Weak Decay Constant of Pseudoscalar Mesons in a QCD-Inspired Model

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Received on 15 August, 2003.

We show that a linear scaling between the weak decay constants of pseudoscalars and the vector meson masses is supported by the available experimental data. The decay constants scale as $f_m/f_\pi = M_V/M_\rho$ (f_m decay constant and M_V vector meson ground state mass). This simple form is justified within a renormalized light-front QCD-inspired model for quark-antiquark bound states.

1 Introduction

Effective theories applied to describe hadrons, which are inspired by Quantum Chromodynamics [1, 2, 3], can be useful in indicating direct correlations between different hadronic properties. In this way, it is possible to pin down the relevant dependence of the observables with some physical scales that otherwise would have no simple reason to show a direct relation, besides being properties of the same underlying theory. For example, if one points out a systematic dependence of a hadron observable with its mass even in a phenomenological model, this fact may be regarded as an useful guide for presenting results obtained in Lattice QCD. In fact, systematic correlations between different meson properties with mass scales were found from the solution of Dyson-Schwinger equations [4].

One intriguing aspect is the dependence of the weak decay constant of the pseudoscalar meson with its mass. For light mesons up to D, the weak decay constant tends to increase with the mass, while numerical simulations of quenched lattice-QCD indicate that $f_D > f_B$ [5], which is still maintained with two flavor sea quarks [5, 6]. General arguments, within Dyson-Schwinger formalism for QCD in the heavy quark limit, says that the weak decay constant should be inversely proportional to $\sqrt{M_m}$ [7] (M_m is the pseudoscalar meson mass). Effective QCD inspired models valid for low energy scales can also be called to help to investigate this subtle point. In these models [2, 3, 8], the interaction is flavor independent, while the masses of constituent quarks can be changed, which naturally implies in correlations between observables and masses.

Our aim here, is to investigate the pseudoscalar weak decay constant within a QCD inspired model [8]. The mass operator equation for the valence component of the meson light-front wave function, described as a bound system of a constituent quark and antiquark of masses m_1 and m_2, was derived in the effective one-gluon-exchange interaction approximation [1]. In a simplified form [2, 3], the effective mass operator equation is written as:

$$M_m^2\psi_m(x,\vec{k}_\perp) = \left[\frac{\vec{k}_\perp^2+m_1^2}{x}+\frac{\vec{k}_\perp^2+m_2^2}{1-x}\right]\psi_m(x,\vec{k}_\perp) - \int dx'd\vec{k}'_\perp \xi(x,x')\left(\frac{4m_1m_2}{3\pi^2}\frac{\alpha}{Q^2}-\lambda\right)\psi_m(x',\vec{k}'_\perp), \quad (1)$$

where the phase space factor is

$$\xi(x,x') = \frac{\theta(x')\theta(1-x')}{\sqrt{x(1-x)x'(1-x')}} \ ,$$

and ψ_m is the projection of the light-front wave-function in the quark-antiquark Fock-state component. The mean square momentum transfer $((k'_1-k_1)^2+(k'_2-k_2)^2)/2$ gives $-Q^2$ (k_i and k'_i are the quark four-momenta). The coupling constant α defines the strength of the Coulomb-like potential and λ is the bare coupling constant of the Dirac-delta hyperfine interaction. The energy transfer in Q^2 is left out. Confinement comes through the binding of the constituents in the meson, which in practice keeps the quarks inside the mesons.

The mass operator equation (1) needs to be regularized and renormalized in order to give physical results, such development has been performed in Ref. [8]. In that work, it was obtained the renormalized form of the equation for the bound state mass, which is i) invariant under renormalization group transformations, ii) the physical input is given by the pion mass and radius, and iii) no regularization parameter.

In the work of Ref. [8], the quark mass was changed to allow the study of mesons with one light antiquark plus a strange, charm or bottom quark. The masses of the constituent quarks were within the range of 300 up to 5000 MeV. A mass of 384 MeV was obtained for the up and down quarks from the rho meson mass, which in the model is weakly bound. The Dirac-delta interaction comes from an effective hyperfine interaction which splits the pseudo-scalar and vector meson states. In the singlet channel the hyperfine interaction is attractive, which is not valid for spin one mesons. In the model, the Dirac-delta interaction mock up short-range physics which are brought by the empirical value of the pion mass, and from that a reasonable description of the binding energies of the constituent quarks forming the pseudoscalar mesons was found [8]. The model, without the Coulomb like interaction, was also able to describe the binding energies of the ground state of spin 1/2 baryons containing two light quarks and a heavy one [9].

Within the effective model of Eq.(1), the low-lying vector mesons are weakly bound systems of constituent quarks while the pseudo-scalars are more strongly bound [8]. This allows to calculate the masses of constituent quarks directly from the masses of the ground state of vector mesons [9]:

$$\begin{aligned} m_u &= \frac{1}{2}M_\rho = 384\ MeV\ , \\ m_s &= M_{K^*} - \frac{1}{2}M_\rho = 508\ MeV\ , \\ m_c &= M_{D^*} - \frac{1}{2}M_\rho = 1623\ MeV\ , \\ m_b &= M_{B^*} - \frac{1}{2}M_\rho = 4941\ MeV\ , \end{aligned} \tag{2}$$

where it is used the values of 768 MeV, 892 MeV, 2007 MeV and 5325 MeV for the ρ, K^*, D^* and B^* masses, respectively [10].

Here, we use the effective model to predict a physical property directly related to the wave-function of the ground state of the pseudo scalar mesons. We calculate the weak decay constants (f_m) of K^+, D^+, D_s^+, for which experimental values are known [10]. Besides the constituent quark masses from Eq.(2) and the pion mass, our calculation needs as input the pion weak decay constant, $f_\pi = 92.4 \pm 0.07 \pm 0.25$ MeV [10]. The eigenfunction of the interacting mass squared operator from Eq.(1), for large transverse momentum, behaves as the asymptotic wave-function, which decreases slowly as $1/\vec{p}_\perp^{\,2}$. Therefore, in the calculation of the weak decay constants it is necessary to regulate the logarithmic divergence in the transverse momentum integration and take care of the cut-off dependence to be able to give an unique answer. One has to consider that the pion decay constant provides the short-range information contained in the pion wave function, which we suppose to be the same for all pseudo-scalars. Here, we just write the divergent integral in the transverse momentum in terms of f_π and from that obtain the other decay constants.

2 Meson light-front wave function

The valence component of the meson (m) wave function is the solution of Eq.(1). In the approximation where the Coulomb-like interaction is considered in lowest order, the pseudo-scalar meson wave function is given by [8]

$$\begin{aligned} \psi_m(x,\vec{k}_\perp) &= \frac{1}{\sqrt{x(1-x)}}\frac{G_m}{M_m^2 - M_0^2} \\ &\times \left[1 - \int \frac{dx' d\vec{k}'_\perp \theta(x')\theta(1-x')}{\sqrt{x'(1-x')}} \left(\frac{4m_1 m_2}{3\pi^2}\frac{\alpha}{Q^2}\right)\right. \\ &\left.\times \frac{1}{M_m^2 - {M'}_0^2}\right] , \end{aligned} \tag{3}$$

where

$$M_0^2 = \frac{\vec{k}_\perp^2 + m_1^2}{x} + \frac{\vec{k}_\perp^2 + m_2^2}{1-x} , \tag{4}$$

in the frame in which the meson has zero transverse momentum. (${M'}_0^2$ is obtained from M_0^2 by substitution of $\vec{k}_\perp$ by $\vec{k}'_\perp$ and x by x'.) The overall normalization of the $q\overline{q}$ Fock-component of the meson wave-function (3) is G_m.

In this first calculation of the decay constant within this model, we are going to assume the dominance of the asymptotic form of the meson wave function and simply use

$$\psi_m(x,\vec{k}_\perp) = \frac{1}{\sqrt{x(1-x)}}\frac{G_m}{M_m^2 - M_0^2} . \tag{5}$$

To obtain the pseudoscalar decay constants, we follow Ref. [11]. To construct the observables in terms of the meson wave function, one has to account for the coupling of the spin of the quarks, which is described by an effective Lagrangian density with a pseudo-scalar coupling between the quark ($q_1(\vec{x})$ and $q_2(\vec{x})$) and meson ($\Phi_m(\vec{x})$) fields [11]

$$\mathcal{L}_{eff}(\vec{x}) = -iG_m \Phi_m(\vec{x})\, \overline{q}_1(\vec{x})\gamma^5 q_2(\vec{x}) + h.c. , \tag{6}$$

the coupling constant is G_m. From the effective Lagrangian above one can derive meson observables and write them in terms of the light-front asymptotic wave function, Eq. (5). To achieve this goal, it is necessary to eliminate the relative x^+-time ($x^+ = t + z$) between the constituents in the physical amplitude, which then allows to write the meson observable in terms of the wave function [11].

3 Results for the weak decay constant of pseudoscalar mesons

The pseudoscalar meson weak decay constant is calculated from the matrix element of the axial current $A^\mu(0)$, between the vacuum state $|0\rangle$ and the meson state $|q_m\rangle$ with four momentum q_m [10]:

$$\langle 0 \mid A^\mu(0) \mid q_m\rangle = \imath\sqrt{2} f_m q_m^\mu , \tag{7}$$

where $A^\mu(\vec{x}) = \overline{q}(\vec{x})\gamma^\mu\gamma^5 q(\vec{x})$.

Using the pseudoscalar Lagrangian, Eq. (6), one can calculate the matrix element of the axial current, which is expressed by a one-loop diagram, and written as:

$$\begin{aligned} &\imath\sqrt{2}M_m f_m = \\ &N_c G_m \int \frac{d^4k}{(2\pi)^4} Tr\left[\gamma^+\gamma^5 S_2(k-q_m)\gamma^5 S_1(k)\right] , \end{aligned} \tag{8}$$

where $\gamma^+ = \gamma^0 + \gamma^3$, $N_c = 3$ is number of colors and $S_i(p) = \imath/(\not{p} - m_i + \imath\epsilon)$ is the propagator of the quark field.

TABLE I. Results for the pseudoscalar meson weak decay constants f_m calculated with Eq.(12). The inputs for the model are vector meson ground state mass (M_v) and f_π given in the table. All masses and decay constants are in MeV. Experimental values from [10]. (†The experimental mass of D_s^{*+} is quite near to the model $c\bar{s}$ vector meson mass which is given by $m_c + m_s = 2131$ MeV.)

$q\bar{q}$	M_m^{exp}	M_v^{exp}	f_m^{model}	f_m^{exp}
$\pi^+(u\bar{d})$	140	771 (ρ)	92.4	$92.4 \pm .07 \pm 0.25$
$K^+(u\bar{s})$	494	892 (K^*)	107	$113.0 \pm 1.0 \pm 0.31$
$D^+(c\bar{d})$	1869	2010 (D^{*+})	241	$212^{+127+6}_{-106-28}$
$D_s^+(c\bar{s})$	1969	$2112^\dagger$ (D_s^{*+})	253	$201 \pm 13 \pm 28$

By integration over k^- in Eq.(8), the relative light-front time between the quarks is eliminated and one derives the expression of f_m, suitable for the introduction of the meson light-front wave function. Performing the Dirac algebra and integrating analytically over k^-, one obtains in the meson rest frame

$$f_m = -\frac{\sqrt{2}}{8\pi^3} N_c \int_0^1 dx\,(m_1(1-x) + m_2 x)$$
$$\times \int dk_\perp^2 \frac{G_m}{x(1-x)M_m^2 - k_\perp^2 - m_1(1-x) - m_2 x} , \quad (9)$$

where $x = k^+/q_m^+$ is the light-cone momentum fraction.

One can write Eq.(9) in terms of the valence component of the pseudoscalar meson wave function as:

$$f_m = \frac{\sqrt{2}}{8\pi^3} N_c \int_0^1 \frac{dx}{\sqrt{x(1-x)}}\,(m_1(1-x) + m_2 x)$$
$$\times \int dk_\perp^2 \psi_m(x, \vec{k}_\perp) \,. \quad (10)$$

The above expression is general, and one can use it to calculate the decay constant of any pseudoscalar meson state from the eigenfunction of the squared mass operator, which is a solution of Eq.(1).

We observe that Eq.(9), written in terms of the asymptotic part of the valence wave function has a logarithmic divergence in the transverse momentum integration due to the slow decrease of the wave function. From a physical point of view, one could think that the regularization scale is larger than the masses of the quarks and the divergent transverse momentum integration, will be defined through the value of f_π, for example. Therefore, one has:

$$f_m = C \int_0^1 dx\,((1-x)m_1 + x m_2) \,, \quad (11)$$

and C determined by f_π. One observe as well that, $f_m \propto m_1 + m_2$, which in our model is the vector meson mass, thus one immediately gets:

$$\frac{f_m}{f_\pi} = \frac{M_v}{M_\rho} \,. \quad (12)$$

The numerical results of Eq.(12) are shown in Table I. It is verified that a reasonable description of the weak decay constants of the pseudoscalar mesons is possible within the effective light-front model. However, we have made use only of the asymptotic form of the wave function and one needs to investigate the decay constant with more refined wave functions, eigenstates of the squared mass operator, Eq.(1), which includes the dynamics of the effective quarks. Therefore, the results which are overestimating the heavier meson decay constants, can be an indication that a more elaborated wave function is needed, although one cannot discard that other mechanisms could be relevant[7].

Also, we intend to perform the evaluation of the weak decay constants using a more sophisticated version of the model, where confinement is included, which so far was shown to describe the S-wave meson spectrum [13].

In summary, we have shown the existence of a direct proportionality between the weak decay constants and the masses of the vector mesons ground states, which can provide an useful tool in the systematic study of these quantities.

Acknowledgments: We thank CNPq and FAPESP for financial support.

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