$f_0(1370)$ Decay in the Fock-Tani Formalism

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We investigate the two-meson decay modes for $f_0(1370)$. In this calculation we consider this resonance as a glueball. The Fock-Tani formalism is introduced to calculate the decay width.

Keywords: Glueballs; Fock-Tani formalism; Meson decay

I. INTRODUCTION

The gluon self-coupling in QCD opens the possibility of existing bound states of pure gauge fields known as glueballs. Even though theoretically acceptable, the question still remains unanswered: do bound states of gluons actually exist? Glueballs are predicted by many models and by lattice calculations. In experiments glueballs are supposed to be produced in gluon-rich environments. The most important reactions to study gluonic degrees of freedom are radiative J/ψ decays, central productions processes and antiproton-proton annihilation.

Numerous technical difficulties have so far been present in our understanding of their properties in experiments, largely because glueball states can mix strongly with nearby $q\bar{q}$ resonances [1],[2].

The best estimate for the masses of glueballs comes from lattice gauge calculations, which in the quenched approximation show [3] that the lightest glueball has $J^{PC} = 0^{++}$ and that its mass should be in the range $1.45 - 1.75$ GeV.

Constituent gluon models have received attention recently, for spectroscopic calculations. For example, a simple potential model, namely the model of Cornwall and Soni [4],[5] has been compared consistently to lattice and experiment [6],[7]. In the present we shall apply the Fock-Tani formalism [8] to glueball decay by defining an effective constituent quark-gluon Hamiltonian. In particular the resonance $f_0(1370)$ shall be considered.

II. THE FOCK-TANI FORMALISM

Now let us to apply the Fock-Tani formalism in the microscopic Hamiltonian to obtain an effective Hamiltonian. In the Fock-Tani formalism we can write the glueball and the meson creation operators in the following form

$$G^{\dagger}_{\alpha} = \frac{1}{\sqrt{2}}\Phi^{\mu\nu}_{\alpha}\, a^{\dagger}_{\mu} a^{\dagger}_{\nu}\,. \;\; ; \;\; M^{\dagger}_{\beta} = \Psi^{\mu\nu}_{\beta}\, q^{\dagger}_{\mu}\bar{q}^{\dagger}_{\nu}. \tag{1}$$

The indexes α and β are the glueball and meson quantum numbers: $\alpha = \{\text{space, spin}\}$ and $\beta = \{\text{space, spin, isospin}\}$. The gluon creation $a^{\dagger}_{\nu}$ and annihilation a_{μ} operators obey the following commutation relations $[a_{\mu}, a_{\nu}] = 0$ and $[a_{\mu}, a^{\dagger}_{\nu}] = \delta_{\mu\nu}$. While the quark creation $q^{\dagger}_{\nu}$, annihilation q_{μ}, the antiquark creation $\bar{q}^{\dagger}_{\nu}$ and annihilation $\bar{q}_{\mu}$ operators obey the following anticommutation relations $\{q_{\mu}, q_{\nu}\} = \{\bar{q}_{\mu}, \bar{q}_{\nu}\} = \{q_{\mu}, \bar{q}_{\nu}\} = \{q_{\mu}, \bar{q}^{\dagger}_{\nu}\} = 0$ and $\{q_{\mu}, q^{\dagger}_{\nu}\} = \{\bar{q}_{\mu}, \bar{q}^{\dagger}_{\nu}\} = \delta_{\mu\nu}$. In (1) $\Phi^{\mu\nu}_{\alpha}$ and $\Psi^{\mu\nu}_{\alpha}$ are the bound-state wave-functions for two-gluons and two-quarks respectively. The composite glueball and meson operators satisfy non-canonical commutation relations

$$[G_{\alpha}, G_{\beta}] = 0 \;\; ; \;\; [G_{\alpha}, G^{\dagger}_{\beta}] = \delta_{\alpha\beta} + \Delta_{\alpha\beta}$$
$$[M_{\alpha}, M_{\beta}] = 0 \;\; ; \;\; [M_{\alpha}, M^{\dagger}_{\beta}] = \delta_{\alpha\beta} + \Delta_{\alpha\beta} \tag{2}$$

The “ideal particles” which obey canonical relations

$$[g_{\alpha}, g_{\beta}] = 0 \;\; ; \;\; [g_{\alpha}, g^{\dagger}_{\beta}] = \delta_{\alpha\beta}$$
$$[m_{\alpha}, m_{\beta}] = 0 \;\; ; \;\; [m_{\alpha}, m^{\dagger}_{\beta}] = \delta_{\alpha\beta}. \tag{3}$$

This way one can transform the composite state $|\alpha\rangle$ into an ideal state $|\alpha)$, in the glueball case for example we have

$$|\alpha) = U^{-1}(-\frac{\pi}{2})\, G^{\dagger}_{\alpha}\,|0\rangle = g^{\dagger}_{\alpha}\,|0\rangle$$

where $U = \exp(tF)$ and F is the generator of the glueball transformation given by

$$F = \sum_{\alpha} g^{\dagger}_{\alpha}\tilde{G}_{\alpha} - \tilde{G}^{\dagger}_{\alpha} g_{\alpha} \tag{4}$$

with

$$\tilde{G}_{\alpha} = G_{\alpha} - \frac{1}{2}\Delta_{\alpha\beta}G_{\beta} - \frac{1}{2}G^{\dagger}_{\beta}[\Delta_{\beta\gamma}, G_{\alpha}]G_{\gamma}.$$

In order to obtain the effective potential one has to use (4) in a set of Heisenberg-like equations for the basic operators $g, \tilde{G}, a$

$$\frac{dg_{\alpha}(t)}{dt} = [g_{\alpha}, F] = \tilde{G}_{\alpha} \;\; ; \;\; \frac{d\tilde{G}_{\alpha}(t)}{dt} = [\tilde{G}_{\alpha}(t), F] = -g_{\alpha}\,.$$

The simplicity of these equations are not present in the equations for a

$$\begin{aligned}\frac{da_\mu(t)}{dt} &= - \sqrt{2}\Phi_\beta^{\mu\nu} a_\nu^\dagger g_\beta + \frac{\sqrt{2}}{2}\Phi_\beta^{\mu\nu} a_\nu^\dagger \Delta_{\beta\alpha} g_\beta \\ &+ \Phi_\alpha^{\star\mu\gamma}\Phi_\beta^{\gamma\mu'}(G_\beta^\dagger a_{\mu'} g_\beta - g_\beta^\dagger a_{\mu'} G_\beta) \\ &- \sqrt{2}(\Phi_\alpha^{\mu\rho'}\Phi_\rho^{\mu'\gamma'}\Phi_\gamma^{\star\gamma'\rho'} + \Phi_\alpha^{\mu'\rho'}\Phi_\rho^{\mu\gamma'}\Phi_\gamma^{\star\gamma'\rho'}) \\ &\times G_\gamma^\dagger a_{\mu'}^\dagger G_\beta g_\beta.\end{aligned}$$

The solution for these equation can be found order by order in the wave functions. For zero order one has $a_\mu^{(0)} = a_\mu$, $g_\alpha^{(0)}(t) = G_\alpha \sin t + g_\alpha \cos t$ and $G_\beta^{(0)}(t) = G_\beta \cos t - g_\beta \sin t$. In the first order $g_\alpha^{(1)} = 0$, $G_\beta^{(1)} = 0$ and $a_\mu^{(1)}(t) = \sqrt{2}\Phi_\beta^{\mu\nu} a_\nu^\dagger g_\beta$. If we repeat a similar calculation for mesons let us to obtain the following equations solution: $q_\mu^{(0)} = q_\mu$, $\bar{q}_\mu^{(0)} = \bar{q}_\mu$, $q_\mu^{(1)}(t) = \Psi_\beta^{\mu\nu}\bar{q}_\nu^\dagger m_\beta$ and $\bar{q}_\mu^{(1)}(t) = -\Psi_\beta^{\mu\nu} q_\nu^\dagger m_\beta$.

III. THE MICROSCOPIC MODEL

The microscopic model adopted here must contain explicit quark and gluon degrees of freedom, so we obtain a microscopic Hamiltonian of the following form

$$\begin{aligned}H &= g^2 \int d^3x d^3y \Psi^\dagger(\vec{x})\gamma^0\gamma^i A_i^a(\vec{x})\frac{\lambda^a}{2}\Psi(\vec{x}) \\ &\times \Psi^\dagger(\vec{y})\gamma^0\gamma^j A_j^b(\vec{y})\frac{\lambda^b}{2}\Psi(\vec{y})\end{aligned} \quad (5)$$

Where the quark and the gluon fields are respectively [9]

$$\Psi(\vec{x}) = \sum_s \int \frac{d^3k}{(2\pi)^3}[u(\vec{k},s)q(\vec{k},s) + v(-\vec{k},s)\bar{q}^\dagger(-\vec{k},s)]e^{i\vec{k}\cdot\vec{x}} \quad (6)$$

and

$$A_i^a(\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{\vec{k}}}}[a_i^a(\vec{k}) + a_i^{a\dagger}(-\vec{k})]e^{i\vec{k}\cdot\vec{x}} \quad (7)$$

We choose this Hamiltonian due to its form that allow to obtain a operators structure of this type $q^\dagger\bar{q}^\dagger q^\dagger\bar{q}^\dagger aa$.

IV. THE FOCK-TANI FORMALISM APPLICATION

Now we are going to apply the Fock-Tani formalism to the microscopic Hamiltonian

$$H_{FT} = U^{-1}HU \quad (8)$$

which gives rise to an effective interaction H_{FT}. To find this Hamiltonian we have to calculate the transformed operators for quarks and gluons by a technique known as *the equation of motion technique*. The resulting H_{FT} for the glueball decay $G \to mm$ is represented by two diagrams which appear in Fig. (1).

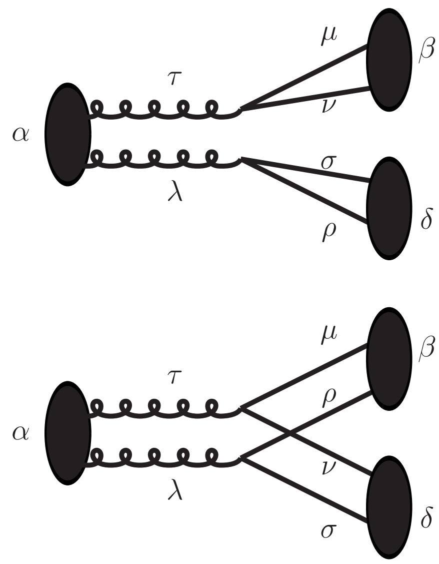

FIG. 1: Diagrams for glueball decay

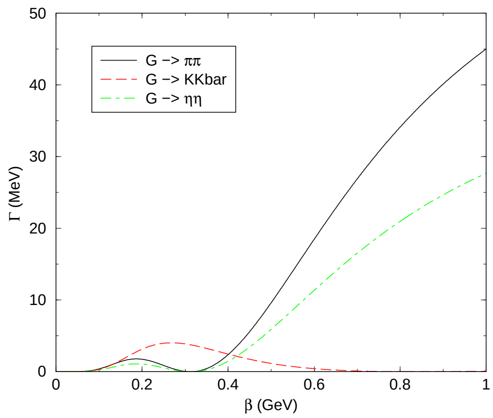

FIG. 2: Decay width for $f_0(1370)$

Analyzing these diagrams, of Fig. (1), it is clear that in the first one there is no color conservation. The glueball's wave-function Φ is written as a product

$$\Phi_\alpha^{\mu\nu} = \chi_{A_\alpha}^{s_\mu s_\nu}\, C^{c_\mu c_\nu}\, \Phi_{\vec{P}_\alpha}^{\vec{p}_\mu \vec{p}_\nu}, \quad (9)$$

$\chi_{A_\alpha}^{s_\mu s_\nu}$ is the spin contribution, with $A_\alpha \equiv \{S_\alpha, S_\alpha^3\}$, where S_α is the glueball's total spin index and S_α^3 the index of the spin's third component; $C^{c_\mu c_\nu}$ is the color component given by $\frac{1}{\sqrt{8}}\delta^{c_\mu c_\nu}$ and the spatial wave-function is

$$\Phi_{\vec{P}_\alpha}^{\vec{p}_\mu \vec{p}_\nu} = \delta^{(3)}(\vec{P}_\alpha - \vec{p}_\mu - \vec{p}_\nu)\left(\frac{1}{\pi b^2}\right)^{\frac{3}{4}} e^{-\frac{1}{8\beta^2}(\vec{p}_\mu - \vec{p}_\nu)^2}. \quad (10)$$

The expectation value of r^2 gives a relation between the *rms* radius r_0 and β of the form $\beta = \sqrt{1.5}/r_0$. The meson wave function Ψ is similar with parameter b replacing β. To determine the decay rate, we evaluate the matrix element between the states $|i\rangle = g^\dagger_\alpha|0\rangle$ and $|f\rangle = m^\dagger_\beta m^\dagger_\gamma|0\rangle$ which is of the form

$$\langle f \mid H_{FT} \mid i\rangle = \delta(\vec{p}_\alpha - \vec{p}_\beta - \vec{p}_\gamma)h_{fi}. \quad (11)$$

The h_{fi} decay amplitude can be combined with a relativistic phase space to give the differential decay rate [10]

$$\frac{d\Gamma_{\alpha\to\beta\gamma}}{d\Omega} = 2\pi\frac{PE_\beta E_\gamma}{M_\alpha}\,|h_{fi}|^2 \quad (12)$$

After several manipulations we obtain the following result

$$h_{fi} = \frac{8\alpha_s}{3\pi}\left(\frac{1}{\pi b^2}\right)^{3/4}\int dq\,\frac{q^2}{\sqrt{q^2+m_g^2}} \times\left(1-\frac{q^2}{4m_q^2}-\frac{q^2}{4m_s^2}\right)e^{-(\frac{1}{2b^2}+\frac{1}{4\beta^2})q^2} \quad (13)$$

Finally one can write the decay amplitude for the f_0 into two mesons

$$\Gamma_{f_0\to M_1M_2} = \frac{512\alpha_s^2}{9}\,\frac{PE_{M_1}E_{M_2}}{M_{f_0}}\left(\frac{1}{\pi b^2}\right)^{3/2} I^2 \quad (14)$$

where

$$I = \int dq\,\frac{q^2}{\sqrt{q^2+m_g^2}}\left(1-\frac{q^2}{4m_q^2}-\frac{q^2}{4m_s^2}\right)e^{-(\frac{1}{2b^2}+\frac{1}{4\beta^2})q^2} \quad (15)$$

with m_q the u and d quark mass and m_s the mass of the s quark. The decays that are studied are for the following processes $f\to\pi\pi$, $f\to K\bar{K}$ and $f\to\eta\eta$. The parameters used are $b = 0.34$ GeV, $m_q = 0.33$, $m_q/m_s = 0.6$, $\alpha_s = 0.6$. Experimental data is still uncertain for this resonance. There is a large interval for the full width $\Gamma = 200$ to 500 MeV and the studied decay channels are seen, but still with no estimation.

V. CONCLUSIONS

The Fock-Tani formalism is proven appropriate not only for hadron scattering but for decay. The example decay process $f_0(1370)\to\pi\pi$; $K\bar{K}$ and $\eta\eta$ in the Fock-Tani formalism is studied. The same procedure can be used for other $f_0\,(M)$ and for heavier scalar mesons and compared with similar calculations which include mixtures.

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