X (3872) in QCD Sum Rules

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QCD spectral sum rules is used to test the nature of the meson $X(3872)$, assumed to be an exotic four-quark ($c\bar{c}q\bar{q}$) state with $J^{PC} = 1^{++}$. For definiteness, the current proposed recently by Maiani et al [1] is used, at leading order in α_s, considering the contributions of higher dimension condensates. The value $M_X = (3.94 \pm 0.11)$ GeV is found which is compatible, within the errors, with the experimental candidate X(3872). The uncertainties of our estimates are mainly due to the one from the c quark mass.

Keywords: QCD sum rules; Tetraquarks; Axial vector mesons

I. INTRODUCTION

In august 2003, there's been a report by BELLE[2] of a narrow resonance in $B^+ \to X(3872)K^+ \to J/\psi\,\pi^+\pi^-K^+$ decays. It has been soon after confirmed by D0 [3], CDF II [4] and BABAR [5]. The collective data from these experiments give an averaged mass of $m_X = 3871,9 \pm 0,5$ MeV [6]. The width has been stablished by BELLE at a upper limit of $2,3$ MeV at 90% confidence level and the most probable quantum numbers are: $J^{PC} = 1^{++}$ [6].

BELLE has also found the resonance in the channel: $X(3872) \to J/\psi\,\pi^+\pi^-\pi^0$. The relative strength of this decay mode is given by[7]:

$$\frac{Br\left(X \to \pi^+\pi^-\pi^0\,J/\psi\right)}{Br\left(X \to \pi^+\pi^-\,J/\psi\right)} = 1.0 \pm 0.4 \pm 0.3 \qquad (1)$$

This strong isospin violation makes it difficult to understand the X as a charmoniun. Besides, the charmoniun spectrum state with correct quantum numbers, $\chi_1^{'}(3925)$, has both the mass and the decay width (≈ 16 MeV) too high to be gracefully identified with the observed resonance[6].

The anomalous nature of the X has led to many speculations: tetraquark [1, 8], cusp [9], hybrid [10], or glueball [11]. Another explanation is that the $X(3872)$ is a $D\bar{D}^*$ bound state [12–16], as predicted before its discovery.

In this work the QCD spectral sum rules (the Borel/Laplace Sum Rules (SR) [17–19] and Finite Energy Sum Rules (FESR) [19–21]) will be used to study the two-point functions of the axial vector meson, $X(3872)$, assumed to be a four-quark state. In previous calculations, the Sum Rule (SR) approach was used to study the light scalar mesons [22–25] and the $D_{sJ}^+(2317)$ meson [26, 27], considered as four-quark states and a good agreement with the experimental masses was obtained. However, the tests were not decisive as the usual quark–antiquark assignments also provide predictions consistent with data and more importantly with chiral symmetry expectations [19, 23, 28, 29]. In the four-quark scenario, scalar mesons can be considered as S-wave bound states of diquark-antidiquark pairs, where the diquark was taken to be a spin zero color anti-triplet. Ref. [1] will be followed here, and the $X(3872)$ will be considered as a $J^{PC} = 1^{++}$ state with the symmetric spin distribution: $[cq]_{S=1}[\bar{c}\bar{q}]_{S=0} + [cq]_{S=0}[\bar{c}\bar{q}]_{S=1}$. Therefore, the corresponding lowest-dimension interpolating operator for describing X_q is given by:

$$j_\mu = \frac{i\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}}[(q_a^TC\gamma_5c_b)(\bar{q}_d\gamma_\mu C\bar{c}_e^T) + (q_a^TC\gamma_\mu c_b)(\bar{q}_d\gamma_5 C\bar{c}_e^T)]\,, \qquad (2)$$

where a, b, c, ... are color indices, C is the charge conjugation matrix and q denotes a u or d quark.

II. THE QCD EXPRESSION OF THE TWO-POINT CORRELATOR

The SR are constructed from the two-point correlation function

$$\begin{aligned}\Pi_{\mu\nu}(q) &= i\int d^4x\, e^{iq.x}\langle 0|T[j_\mu(x)j_\nu^\dagger(0)]|0\rangle = \\ &= -\Pi_1(q^2)(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + \Pi_0(q^2)\frac{q_\mu q_\nu}{q^2}. \end{aligned} \qquad (3)$$

Since the axial vector current is not conserved, the two functions, Π_1 and Π_0, appearing in Eq. (3) are independent and have respectively the quantum numbers of the spin 1 and 0 mesons.

The fundamental assumption of the sum rules approach is the principle of duality. Specifically, we assume that there is an interval over which the correlation function may be equivalently described at both the quark and the hadron levels. Therefore, on one hand, we calculate the correlation function at the quark level in terms of quark and gluon fields. On the other hand, the correlation function is calculated at the hadronic level introducing hadron characteristics such as masses and coupling constants. At the quark level, the complex structure of the QCD vacuum leads us to employ the Wilson's operator product expansion (OPE). The calculation of the phenomenological side proceeds by inserting intermediate states for the meson X. Parametrizing the coupling of the axial vector meson 1^{++}, X, to the current, j_μ, in Eq. (2) in terms of the meson decay constant f_X as:

$$\langle 0|j_\mu|X\rangle = \sqrt{2}f_X M_X^4\epsilon_\mu\,, \qquad (4)$$

the phenomenological side of Eq. (3) can be written as

$$\Pi_{\mu\nu}^{phen}(q^2) = \frac{2f_X^2M_X^8}{M_X^2 - q^2}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_X^2}\right) + \cdots\,, \qquad (5)$$

where the Lorentz structure projects out the 1^{++} state. The dots denote higher axial-vector resonance contributions that will be parametrized, as usual, through the introduction of a continuum threshold parameter s_0.

The OPE side will be evaluated at leading order in α_s and the contributions of condensates will be considered up to dimension five.

The correlation function, Π_1, in the OPE side can be written as a dispersion relation:

$$\Pi_1^{OPE}(q^2) = \int_{4m_c^2}^{\infty} ds \frac{\rho(s)}{s-q^2} , \tag{6}$$

where the spectral density is given by the imaginary part of the correlation function: $\pi\rho(s) = \mathrm{Im}[\Pi_1^{OPE}(s)]$. After making an inverse-Laplace (or Borel) transform of both sides, and transferring the continuum contribution to the OPE side, the sum rule for the axial vector meson X up to dimension-five condensates can be written as:

$$2 f_X^2 M_X^8 e^{-M_X^2/M^2} = \int_{4m_c^2}^{s_0} ds\, e^{-s/M^2}\, \rho(s) , \tag{7}$$

where $\rho(s)$ is the sum of the $\rho^{Dim}(s)$ below:

$$\rho^{pert}(s) = \frac{1}{2^{10}\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3}(1-\alpha-\beta) \times$$
$$\times (1+\alpha+\beta)\left[(\alpha+\beta)m_c^2 - \alpha\beta s\right]^4 ,$$
$$\rho^{\langle\bar{q}q\rangle}(s) = -\frac{m_c\langle\bar{q}q\rangle}{2^5\pi^4} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta}(1+\alpha+\beta) \times$$
$$\times \left[(\alpha+\beta)m_c^2 - \alpha\beta s\right]^2 ,$$
$$\rho^{\langle G^2\rangle}(s) = \frac{\langle g^2G^2\rangle}{2^9 3\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^2} \left[(\alpha+\beta)m_c^2 - \alpha\beta s\right] \times$$
$$\times \left[\frac{m_c^2(1-(\alpha+\beta)^2)}{\beta} - \frac{(1-2\alpha-2\beta)}{2\alpha}\left[(\alpha+\beta)m_c^2 - \alpha\beta s\right]\right], \tag{8}$$

where the integration limits are given by $\alpha_{min} = (1-\sqrt{1-4m_c^2/s})/2$, $\alpha_{max} = (1+\sqrt{1-4m_c^2/s})/2$ and $(\beta_{min} = \alpha m_c^2)/(s\alpha - m_c^2)$. The dominant contributions from the dimension-five condensates have also been included :

$$\rho^{mix}(s) = \frac{m_c\langle\bar{q}g\sigma.Gq\rangle}{2^6\pi^4} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \Bigg[-\frac{2}{\alpha}(m_c^2 - \alpha(1-\alpha)s) +$$
$$+ \int_{\beta_{min}}^{1-\alpha} d\beta \left[(\alpha+\beta)m_c^2 - \alpha\beta s\right] \left(\frac{1}{\alpha} + \frac{\alpha+\beta}{\beta^2}\right) \Bigg], \tag{9}$$

III. LSR PREDICTIONS OF M_X

In order to extract the mass M_X without worrying about the value of the decay constant f_X, one must take the derivative of Eq. (7) with respect to $1/M^2$, divide the result by Eq. (7) and obtain:

$$M_X^2 = \frac{\int_{4m_c^2}^{s_0} ds\, e^{-s/M^2}\, s\, \rho(s)}{\int_{4m_c^2}^{s_0} ds\, e^{-s/M^2}\, \rho(s)} . \tag{10}$$

In the numerical analysis of the sum rules, the values used for the quark masses and condensates are (see e.g. [19, 30–32]): $m_c = (1.23 \pm 0.05)$ GeV, $m_u = 2.3$ MeV , $m_d = 6.4$ MeV, $m_q = (m_u + m_d)/2 = 4.3$ MeV, $\langle\bar{q}q\rangle = -(0.23 \pm 0.03)^3$ GeV^3, $\langle\bar{q}g\sigma.Gq\rangle = m_0^2\langle\bar{q}q\rangle$ with $m_0^2 = 0.8$ GeV^2 and $\langle g^2G^2\rangle = 0.88$ GeV^4. The sum rules were evaluated in the range $1.6 \leq M^2 \leq 2.8$ for three values of s_0: $s_0^{1/2} = 4.1$ GeV, $s_0^{1/2} = 4.3$ GeV and $s_0^{1/2} = 4.5$ GeV.

Comparing the relative contribution of each term in Eqs. (8) and (9), to the right hand side of Eq. (7) there is a quite good OPE convergence for $M^2 > 1.9$ GeV^2. This analysis allows us to determine the lower limit constraint for M^2 in the sum rules window [33].

The upper limit constraint for M^2 is determined by imposing that the QCD continuum contribution should be smaller than the pole contribution. The maximum value of M^2 for which this constraint is satisfied depends on the value of s_0. The comparison between pole and continuum contributions gives $M^2 < 3.0$ for $s_0^{1/2} = 4.5$ GeV, $M^2 < 2.6$ for $s_0^{1/2} = 4.3$ GeV and $M^2 < 2.3$ GeV^2 for $s_0^{1/2} = 4.1$ GeV.

Figure 1 shows the X meson mass obtained from Eq. (10), in the relevant sum rules window, with the upper and lower validity limits indicated. From Fig. 1 we see that the results are reasonably stable as a function of M^2. In the numerical analysis the range of M^2 values from 2.1 GeV^2 until the one allowed by the sum rule window criteria shall then be considered for each value of s_0 .

Varying the QCD parameters inside the limits shown before and taking into account the range of values for M^2 and s_0 we get:

$$M_X = (3.94 \pm 0.17)\,\mathrm{GeV} \tag{11}$$

The error is due to the combined effect of M^2, s_0, $\langle\bar{q}q\rangle$, and m_c (m_c having the greatest effect).

IV. FESR PREDICTION FOR M_X

As an alternative, one can use the FESR, which can be obtained from Eq. (7) by taking the limit $1/M^2 \to 0$ and equating the same power in $1/M^2$ in the two sides of the sum rules to get n equations:

$$2 f_X^2 M_X^8 M_X^{2n} = \int_{4m_c^2}^{s_0} ds\, s^n \rho(s), \quad n = 0,1,2... \tag{12}$$

Finally, dividing two subsequent equations (with n and $n+1$) one can obtain the mass M_X for any chosen value of n (which, formally, is expected to be the same for any n):

$$M_X^2 = \frac{\int_{4m_c^2}^{s_0} ds\, s^{n+1}\rho(s)}{\int_{4m_c^2}^{s_0} ds\, s^n \rho(s)}, \quad n = 0,1,2... \tag{13}$$

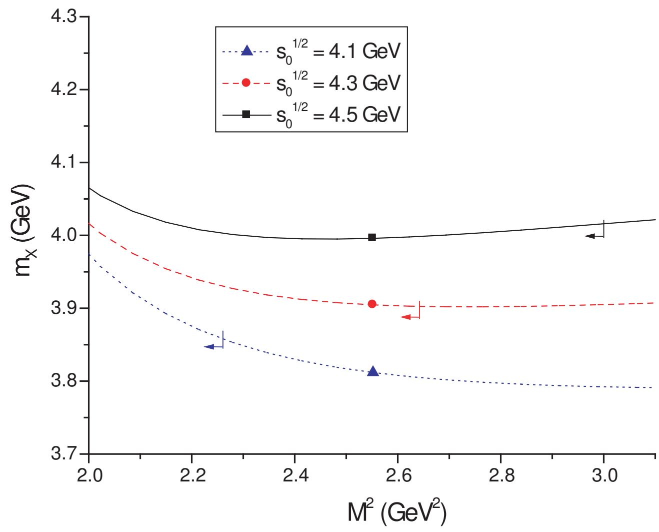

FIG. 1: The X meson mass as a function of the sum rule parameter (M^2) for different values of the continuum threshold: $s_0^{1/2} = 4.5$ GeV (solid line), $s_0^{1/2} = 4.3$ GeV (dashed line) and $s_0^{1/2} = 4.1$ GeV (dotted line). The arrows indicate the region allowed for the sum rules: the lower limit (cut below 2.0 GeV^2) is given by OPE convergence requirement and the upper limit by the dominance of the QCD pole contribution.

In contrast to the previous method, the FESR have the advantage of giving correlations between the mass and the continuum threshold s_0, which can be used to avoid inconsistencies in the determination of these parameters. Ideally, one looks at a minimum in the function $M_X(s_0)$, which would provide a good criteria for fixing both s_0 and M_X. The results for different values of n are very similar, therefore, in Fig. 2, only the results for $n = 0$ and $n = 1$ are shown. One can see in Fig. 2 that there is no stability in s_0, which presumably indicates the important role of the QCD continuum in the analysis.

The FESR results agree with LSR ones only for a small value of s_0: $s_0^{1/2} \approx 4.2$ GeV, but since no stability was found we consider the LSR to be more reliable in this case.

V. CONCLUSIONS

We have presented a QCD spectral sum rules analysis of the two-point function of the $X(3872)$ meson considered as a four quark state. We find that the sum rules result in Eq. (11) is compatible with experimental data. An improvement of this result needs an accurate determination of m_c.

Once the mass of the $X(3872)$ is understood, it remains to explain why it is so narrow. There are presumably many multiquark states, but most of them are very broad and cannot be singled out from the continuum. In a recent study [34], based on the same interpolating field as the one used here, it was shown that, in order to explain the small width of the $X(3872)$, one has to choose a particular set of diagrams contributing to its decay. However, it will be desirable if this investigation can be checked from alternative approaches, like e.g. lattice calculations. If confirmed, this method can be straightforwardly repeated to a variety of currents for understanding the width and the internal structure of the $X(3872)$.

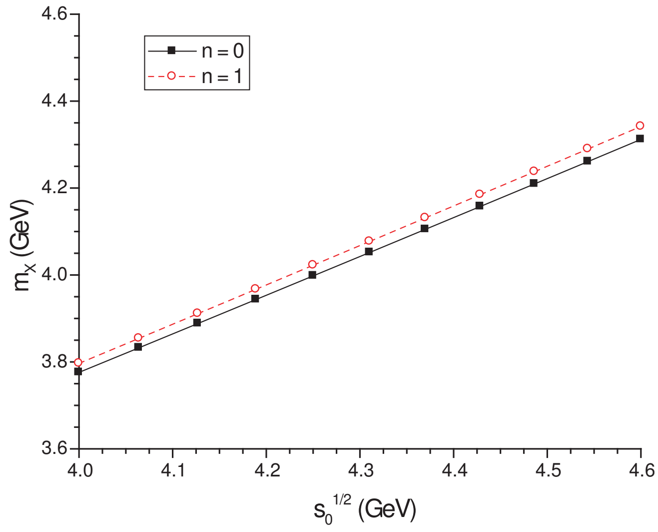

FIG. 2: The FESR results in Eq. (13) for M_X as a function of s_0 for $n = 0$ and $n = 1$.

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