2\pi)^4}\frac{K_\mu K_\nu}{[K^2-M^2]^3}, \quad (46)$$

where $-i\tilde{\Sigma}(p)$ stands for the finite part. We may at this point use the one parameter Pauli- Villars Regularization to evaluate the divergent integrals. We would get precisely the same result as the one we got by using the regularization from the very beginning. We note, then, that the key result behind obtaining or not an adequate physical result for the self energy is intimately connected to respecting or not the difference Eq.(1). The virtue of Dimensional Regularization wherever it applies is in the fact that D. R. preserves such relation and we note that it is only at this point where the extension to 2w dimensions is really necessary, since the rest of the divergence content can be maintained in $I_{\log}(M^2)$. The problem with the one parameter P. V. regularization is that relation (1) is not respected. The subsequent introduction of parameters to actally evaluate the integrals renders the determination of the finite contribution ambiguous.

When evaluating the polarization tensor we needed the two other consistency conditions (2) and (3), and they have been decisive for preserving gauge invariance. Also because of relation (3), it was cancelled the quadratic divergence. The calculations with one parameter Pauli-Villars regularization there would be violation of gauge invariance precisely due to the non fullfilment of these relations.

In conclusion, the method presented here has the virtue of isolating the divergent content of the amplitude in an unambiguous way, which is not possible with another techniques. Besides, it made clear the origin of the problems.

The present work is meant as a test of the new prescription which, although applicable to renormalizable theories, should be most specially useful in the context of nonrenormalizable models work along these lines is presently under way.

Acknowledgements: This work has been partially supported by FAPEMIG and CNPq.

References

1 S.P. Klevansky, Rev. Mod. Phys. **64**, 649 (1992).

2 O. A. Battistel, PhD Thesis.

3 G. t'Hooft and M. Veltman, Nuclear Physics B**44**, 189 (1972a).

4 A. L. Mota, PhD Thesis.

5 L. H. Ryder, *Quantum Field Theory*, Cambridge University Press (second edition).

6 W. Pauli and F. Villars, Rev. Mod. Phys. **21**, 434 (1949).