Coherent Properties and Rabi Oscillations in Two-Level Donor Systems

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Coherent properties and Rabi oscillations in two-level donor systems, under terahertz excitation, are theoretically investigated. Here we are concerned with donor states in bulk *GaAs* and *GaAs* − (*Ga,Al*)*As* quantum dots. We study confinement effects, in the presence of an applied magnetic field, on the electronic and on-center donor states in *GaAs* − (*Ga,Al*)*As* dots, as compared to the situation in bulk *GaAs*, and estimate some of the associated decay rate parameters. Using the optical Bloch equations with damping, we study the time evolution of the 1s and 2p$_+$ states in the presence of an applied magnetic field and of a terahertz laser. We also discuss the role played by the distinct dephasing rates on the photocurrent and calculate the electric dipole transition moment. Results indicate that the Rabi oscillations are more robust as the total dephasing rate diminishes, corresponding to a favorable coherence time.

One of the proposals concerning new solid-state quantum computers (QC) is the possibility of using quantum dots (QDs) as the basic architecture for their implementation [1]. In that case, discrete electronic charge or spin states are the qubits responsible for encoding quantum information [2–4]. One crucial point is that the model-qubit system operates under the conditions that decoherence processes are weak and single-qubit and two-qubit unitary operations are controlled. This implies that a QC would be effective only if the decoherence times are much longer than the time involved in the single- and two-qubit operations. The use of laser pulses in controlling the qubit operations may overcome this limitation. Coherent optical excitations in two-level donor systems in bulk *GaAs* [2], under applied magnetic fields, were converted into deterministic photocurrents. The 1*s* and 2*p$_+$* donor states are the model qubits coherently manipulated by laser radiation. A more favorable situation concerning the coherence time may be obtained if the excited donor state lies below the continuum. Donor-doped QDs which exhibit pronounced confining effects are then natural candidates to both theoretical [5] and experimental investigations.

Here we investigate the confinement effects of a model spherical QD, under applied magnetic field, on the electronic and on-center donor states in *GaAs* − (*Ga,Al*)*As* QDs. We investigate the conditions in which one may obtain a bound 2*p$_+$* state in contrast to the resonant one in the study by Cole *et al* [2], and using the optical Bloch equations with damping terms [6], we study the time evolution of the 1s and 2*p$_+$* donor states under the action of a terahertz laser.

The on-center donor Hamiltonian for a spherical *GaAs* − (*Ga,Al*)*As* QD, in the effective-mass approximation, is given by

$$H = -\nabla^2 + \gamma l_z + \gamma^2 \rho^2/4 + V_b(r) - 2/r, \qquad (1)$$

where $l_z = \frac{1}{i}\frac{\partial}{\partial \phi}$, $V_b(r)$ is the QD barrier potential, $\gamma = \frac{e\hbar B}{2m^*cR^*} = $

$\mu_B^* B/R^* = (a_0^*/l_B)^2$ is the ratio of the magnetic and Coulomb energies (for donors in *GaAs*, $\gamma = 1$ corresponds to an applied magnetic field of $\approx 6.9\,T$), $R^* \approx 5.9\,meV$ is the GaAs donor effective Rydberg, $l_B = (\hbar c/eB)^{1/2}$ is the magnetic length (or cyclotron radius), and a_0^* and μ_B^* are the effective Bohr radius and effective Bohr magneton, respectively. Using hydrogenic-like envelope wave functions [7], the 1s and 2p$_\pm$ energies may then be variationally obtained as a function of the *z*-direction applied magnetic field.

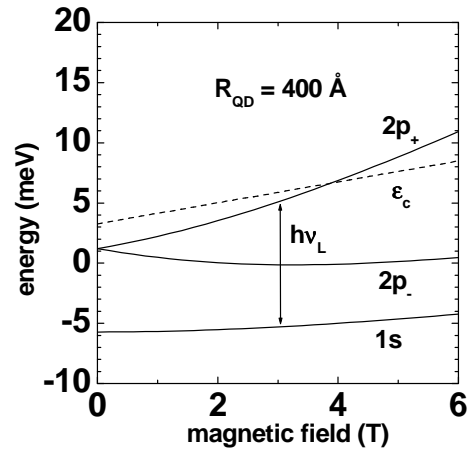FIG. 1: Magnetic-field dependence of $\varepsilon_c = \varepsilon_o + \gamma$ and of the energies of 1s, 2p$_-$, and 2p$_+$ donor states in a R=400 Å *GaAs* − *Ga$_{0.7}$Al$_{0.3}$As* QD. The energy $h\nu_L$ for a 2.52 THz free-electron laser is shown as a 1s-2p$_+$ transition.

The magnetic-field dependence of the energies of donor states 1s, 2p$_-$, 2p$_+$, and of $\varepsilon_c = \varepsilon_0 + \gamma$, for a $R = 400$ Å *GaAs* − *Ga$_{0.7}$Al$_{0.3}$As* spherical QD are shown in Fig. 1. Notice that ε_0 is the energy of the lowest confined non-occupied electronic state. The arrow shows the 1s-2p$_+$ transition energy
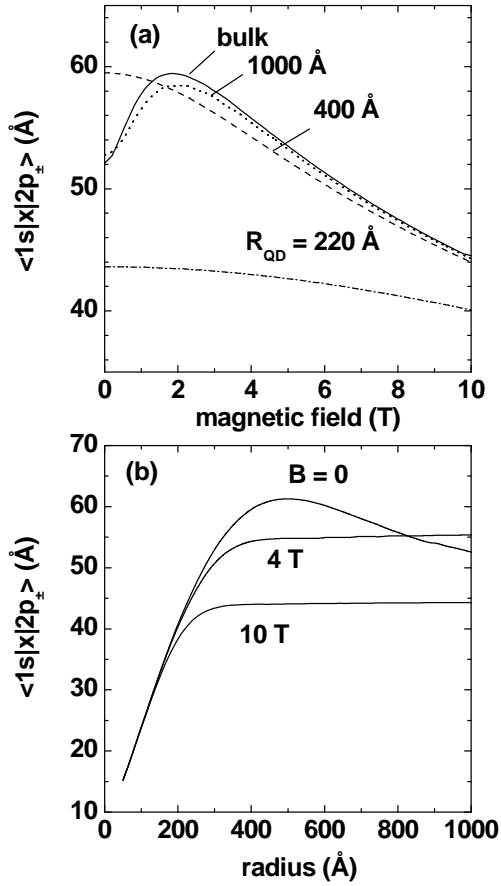
FIG. 2: Magnetic-field (a) and radius (b) dependence of the $\langle 1s|x|2p_\pm \rangle$ matrix elements for bulk *GaAs* and different *GaAs* − *Ga*$_{0.7}$*Al*$_{0.3}$*As* spherical QDs.

corresponding to $2.52\,THz$, which is the free-electron laser frequency used in the experimental measurements by Cole *et al* [2]. The confinement effects due to the QD are such that a magnetic field of $\approx 3.0\ T$ tunes the THz radiation to the corresponding $1s$-$2p_+$ transition, with the $2p_+$ below the continuum states, and this source of decoherence is removed [2].

The *x*-component of the corresponding 1s-2p$_\pm$ dipole matrix element, $d_{12}^x = \langle 1s|x|2p_\pm \rangle$, and the Rabi frequency $\Omega_R = E_{THz}d_{12}^x/\hbar$, where E_{THz} is the amplitude of the terahertz electric field (in the *x*-direction), are then calculated. Fig. 2 shows the $\langle 1s|x|2p_\pm \rangle$ matrix elements as a function of the applied magnetic field and of the dot radius. Notice that the d_{12}^x matrix-elements results for a *GaAs* − *Ga*$_{0.7}$*Al*$_{0.3}$*As* spherical QD of radius R = 1000 Å are essentially the same as for bulk GaAs, as expected. In the bulk regime, for small values of applied magnetic fields, the $\langle 1s|x|2p_\pm \rangle$ matrix elements increase with increasing magnetic fields which can be related to the magnetic-field confinement effects being stronger for the $2p_\pm$ state as compared to the $1s$ state. This leads to a larger overlap between $1s-$ and $2p_\pm-$like wave functions and therefore to a larger value of the d_{12}^x matrix-elements. One notices the existence of a maximum around $2-3T$, which may be
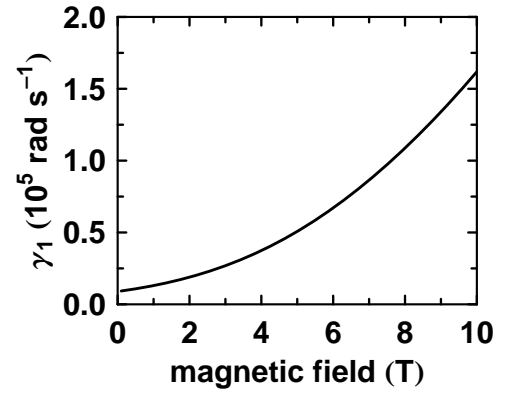

FIG. 3: 1s-2p$_+$ recombination rate (γ_1) as a function of the magnetic field for a R = 400 Å *GaAs* − *Ga*$_{0.7}$*Al*$_{0.3}$*As* QD.

traced back to the fact that, with increasing values of the magnetic field, the Landau magnetic length and Bohr radius (i.e., magnetic and Coulomb energies) become comparable.

The time evolution of the elements of the density matrix within a two-level model for the donor-QD system are obtained via standard procedures [5, 6], from the set of optical Bloch equations, i.e.,

$$\frac{d\rho_{11}}{dt} = -i\Omega_R \cos(\omega_L t)(\rho_{21} - \rho_{12}) + \gamma_1 \rho_{22} \qquad (2)$$

$$\frac{d\rho_{22}}{dt} = +i\Omega_R \cos(\omega_L t)(\rho_{21} - \rho_{12}) - (\gamma_1 + \gamma_3)\rho_{22}$$

$$\frac{d\rho_{12}}{dt} = +i\omega_{21}\rho_{12} + i\Omega_R \cos(\omega_L t)(\rho_{11} - \rho_{22}) - \gamma_2 \rho_{12}$$

$$\frac{d\rho_{21}}{dt} = -i\omega_{21}\rho_{21} - i\Omega_R \cos(\omega_L t)(\rho_{11} - \rho_{22}) - \gamma_2 \rho_{21}$$

where ω_L is the THz laser frequency, and ω_{21} is the energy separation of the 1s and 2p$_+$ impurity levels. The parameters γ_1, γ_2, and γ_3 are recombination rates as introduced phenomenologically in Cole *et al* [2].

To calculate the time evolution of the photosignal corresponding to the 1s-2p$_+$ transition, we first estimate the recombination rates. The parameter γ_1, giving the rate of spontaneous emission of photons due to $2p_+ \rightarrow 1s$ transitions, may be obtained by

$$\gamma_1 = \frac{2e^2}{3\pi\varepsilon_o \hbar c^3}\,\omega_{21}^3\,|\langle 1s|x|2p_+ \rangle|^2. \qquad (3)$$

Figure 3 shows that the 1s-2p$_+$ recombination rate for the R = 400 Å *GaAs* − *Ga*$_{0.7}$*Al*$_{0.3}$*As* QD increases with the magnetic field. However, the calculated values are negligible in the THz range of the oscillation frequencies of the problem, and may be neglected. The dephasing rate γ_2 at the lowest THz field is estimated from far-infrared measurements as $\gamma_2 = 6.0 \times 10^{10}$ rad s^{-1} [8], and the ionization rate γ_3 is set as $\gamma_3 = 0$, since the 2p$_+$ excited donor state lies below the first Landau level [2].

Calculated results are shown in Fig. 4 (a), at resonance, for a R = 400 Å *GaAs* − *Ga*$_{0.7}$*Al*$_{0.3}$*As* spherical QD (B $\approx 3.0T$
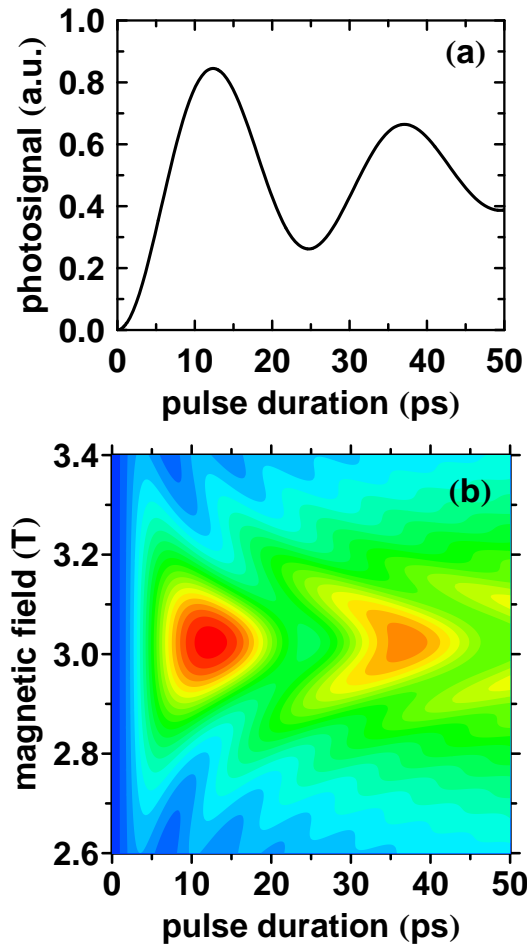
FIG. 4: (a) Theoretical 1s-2p$_+$ Rabi oscillations, at resonance, for a fixed THz electric field in the case of a R = 400 Å $GaAs - Ga_{0.7}Al_{0.3}As$ spherical QD; (b) Contour plot of photocurrent versus pulse duration and magnetic field at the same fixed THz electric field ($\gamma_{1,3} = 0$ and $\gamma_2 = 0.6 \times 10^{11} \, rad/s$). The THZ field is $E_{THz} = 3 \times 10^4 \, V/m$.

and $E_{THz} = 3 \times 10^4 \, V/m$). One clearly notices that the displayed Rabi oscillations are more robust, as compared with the experiment by Cole *et al* [2] in doped bulk *GaAs*, and therefore it suggests that a donor-doped QD leads to a favorable coherence time so that qubit operations may be efficiently controlled. The corresponding contour plot of the photocurrent for varying pulse duration and applied magnetic field is depicted in Fig. 4 (b).

In summary, we have discussed the possible conditions under which decoherence is weak and qubit operations are efficiently controlled in QDs. Using the optical Bloch equations with damping, we are able to investigate, in a phenomenolog-

ical manner, the coherence effects on Rabi oscillations associated to donor states confined in $GaAs - (Ga,Al)As$ QDs in the presence of an applied magnetic field and under a terahertz laser. The pronounced confining effects of semiconductor QDs are shown to provide better coherence-time conditions for the Rabi oscillations.

Acknowledgments

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