Effect of Event-by-Event Fluctuations on Hydrodynamical Evaluation of Elliptical Flow

R. Andrade[1], F. Grassi[1], Y. Hama[1], T. Kodama[2], and O. Socolowski Jr.[3]
[1]*Instituto de Física, Universidade de São Paulo, Brazil*
[2]*Instituto de Física, Universidade Federal do Rio de Janeiro, Brazil*
[3] *Departamento de Fisica, Instituto Tecnológico de Aeronáutica - CTA, Brazil*

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Elliptic flow at RHIC is computed event-by-event with NeXSPheRIO. We show that when symmetry of the particle distribution in relation to the reaction plane is assumed, there is a disagreement between the true and reconstructed elliptic flows. We suggest a possible way to take into account the asymmetry and get good agreement between these elliptic flows.

Keywords: Relativistic nuclear collisions; Hydrodynamics; Elliptic flow

Hydrodynamics is one of the main tools to study the collective flow in high-energy nuclear collisions. Here we discuss results obtained with the hydrodynamical code NeXSPheRIO. It is a junction of two codes: NeXus and SPheRIO. The SPheRIO code is used to compute the hydrodynamical evolution. It is based on Smoothed Particle Hydrodynamics, a method originally developed in astrophysics and adapted to relativistic heavy ion collisions [1]. Its main advantage is that any geometry in the initial conditions can be incorporated. The NeXus code is used to compute the initial conditions $T_{\mu\nu}$, j^μ and u^μ on a proper time hypersurface [2]. NeXSPheRIO is run many times, corresponding to many different events or initial conditions. At the end, an average over final results is performed. This mimics experimental conditions. This is different from the canonical approach in hydrodynamics where initial conditions are adjusted to reproduce some selected data and are very smooth. This code has been used to study a range of problems concerning relativistic nuclear collisions [3–8]. Here a calculation of elliptic flow is performed [9].

In a hydrodynamical code, the impact parameter $\vec{b}$ is usually known. The theoretical, or true, elliptic flow parameter at a given pseudo-rapidity η is defined as

$$\langle v_2^b(\eta)\rangle = \langle \frac{\int d^2N/d\phi d\eta \cos[2(\phi-\phi_b)]\, d\phi}{\int d^2N/d\phi d\eta\, d\phi}\rangle \tag{1}$$

ϕ_b is the angle between $\vec{b}$ and some fixed reference axis. The average is performed over all events in the centrality bin.

Experimentally, the impact parameter angle ϕ_b is not known. In the so-called standard method, an approximation, ψ_2, is estimated. Elliptic flow parameter with respect to this angle, $v_2^{obs}(\eta)$, is calculated. Then a correction is applied to $v_2^{obs}(\eta)$ to account for the reaction plane resolution, leading to the experimentally reconstructed elliptic flow parameter $v_2^{rec}(\eta)$. For example in a Phobos-like way [10, 11]

$$\langle v_2^{rec}(\eta)\rangle = \langle \frac{v_2^{obs}(\eta)}{\sqrt{\langle\cos[2(\psi_2^{<0}-\psi_2^{>0})]\rangle}}\rangle \tag{2}$$

where

$$v_2^{obs}(\eta) = \frac{\sum_i d^2N/d\phi_i d\eta \cos[2(\phi_i-\psi_2)]}{\sum_i d^2N/d\phi_i d\eta} \tag{3}$$

and

$$\psi_2 = \frac{1}{2}\tan^{-1}\frac{\sum_i \sin 2\phi_i}{\sum_i \cos 2\phi_i} \tag{4}$$

In the hit-based method, $\psi_2^{<0}$ and $\psi_2^{>0}$ are determined for subevents $\eta < 0$ and > 0 respectively and if v_2 is computed for a positive (negative) η, the sums in ψ_2, eq. (4), are over particles with $\eta < 0$ ($\eta > 0$). In the track-based method, $\psi_2^{<0}$ and $\psi_2^{>0}$ are determined for subevents $2.05 <\mid \eta \mid< 3.2$, the sums in ψ_2, eq. (4), are over particles in both sub-events, v_2 is obtained for particles around $0 < \eta < 1.8$ and reflected (to account for the different multiplicities between a subevent and the sums in eq. (4), there is also an additional $\sqrt{2\alpha}$ with $\alpha \sim 1$, in the reaction plane correction in eq. (2)). Since both methods are in agreement but only the hit-based method covers a large pseudo-rapidity interval, we use this latter method.

We want to check whether the theoretical and experimental estimates are in agreement, i.e., $\langle v_2^b(\eta)\rangle = \langle v_2^{rec}(\eta)\rangle$. A necessary condition for this, from eq. (2), is, $\langle v_2^b(\eta)\rangle \geq \langle v_2^{obs}(\eta)\rangle$. In Fig. 1, we show the results for $\langle v_2^b(\eta)\rangle$ (solid line) and $\langle v_2^{obs}(\eta)\rangle$ (dashed line).

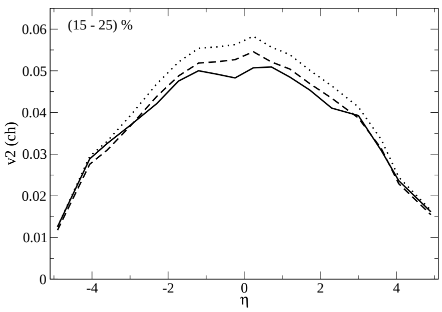

FIG. 1: Comparison between various ways of computing v_2 using NeXSPheRIO for Phobos 15-25% centrality window[11]: solid line is v_2^b, obtained using the known impact parameter angle ϕ_b, dashed (dotted) line is v_2^{obs} (v_2^{rec}), obtained using the reconstructed impact parameter angle ψ_2 without (with) reaction plane correction.

We see that $\langle v_2^b(\eta)\rangle \leq \langle v_2^{obs}(\eta)\rangle$ for most η's. So, as shown also in the figure, dividing by a cosine to get $\langle v_2^{rec}(\eta)\rangle$ (dotted curve) makes the disagreement worse: $\langle v_2^b(\eta)\rangle$ and $\langle v_2^{rec}(\eta)\rangle$ *are* different. This is true for all three Phobos centrality windows and more pronounced in the most central window.

Since the standard way to include the correction for the reaction plane resolution (eq. (2)) seems inapplicable, we need to understand why. When we look at the distribution $d^2N/d\phi d\eta$ obtained in a NeXSPheRIO event (presumably also in a true event), it is not symmetric with respect to the reaction plane. (We recall that the reaction plane is the plane defined by the impact parameter vector and the beam axis.) This happens because i) the incident nuclei have a granular structure, ii) the number of produced particles is finite. The symmetry might be better with respect to the plane with inclination ψ_2 in relation to the reference axis and containing the beam axis. Therefore we must write for each event

$$\frac{d^2N}{d\phi d\eta} = v_0(\eta)[1+\sum_n 2v_n^b(\eta)\cos(n(\phi-\phi_b))+\sum_n 2v_n^{\prime b}(\eta)\sin(n(\phi-\phi_b))] \quad (5)$$

$$= v_0(\eta)[1+\sum_n 2v_n^{obs}(\eta)\cos(n(\phi-\psi_2))+\sum_n 2v_n^{\prime obs}(\eta)\sin(n(\phi-\psi_2))] \quad (6)$$

It follows that

$$v_2^{obs}(\eta) = v_2^b(\eta)\cos[2(\psi_2-\phi_b)]+v_2^{\prime b}(\eta)\sin[2(\psi_2-\phi_b)] \quad (7)$$

Due to the sine term [9], we can indeed have $\langle v_2^{obs}(\eta)\rangle > \langle v_2^b(\eta)\rangle$, and therefore $\langle v_2^{rec}(\eta)\rangle > \langle v_2^b(\eta)\rangle$ as in Fig. 1 (for a more detailed discussion, see [9]).

In the standard approach, it is *assumed* that $d^2N/d\phi d\eta$ is symmetric with respect to the reaction plane and there are no sine terms in the Fourier decomposition in (eq. (5)); eq. (7) leads to (for the hit-based or track-based method)

$$\langle v_2^b(\eta)\rangle = \langle v_2^{obs}(\eta)\rangle/\langle\cos[2(\psi_2-\phi_b)]\rangle \quad (8)$$

and eq. (2) follows. However as explained above, the use of NeXus initial conditions leads to $d^2N/d\phi d\eta$ not symmetric with respect to the reaction plane (and presumably this is also the case in each real event), so eq. (8) and (2) are not valid.

As already mentioned, the symmetry might be better with respect to the plane with inclination ψ_2 in relation to the reference axis and containing the beam axis. From (5) and (6), we have

$$v_2^b(\eta) = v_2^{obs}(\eta)\times\cos[2(\psi_2-\phi_b)]+v_2^{\prime obs}(\eta)\times\sin[2(\psi_2-\phi_b)]. \quad (9)$$

If the symmetry is perfect $v_2^{\prime obs}=0$. Otherwise, $\langle v_2^{\prime obs}(\eta)\times\sin[2(\psi_2-\phi_b)]\rangle = 0$ (for a more detailed discussion, see [9]). So whether the symmetry is perfect or approximate, $\langle v_2^b(\eta)\rangle \sim \langle v_2^{obs}(\eta)\times\cos[2(\psi_2-\phi_b)]\rangle$ and instead of eq. (2) we would have

$$\langle v_2^{Rec}(\eta)\rangle = \left\langle v_2^{obs}(\eta)\times\sqrt{\langle\cos[2(\psi_2^{<0}-\psi_2^{>0})]\rangle}\right\rangle \quad (10)$$

In Fig. 2, we show $\langle v_2^{Rec}(\eta)\rangle$ (dash-dotted line) and $\langle v_2^b(\eta)\rangle$ (solid line). We see that the agreement between both methods is improved compared to Fig. 1.

We have also computed the elliptic flow parameter as function of transverse momentum for charged hadrons with $0 < \eta < 1.5$ for the 50% most central collisions. We found that $\langle v_2^b(p_\perp)\rangle$ computed as in eq. (1) is well approximated by $\langle v_2^{Rec}(p_\perp)\rangle$ computed as in eq. (10).

In summary, from Fig. 1, elliptic flow estimated from the standard method with reaction plane correction is an overestimate of true elliptic flow ($v_2^{rec} > v_2^b$). From Fig. 2, using a method that takes into account the more symmetrical nature of particle distribution in relation to the plane with inclination ψ_2 with respect to the reference axis and containing the beam axis (rather than with respect to the reaction plane), we get a better agreement between reconstructed and true elliptic flows ($v_2^{Rec} \sim v_2^b$).

As for overestimating the true elliptic flow, a similar conclusion was reached in [12] and [13]. In both papers, it is found and expected that there will be differences between v_2^b, calculated using the known quantity $\vec{b}$, and v_2^{rec}, calculated with the reaction plane method or two-particle cumulant method both because of the so-called non-flow correlations (overall momentum conservation, resonance decays, jet production,etc) and event-by-event fluctuations (mostly eccentricity fluctuations). In principle, higher-order cumulant methods take care of non-flow effects. If there is still disagreement between the true elliptic flow and higher-order cumulant methods, as in [12], then fluctuations are important. If there is agreement as in [13], then non-flow effects are important and not fluctuations. In addition to the disagreement between their conclusions, [12] and [13] do not (neither are expected to) reproduce the RHIC data. So an interesting question is whether a more accepted hydrodynamical description would lead to a sizable effect. Using NeXSPheRIO, we found that true elliptic flow $v_2^b(\eta=0)$ is overestimated by $\sim$ 15-30 % (according to centrality) with the reaction plane method, and $v_2^b(p_\perp)$ by $\sim$ 30% at $p_\perp$=0.5 GeV. In our case, since $\langle v_2^b\rangle \sim \langle v_2^{Rec}\rangle$, a large part of the difference between the true $\langle v_2^b\rangle$ and reconstructed $\langle v_2^{rec}\rangle$ is due to the (wrong) assumption of symmetry of the particle distribution around the reaction plane, made to obtain $\langle v_2^{rec}\rangle$.

Finally, we would like to emphasize that it is important to have precise experimental determination of elliptic flow, in particular free from the assumption of symmetry that we dis-

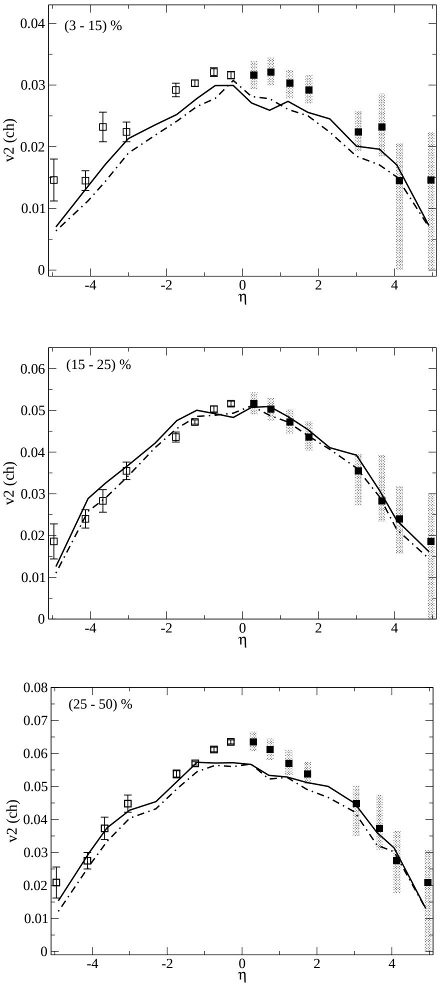

FIG. 2: Comparison between true elliptic flow v_2^b (solid line) and suggested method to compute reconstructed elliptic flow from data v_2^{Rec} (dash-dotted) for the three Phobos centrality windows[11]. Squares represent Phobos data (black error bars are 1 σ statistical errors and grey bands, systematic uncertainties at $\sim$90% confidence level).

cussed. Elliptic flow teaches us about the initial conditions and thermalization, in principle. In this manner [14], Hirano had noted that with his hydrodynamical code plus freeze out, he could not reproduce $v_2(\eta)$, in particular at large η, perhaps signalizing lack of thermalization. In subsequent works [15, 16], Hirano and collaborators argued that agreement with $v_2(\eta)$ data could be obtained with adequate initial conditions, a similar hydrodynamical code and, instead of freeze out, a transport code providing hadronic dissipation. However these conclusions would be affected if $v_2(\eta)$ data were lower, as we think they should be. (Incidentally, though our objective was not to reproduce data, note that our model with freeze out (no transport code) reproduces reasonably both the $v_2(\eta)$ data as in [16] (Fig. 3) and the $v_2(p_\perp)$ data (not shown).) Therefore, to know e.g. what the initial conditions are or if there is viscosity and in what phase, we need to settle the question of whether event-by-event fluctuations are important and take them into account in the experimental analysis.

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