Brazilian Relativistic $O(q^4)$ Two-Pion Exchange Nucleon-Nucleon Potential: Parametrized Version

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In our recent works we derived a chiral $O(q^4)$ two-pion exchange nucleon-nucleon potential (TPEP) formulated in a relativistic baryon (RB) framework, expressed in terms of the so called low energy constants (LECs) and functions representing covariant loop integrations. In order to facilitate the use of the potential in nuclear applications, we present a parametrized version of our configuration space TPEP.

Keywords: Two-pion exchange; Nucleon-nucleon potential; Relativistic baryon; Chiral symmetry; Chiral perturbation theory

I. INTRODUCTION

The one-pion exchange NN potential $(OPEP)$ is simple, has been well established long ago, and dominates completely partial waves with orbital momentum $L \geq 5$. The two-pion exchange potential $(TPEP)$, on the other hand, is rather complex and has become free of important ambiguities only in the 1990s, after the systematic use of chiral symmetry in its theoretical description [1–11].

Chiral perturbation theory is based on the existence of a characteristic scale q, set by both pion four-momenta and nucleon three-momenta, such that $q < 1$ GeV. Due to this technique, nowadays one understands rather well the internal hierarchies of the NN potential in terms of chiral layers. Leading terms of the chiral $TPEP$ are of order $O(q^2)$ and expansions which go up to $O(q^4)$ are already available. One of them was produced recently by our group [10, 11]. We departed from a relativistic Lagrangian and evaluated the relevant Feynman diagrams covariantly, without resorting to heavy baryon approximations. The so obtained T-matrix was then transformed into a potential, expressed in terms of covariant loop integrals and observable parameters. Without loss of generality, one may choose these parameters to be either the subthreshold coefficients extracted from πN scattering [12] or the low-energy constants $(LECs)$ present in the effective Lagrangian.

The research on the $TPEP$ performed in the last decade has set its conceptual foundations on a rather solid basis, comparable to that of the OPEP in the late sixties. On the other hand, the $TPEP$ depends on several $LECs$, which must be extracted from either πN scattering data or direct fits of NN phase shifts. In general, this last kind of procedure tends to be computationally heavy, for theoretical results are usually given as cumbersome expressions. In order to make applications easier, in this work we present a parametrized version of our $O(q^4)$ relativistic configuration space $TPEP$, which is numerically accurate for distances larger than 1 fm. It is based on the theoretical expressions derived in Ref. [11].

II. THEORETICAL POTENTIAL

The $O(q^4)$ relativistic expansion of the TPEP produced in refs.[10, 11] was based on the evaluation of three families of diagrams given in Fig. 1. The first of them involves only pion and nucleon degrees of freedom into single loops and corresponds to the minimal realization of chiral symmetry[3]. It includes the subtraction of the iterated OPEP and yields the terms in the profile functions given below which are proportional to just g_A^4/f_π^4, g_A^2/f_π^4 or $1/f_\pi^4$. Terms proportional to $1/f_\pi^6$, on the other hand, come from two-loop processes, either in the form of t-channel contributions from the second family or s and u-channel terms embodied in the subthreshold coefficients of the third family. Finally, the third group of diagrams includes chiral corrections associated with other degrees of freedom, hidden within the LECs c_i and d_i, and gives rise to contributions which are proportional to either $(LEC)/f_\pi^4$ or $(LEC)^2/f_\pi^4$.

The configuration space potential has the isospin structure

$$V(\boldsymbol{r}) = V^+(\boldsymbol{r}) + \boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\, V^-(\boldsymbol{r})\,, \tag{1}$$

with

$$V^\pm(\boldsymbol{r}) = V_C^\pm + V_{LS}^\pm \Omega_{LS} + V_T^\pm\, \Omega_T + V_{SS}^\pm \Omega_{SS} + V_Q^\pm\, \Omega_Q\,, \tag{2}$$

and $\Omega_{LS} = \boldsymbol{L}\cdot(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)})/2$, $\Omega_T = 3\,\boldsymbol{\sigma}^{(1)}\cdot\hat{\boldsymbol{r}}\;\boldsymbol{\sigma}^{(2)}\cdot\hat{\boldsymbol{r}} - \boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}$, $\Omega_{SS} = \boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}$. The form of the operator Ω_Q in configuration space is highly non-local and can be found in ref.[13].

The radial components of the potential are expressed in terms of the following dimensionless profile functions $U_I^\pm$ with $I = C, LS, T, SS$:

$$V_C^\pm(r) \;=\; \tau^\pm\, U_C^\pm(x)\,, \tag{3}$$

$$V_{LS}^\pm(r) \;=\; \tau^\pm\, \frac{\mu^2}{m^2}\,\frac{1}{x}\,\frac{d}{dx}\, U_{LS}^\pm(x)\,, \tag{4}$$

$$V_T^\pm(r) \;=\; \tau^\pm\, \frac{\mu^2}{m^2}\left[\frac{d^2}{dx^2} - \frac{1}{x}\,\frac{d}{dx}\right] U_T^\pm(x)\,, \tag{5}$$

$$V_{SS}^\pm(r) \;=\; -\tau^\pm\, \frac{\mu^2}{m^2}\left[\frac{d^2}{dx^2} + \frac{2}{x}\,\frac{d}{dx}\right] U_{SS}^\pm(x)\,, \tag{6}$$

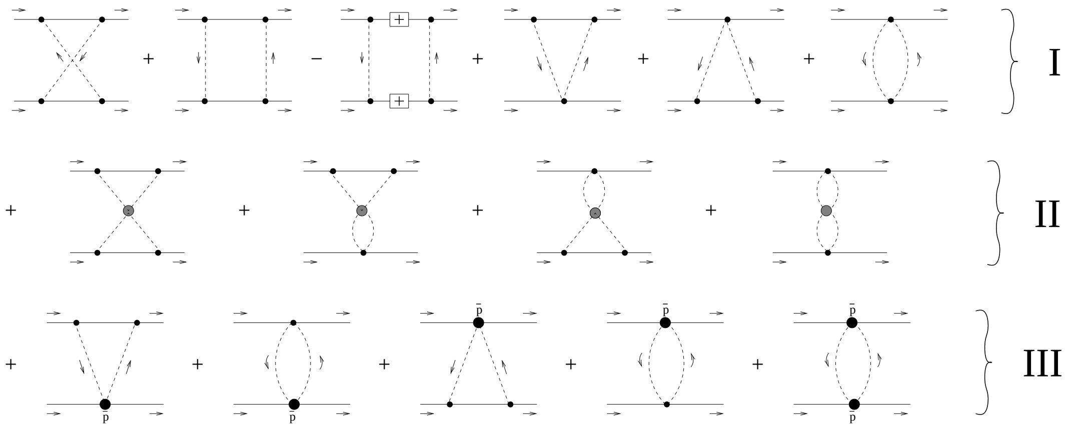

FIG. 1: Dynamics of the relativistic TPEP. The small black dots represent vertices from $\mathcal{L}_{\pi N}^{(1)}$, the big shaded dots, the $\pi\pi$ scattering amplitude, and the big black dots, the πN subthreshold coefficients. The latter contains implicitly other two loop contributions, as well as vertices from $\mathcal{L}_{\pi N}^{(2)}$ and $\mathcal{L}_{\pi N}^{(3)}$.

where $\tau^+ = 3$ and $\tau^- = 2$.

Therefore, the configuration space potential is written in terms of numerical coefficients which multiply dimensionless functions arising form the Fourier transforms of Feynman loop integrals. The former are combinations of external parameters representing the pion and nucleon masses, μ and m, respectively, the pion decay constant f_π, the axial coupling constant g_A, and the LECs c_i and d_i. The latter, denoted by $U_I^\pm$, depend on just μ, m and $x \equiv \mu r$.

III. PARAMETRIZED POTENTIAL

We keep the external quantities as free and parametrize the dimensionless Feynman loop integral functions $U_I^\pm$ and their derivatives in terms of $Z_i \equiv (G_i, H_i, I_i)$. They have the following structure

$$Z_i = \frac{\mu}{(4\pi)^{5/2}} \left(\frac{\mu}{f_\pi}\right)^4 \left[\sum \gamma_i x^n\right] \frac{e^{-2x}}{x^2} \,. \tag{7}$$

which is a combination of an exponential function times a polynomial with coefficients γ_i. This parametrization is more than 1% accurate in the range 0.8 fm $\leq r \leq$ 10 fm.

Using the definition $\alpha \equiv \mu/m$, the profile functions are written as

$$\begin{aligned}
V_C^+ = & \left\{ \frac{3\alpha^2}{2} \left[(4mc_1)^2 H_1 + \frac{1}{5}(mc_2)^2 \left(4H_2 - H_3\right) \right.\right. \\
+ & \; (2mc_3)^2 \left(H_1 - H_3\right) - \frac{16}{3} m^2 c_1 c_2 H_2 \\
- & \; 16 m^2 c_1 c_3 \left(2H_2 - H_1\right) + \left.\left. \frac{4}{3} m^2 c_2 c_3 \left(2H_2 - H_3\right) \right] \right\} \\
- & \left(\frac{g_A}{2}\right)^2 3\alpha \left\{ 8mc_1 \left[I_2 - 2\alpha(H_1 - H_2)\right] + \frac{2mc_2}{3}\alpha \right. \\
\times & \left(3H_1 - 2H_3\right) - 4mc_3 \left.\left[I_1 - I_3 + \alpha\left(2H_1 - 2H_2 - H_3\right)\right] \right\} \\
+ & \left(\frac{g_A}{2}\right)^4 3 \left\{ G_1 + \left(\frac{\mu}{\pi f_\pi}\right)^2 \left[I_6 + \frac{\pi}{4}\left(4I_3 + 6I_2 - 7I_1\right)\right]\right\} \\
+ & \left(\frac{g_A}{2}\right)^6 12 \left(\frac{\mu}{\pi f_\pi}\right)^2 \left(I_1 - I_3\right)
\end{aligned} \tag{8}$$

$$V_{LS}^+ = -\left(\frac{g_A}{2}\right)^2 16\alpha^2 m c_2 H_5 + \left(\frac{g_A}{2}\right)^4 6\alpha G_2 \,, \tag{9}$$

$$\begin{aligned}
V_T^+ = & \left(\frac{g_A}{2}\right)^2 \frac{4\alpha^2}{3} \left(m^2 \tilde{d}_{14} - m^2 \tilde{d}_{15}\right)\left(H_3 - 3H_5\right) \\
& - \left(\frac{g_A}{2}\right)^4 G_3 - \left(\frac{g_A}{2}\right)^6 \left(\frac{\mu}{\pi f_\pi}\right)^2 \left(2H_3 - H_5\right)
\end{aligned} \tag{10}$$

$$\begin{aligned}
V_{SS}^+ = & -\left(\frac{g_A}{2}\right)^2 \frac{8\alpha^2}{3} \left(m^2 \tilde{d}_{14} - m^2 \tilde{d}_{15}\right) H_3 \\
& + \left(\frac{g_A}{2}\right)^4 2G_4 + \left(\frac{g_A}{2}\right)^6 \left(\frac{\mu}{\pi f_\pi}\right)^2 \frac{4}{3} H_3
\end{aligned} \tag{11}$$

$$
\begin{aligned}
V_C^- &= \frac{H_2}{12} - \frac{\mu^2 H_8}{\pi^2 f_\pi^2} + \alpha^2 \Bigg[\frac{m c_4}{6} H_3 + \frac{2m^2}{3}(d_1+d_2) \\
&\times \Big(2H_2 - H_3\Big) + \frac{m^2 d_3}{5}\Big(4H_2 - H_3\Big) + \frac{8}{3} m^2 d_5 H_2 \Bigg] \\
&+ \left(\frac{g_A}{2}\right)^2 \Bigg\{ \Bigg[\frac{2}{3}\Big(5H_2 - 3H_1\Big) - \alpha\Big(I_1 - I_3\Big) \\
&- \alpha^2 \Big(2H1 - 2H2 - H_3\Big) \Bigg] \\
&+ \frac{\alpha^2}{3} \Bigg[2 m c_4 \Big(5H_3 - 12H_1 + 12H_2\Big) \\
&- 8m^2(d_1+d_2)\Big(5H_3 + 2H_2 - 6H_1\Big) \\
&- \frac{4m^2 d_3}{5}\Big(7H_3 - 8H_2\Big) + 32 m^2 d_5 \Big(5H_2 - 3H_1\Big) \Bigg] \\
&- \left(\frac{\mu}{\pi f_\pi}\right)^2 H_7 \Bigg\} + \left(\frac{g_A}{2}\right)^4 \left[2G_5 - \left(\frac{\mu}{\pi f_\pi}\right)^2 H_6 \right] \\
&- \left(\frac{g_A}{2}\right)^6 \left(\frac{\mu}{\pi f_\pi}\right)^2 H_9 \qquad (12)
\end{aligned}
$$

$$
\begin{aligned}
V_{LS}^- &= \frac{\alpha^2}{24}(3 + 16 m c_4) H_5 - \left(\frac{g_A}{2}\right)^2 \Bigg[2\alpha I_4 \\
&+ \alpha^2 \left(1 - \frac{40}{3} m c_4\right) H_5 + 8\alpha^2 m c_4 H_4 \Bigg] \\
&+ \left(\frac{g_A}{2}\right)^4 \alpha G_6 \qquad (13)
\end{aligned}
$$

$$
\begin{aligned}
V_T^- &= -\frac{\alpha^2}{144}(1 + 4 m c_4)^2 \Big(H_3 - 3H_5\Big) + \\
&+ \left(\frac{g_A}{2}\right)^2 \frac{\alpha}{36}(1 + 4 m c_4) \Big\{ 6I_5 + 4\alpha \\
&\times \Big[\Big(2H_3 - 3H_1 + 3H_2\Big) - 3\Big(8H_5 - 3H_4\Big) \Big] \Big\} \\
&- \left(\frac{g_A}{2}\right)^4 \left[\frac{\alpha}{3} G_7 + \left(\frac{\mu}{\pi f_\pi}\right)^2 \frac{1}{24}\Big(I_7 - 2\pi I_5\Big) \right] \\
&+ \left(\frac{\mu}{\pi f_\pi}\right)^2 \left(\frac{g_A}{2}\right)^6 \frac{2I_5}{3} \qquad (14)
\end{aligned}
$$

$$
\begin{aligned}
V_{SS}^- &= \frac{\alpha^2}{72}(1 + 4 m c_4)^2 H_3 - \left(\frac{g_A}{2}\right)^2 \frac{\alpha}{18}(1 + 4 m c_4) \\
&\times \Big[6I_3 + 4\alpha\Big(2H_3 - 3H_1 + 3H_2\Big) \Big] \\
&- \left(\frac{g_A}{2}\right)^4 \left[\frac{2\alpha}{3} G_8 + \frac{1}{12}\left(\frac{\mu}{\pi f_\pi}\right)^2 \Big(I_8 - 2\pi I_3\Big) \right] \\
&+ \left(\frac{g_A}{2}\right)^6 \left(\frac{\mu}{\pi f_\pi}\right)^2 \frac{4I_3}{3} \qquad (15)
\end{aligned}
$$

The fitted functions $Z_i \equiv (G_i, H_i, I_i)$ given by eq. (7) can be re-written as

$$
Z_i = \frac{\mu}{(4\pi)^{5/2}} \left(\frac{\mu}{f_\pi}\right)^4 P_{Z_i} \frac{e^{-2x}}{x^2}\,, \qquad (16)
$$

where $P_{Z_i} \equiv (P_{G_i}, P_{H_i}, P_{I_i}) = \sum \gamma_i x^n$ is the adjusted polynomial, which is given by

$$
\begin{aligned}
P_{H_1} &= -\frac{1}{x^{1/2}} - \frac{3}{16x^{3/2}} + \frac{15}{512x^{5/2}} - \frac{105}{8192x^{7/2}} \\
&+ \frac{0.0069211}{x^{9/2}} - \frac{0.002031054}{x^{11/2}} \qquad (17)
\end{aligned}
$$

$$
\begin{aligned}
P_{H_2} &= +\frac{3}{2x^{3/2}} + \frac{45}{32x^{5/2}} + \frac{315}{1024x^{7/2}} - \frac{0.050879}{x^{9/2}} \\
&+ \frac{0.0105639}{x^{11/2}} \qquad (18)
\end{aligned}
$$

$$
\begin{aligned}
P_{H_3} &= +\frac{6}{x^{3/2}} + \frac{165}{8x^{5/2}} + \frac{8715}{256x^{7/2}} + \frac{27.45483}{x^{9/2}} \\
&+ \frac{5.43256}{x^{11/2}} \qquad (19)
\end{aligned}
$$

$$
P_{H_4} = +\frac{2}{x^{3/2}} + \frac{23}{8x^{5/2}} + \frac{153}{256x^{7/2}} - \frac{0.0723934}{x^{9/2}} \qquad (20)
$$

$$
P_{H_5} = -\frac{3}{x^{5/2}} - \frac{129}{16x^{7/2}} - \frac{3555}{512x^{9/2}} - \frac{1.33605}{x^{11/2}} \qquad (21)
$$

$$
\begin{aligned}
P_{H_6} &= -\frac{1.61597}{x^{1/2}} - \frac{7.96919}{x^{3/2}} - \frac{13.8967}{x^{5/2}} + \frac{12.1099}{x^{7/2}} \\
&+ \frac{44.0911}{x^{9/2}} + \frac{24.1372}{x^{11/2}} \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
P_{H_7} &= \frac{0.08333}{x^{1/2}} - \frac{0.46679}{x^{3/2}} - \frac{2.63262}{x^{5/2}} - \frac{0.52125}{x^{7/2}} \\
&+ \frac{3.12711}{x^{9/2}} + \frac{2.16473}{x^{11/2}} \qquad (23)
\end{aligned}
$$

$$
\begin{aligned}
P_{H_8} &= \frac{0.0104167}{x^{3/2}} - \frac{0.113031}{x^{5/2}} - \frac{0.106544}{x^{7/2}} \\
&+ \frac{0.00197044}{x^{9/2}} + \frac{0.0394014}{x^{11/2}} \qquad (24)
\end{aligned}
$$

$$
\begin{aligned}
P_{H_9} &= \frac{1.33333}{x^{1/2}} + \frac{6.65}{x^{3/2}} + \frac{19.3609}{x^{5/2}} + \frac{30.6421}{x^{7/2}} \\
&+ \frac{24.4818}{x^{9/2}} + \frac{4.86312}{x^{11/2}} \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
P_{G_1} &= 2.83823 - \frac{7.200711}{x} + \frac{38.9637}{x^2} - \frac{55.5164}{x^3} \\
&+ \frac{47.2443}{x^4} - \frac{16.2395}{x^5} \qquad (26)
\end{aligned}
$$

$$P_{G_2} = -\frac{6.12315}{x} - \frac{28.1422}{x^2} - \frac{30.2813}{x^3} + \frac{0.023458}{x^4} - \frac{15.8996}{x^5} + \frac{7.18869}{x^6} \quad (27)$$

$$P_{G_3} = 0.5579 + \frac{17.1039}{x} + \frac{16.8038}{x^2} + \frac{9.94755}{x^3} + \frac{3.40171}{x^4} - \frac{2.7544}{x^5} \quad (28)$$

$$P_{G_4} = 0.569624 + \frac{15.9429}{x} - \frac{4.26031}{x^2} + \frac{15.6445}{x^3} - \frac{5.06641}{x^4} \quad (29)$$

$$P_{G_5} = -0.217221x - 9.98415 - \frac{4.662}{x} - \frac{36.9761}{x^2} + \frac{13.4087}{x^3} - \frac{6.21047}{x^4} \quad (30)$$

$$P_{G_6} = +\frac{7.90985}{x} + \frac{55.9568}{x^2} + \frac{86.3242}{x^3} + \frac{66.9540}{x^4} - \frac{29.5680}{x^5} + \frac{11.8985}{x^6} \quad (31)$$

$$P_{G_7} = 1.69219 + \frac{25.5612}{x} + \frac{6.53589}{x^2} + \frac{160.459}{x^3} - \frac{169.567}{x^4} + \frac{120.612}{x^5} - \frac{36.7881}{x^6} \quad (32)$$

$$P_{G_8} = 1.7661 + \frac{21.2122}{x} - \frac{9.87710}{x^2} + \frac{116.454}{x^3} - \frac{144.344}{x^4} + \frac{103.063}{x^5} - \frac{30.9265}{x^6} \quad (33)$$

$$P_{I_1} = 0.000483761x^2 - 0.0226386x + 1.53346 + \frac{0.0595627}{x} - \frac{0.0913580}{x^2} + \frac{0.0291743}{x^3} \quad (34)$$

$$P_{I_2} = -0.000483761x^2 + 0.0218747x - 1.50179 - \frac{3.31974}{x} - \frac{1.22868}{x^2} + \frac{0.0773095}{x^3} \quad (35)$$

$$P_{I_3} = +0.242214 - \frac{8.87827}{x} - \frac{6.47733}{x^2} - \frac{30.5206}{x^3} \quad (36)$$

$$P_{I_4} = -\frac{0.00147946}{x} + \frac{2.99191}{x^2} + \frac{6.86185}{x^3} + \frac{1.82098}{x^4} \quad (37)$$

$$P_{I_5} = +0.242214 - \frac{8.87383}{x} - \frac{15.4531}{x^2} - \frac{51.1062}{x^3} - \frac{5.46294}{x^4} \quad (38)$$

$$P_{I_6} = -0.07382x^{3/2} + 2.35815x^{1/2} + \frac{12.5677}{x^{1/2}} + \frac{26.0361}{x^{3/2}} + \frac{39.5975}{x^{5/2}} - \frac{14.1886}{x^{7/2}} - \frac{49.6305}{x^{9/2}} \quad (39)$$

$$P_{I_7} = -\frac{6.82062}{x^{3/2}} - \frac{39.5219}{x^{5/2}} - \frac{11.2626}{x^{7/2}} + \frac{48.2308}{x^{9/2}} \quad (40)$$

$$P_{I_8} = -\frac{28.6452}{x^{3/2}} - \frac{101.827}{x^{5/2}} + \frac{28.3771}{x^{7/2}} + \frac{99.2609}{x^{9/2}} \quad (41)$$

For instance, the parametrized function I_8 is given by:

$$I_8 = \frac{\mu}{(4\pi)^{5/2}} \left(\frac{\mu}{f_\pi}\right)^4 \left[-\frac{28.6452}{x^{3/2}} - \frac{101.827}{x^{5/2}} + \frac{28.3771}{x^{7/2}} + \frac{99.2609}{x^{9/2}} \right] \frac{e^{-2x}}{x^2}\,, \quad (42)$$

IV. CONCLUSIONS

Nowadays, the asymptotic expressions for the chiral TPEP have the status of theorems and are written as sums of chiral layers, with little model dependence. These asymptotic functions were our guide to guess what were the correct choice of the parametrized functions to fit the potential profile functions.

The parametrized profile functions given in the preceding section depend explicitly on four well known quantities, namely m, μ, g_A, f_π, and on the less known LECs c_i and d_i. Therefore, the latter may be extracted from fits to data. When doing this, however, one has to bear in mind that, as discussed in Ref. [11], the influence of the LECs over the profile functions is rather uneven. Indeed, their influence over V_C^+, V_{LS}^-,

V_T^-, and V_{SS}^- is rather strong, but barely perceptible in V_C^-, V_{LS}^+, V_T^+ and V_{SS}^+.

We conclude saying that the various channels of the potential are clearly dominated by either nucleonic (minimal model) or non-nucleonic (LECs) degrees of freedom, mainly because we found the role of intermediate $\pi\pi$ scattering (two-loops) to be small.

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