A Multiparton Model for pp / $p\overline{p}$ Inelastic Scattering

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We have applied a geometrical approach to study pp / $p\overline{p}$ inelastic scattering over the range of center mass energies from 44.5 to 900 GeV. The multiplicity distributions are described by multiple parton-parton collisions without free parameters. The output seems to be consistent with data and the results are discussed. Exploring the possible connection between impact parameter representation of the multiplicity distribution and that of the eikonal function, the range of impact parameters for multiple collisions of partons are estimated. The energy dependence of i parton-parton collisions probability is studied and the average multiplicity is related to hadron opacity at each impact parameter.

Keywords: Eikonal approximation; Hadronic multiplicity distribution; $pp/\bar{p}p$ scattering

1. INTRODUCTION

It is well know that the underlying theory of hadronic interactions, the Quantum Chromodynamics (QCD), is not yet able to describe the bulk of experimental data associated with soft processes in hadronic interactions. Phenomenological approaches are used as source of information for adequate theoretical developments. General principles are very important in the nonperturbative sector, in particular unitarity, which regulates the relative strength of elastic and inelastic processes [1]. Thus, since that an approach based exclusively in QCD is still missing, we hope infer some useful information on multiparticle production in $pp/p\overline{p}$ collisions by using a geometrical approach involving both elastic and inelastic scattering related by unitarity. The multiplicity distribution is the most general feature of multiparticle production processes. To be successful, any phenomenological model of these processes should, first of all, fit the experimental values for probabilities of n particle production [2]. Thus, we shall apply an existing generalized phenomenological procedure, referred to as *Fused String Model* (*FSM*) [3], which describes well the overall multiplicity distributions arising from $pp/p\overline{p}$ collisions at ISR (52.6 GeV) and Collider (546 GeV) energies once the eikonal function (Ω) is given, which can be inferred from measurements of the elastic differential cross section. The success of the *FSM* encourages us apply them to study $pp/p\overline{p}$ interactions in others collision energies. Their formalism is based on the idea that particles are produced due to the multiparton collisions and the impact parameter formalism is used.

The plan of the paper is as follows. We shall begin by presenting the basic formalism of the *FSM* in the next section. In Sec. 3 we apply the model to study some aspects of $pp/p\overline{p}$ collisions, namely, overall multiplicity distributions, the range of impact parameters for multiple collisions of partons and the average multiplicity as a function of impact parameter. Discussions of the results and concluding remarks are given in Sec. 4.

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2. THE FUSED STRING MODEL FORMALISM

We review in this section the key points of the *FSM*. The multiplicity distribution is defined [4] by the formula

$$P_n(s) = \frac{\sigma_n(s)}{\sum_{n=0}^{\infty}\sigma_n(s)} = \frac{\sigma_n(s)}{\sigma_{in}(s)}, \quad (1)$$

where σ_n is the cross section of an n-particle process (the so-called topological cross section), σ_{in} is the inelastic cross section and $\sqrt{s}$ the center-of-mass energy. In the *FSM* the incident hadrons are treated as spatially extended objects and their collisions depicted as an ensemble of elementary interaction between quarks and/or gluons (parton-parton collision). It is assumed that in each parton-parton collision there is formation of an string, a object similar to the one in e^-e^+ collisions, in which probably one $q\overline{q}$ pair has triggered the multitude of the final particles. Since the model treats hadrons as extended objects, the impact parameter b is an essential variable in the description of their collisions. Thus, in studies of the collisions between compound objects, such as hadrons, we expect that more than one parton-parton collisions tend to occur in smaller b collisions, where the overlap of hadronic matter is larger and hence two or more strings could be produced. Making the assumption that may occur formation of several strings (i $q\overline{q}$ pairs) and since they are formed in a very small region of typical hadron size, it is natural that they are subsequently fused to from just one larger final chain. By using the eikonal approximation the connection between the elastic and the inelastic channels is established through the unitarity condition and σ_n, in Eq. (1), is decomposed into contributions from each b with weight $G_{in}(s,b)$ [5]

$$P_n(s) = \frac{\sigma_n(s)}{\sigma_{in}(s)} = \frac{\int d^2b\, G_{in}(s,b)\left[\frac{\sigma_n(s,b)}{\sigma_{in}(s,b)}\right]}{\int d^2b\, G_{in}(s,b)}, \quad (2)$$

where $G_{in}(s,b)$ is the inelastic overlap function. To implement the idea of formation of more than just one string, we expanding $G_{in}(s,b)$ in following form

$$G_{in}(s,b) = \sum_{i=1}^{\infty} \frac{[2\Omega(s,b)]^i}{i!} e^{-2\Omega(s,b)} \quad (3)$$

$$\equiv \sum_{i=1}^{\infty} G^{(i)}(s,b), \quad (4)$$

with i representing the number of initial strings created in the interaction. $G^{(i)}$ is the overlap function for formation of i strings, introduced in Ref. [3]. We recall that, in Eq. (2), the quantity in brackets can be interpreted as the probability of producing n particles at b and that it should scale in KNO sense due to its elementary structure [3]. Then, with Eq. (4), the overall multiplicity distribution is given by summing contributions coming from each set of i strings, in general

$$P_n(s) = \sum_{i=1}^{\infty} \frac{\int d^2b \, \frac{G^{(i)}(s,b)}{<n(s,b)>^{(i)}} \left[<n(s,b)>^{(i)} \, \frac{\sigma_n(s,b)}{\sigma_{in}(s,b)} \right]}{\int d^2b \, G_{in}(s,b)}, \quad (5)$$

$< n(s,b) >^{(i)}$ is the average multiplicity of the final fused chain at b and $\sqrt{s}$, originated from i initial strings. This quantity is written as [3]

$$< n(s,b) >^{(i)} = < N(s) > f^{(i)}(s,b), \quad (6)$$

where $< N(s) >$ is the average multiplicity at $\sqrt{s}$ and call $f^{(i)}(s,b)$ multiplicity function of i strings [3]. Now, similarly to KNO, let us introduce for each b the elementary multiplicity distribution

$$\psi\left(\frac{n}{< n(s,b) >^{(i)}}\right) = < n(s,b) >^{(i)} \frac{\sigma_n(s,b)}{\sigma_{in}(s,b)}. \quad (7)$$

Thus, with Eqs. (6) and (7), Eq. (5) becomes

$$\Phi(s,z) = \sum_{i=1}^{\infty} \frac{\int d^2b \, \frac{G^{(i)}(s,b)}{f^{(i)}(s,b)} \, \psi\left(\frac{z}{f^{(i)}(s,b)}\right)}{\int d^2b \, G_{in}(s,b)}, \quad (8)$$

where $\Phi(s,z) = < N(s) > P_n(s)$ and $z = n / < N(s) >$ represents the usual KNO scaling variable. We assume that the i strings formed in the collision are fused to from just one larger final chain and that this final fused chain decays producing particles and, for each value of b, follows a gamma distribution normalized to 2 [5]

$$\psi(z) = 2\,\frac{k^k}{\Gamma(k)} z^{k-1} e^{-kz}, \quad (9)$$

which is characterized by the parameter k. Experimental data for e^-e^+ multiplicity distributions, covering the interval 22.0 GeV $\leq \sqrt{s} \leq$ 161 GeV, were fitted obtaining k=10.775 $\pm$ 0.064 ($\chi^2/N_{DF} = 2.61$) [5]. The gamma distribution is know to arise as a limiting form (with certain correction terms which depend on details of the evolution equation) for the parton number variable, when the dynamical theory (as in QCD case) allows each existing parton to act as a source to emit additional partons (parton branching) [6]. Now, to obtain the multiplicity function $f^{(i)}(s,b)$ in terms of the eikonal Ω we assume that the fractional energy $\sqrt{s'}$, that is deposited for particle production in a collision at b, is equally divided among i number of colliding partons pairs and that this quantity is proportional to Ω. Thus, the total energy $\sqrt{s'}$ of a final fused chain, originated from i initial strings, is written

$$\frac{\sqrt{s'}}{i} \propto \Omega(s,b) \Rightarrow \sqrt{s'} = i\beta(s)\,\Omega(s,b)\,, \quad (10)$$

β is a function of the collision energy and their values increases as the collision energy also increases. In section 3.3 we have estimated the values of β at two different energies. Finally, to obtain the $f^{(i)}(s,b)$ as a function of the Ω, we assume that the average multiplicity originated from i strings depends on the energy $\sqrt{s'}$ in the same way as in e^-e^+ annihilations, which is approximately represented by a power law in $\sqrt{s}$

$$< n(s,b) >^{(i)} = \gamma s'^{A}. \quad (11)$$

Combining the Eqs. (6), (10) and (11) we obtain

$$f^{(i)}(s,b) = i^{2A} \xi(s) [\Omega(s,b)]^{2A}, \quad (12)$$

with $\xi(s)$ determined by the usual normalization conditions on Φ [3] and which also gives $\xi(s)$ as an energy dependent quantity [3]

$$\xi(s) = \frac{\int d^2b \, G_{in}(s,b)}{\int d^2b \, [\Omega(s,b)]^{2A} \, [\sum_{i=1}^{\infty} i^{2A} \, G^{(i)}(s,b)]}. \quad (13)$$

The overall multiplicity distribution, Eq. (8), is explicitly given by

$$\Phi(s,z) = \sum_{i=1}^{\infty} \frac{\int d^2b \, \frac{G^{(i)}(s,b)}{i^{2A}\xi(s)[\Omega(s,b)]^{2A}} \, \psi\left(\frac{z}{i^{2A}\xi(s)[\Omega(s,b)]^{2A}}\right)}{\int d^2b \, G_{in}(s,b)}. \quad (14)$$

We note that, in the last Eq., the weight function is now the overlap function for formation of i strings, $G^{(i)}$, and that, according to Eqs. (3) and (14), the model should contain infinite number of strings irrespective of the incident energy. However, the finite effective partons in the incident hadrons and the finite available energy forbid to form indefinitely large number of strings. In this approach, the effective number of strings involved in the interaction is inferred from multiplicity distribution data. In previous studies, the average multiplicity data in e^-e^+ annihilations were fitted by Eq.(11), giving $A = 0.198 \pm 0.004$ ($\chi^2/N_{DF} = 1.7$) in the interval 10 GeV $< \sqrt{s} \leq$ 183 GeV [5], and $A = 0.200 \pm 0.006$ ($\chi^2/N_{DF} = 0.94$) for the set in the interval 10 GeV $< \sqrt{s} \leq$ 200 GeV [3]. The parameter A is essencially the same in both fits. With the results above and adopting an appropriate parametrization for $\Omega(s,b)$, we have all the necessary elements to test the formalism embodied in Eqs. (14) and (13), making direct comparisons with multiplicity distributions data. In fact, by fixing the value of $A = 0.2$ and adopting Ω from analysis by Henzi and Valin [7], the authors of Ref. [3] computed the overall multiplicity distribution at energies 52.6 and 546 GeV. The theoretical curve at 52.6 GeV showed good agreement with the data assuming formation of up to three initial strings. At 546 GeV the model prediction compared better with the data with up to four strings formation.

2.1. Remarks on FSM

The formalism of *FSM* is based on the idea that, in hadronic interactions, particles are produced due to the parton-parton collisions and with the formation of an object similar to the one in e^-e^+ annihilations. In e^-e^+ collisions probably one $q\bar{q}$ pair has triggered the multitude of final particles ($e^-e^+ \longrightarrow Z/(\gamma) \longrightarrow q\bar{q} \longrightarrow$ hadron). Thus, neglecting the first phase in e^-e^+ collisions ($e^-e^+ \longrightarrow Z/(\gamma)$), we have considered the experimental data on e^-e^+ annihilations as a possible source of information concerning parton-parton interactions in $pp/p\bar{p}$ collisions. With this procedure we are able to predict the hadronic multiplicity distribution in a wide interval of energy without free parameters, which could be adjusted to achieve agreement with experimental results. Specifically, $\Omega(s,b)$ comes from analysis of the elastic scattering, k and A from fits to e^-e^+ data. Despite of the fact that there are some doubts whether such an extremely narrow ψ corresponds to reality, similarities between e^-e^+ and hadronic interactions are often quoted [8]. In addition, Basile et al. [9] found that the average multiplicity in pp collisions is similar to that for e^-e^+ collisions. It was concluded that the multiparticle production mechanism is the same in both processes, controlled mainly by the amount of energy available for particle production.

3. APPLYING THE *FSM*

In next subsections we shall apply the *FSM* formalism to study some quantities associated with the high energy hadronic collisions.

3.1. Multiplicity distributions

The model has been previously used to study multiplicity distributions, Φ, in pp and $p\bar{p}$ collisions at energies 52.6 and 546 GeV, respectively [3]. Now, we would like to investigate the effects of the multiple string formation during the hadronization process on the charged multiplicity distributions arising from $pp/p\bar{p}$ collisions at energies 44.5, 200 and 900 GeV. For this purpose, we have fixed the value of A=0.2 and adopted the Henzi and Valin parametrization for $\Omega(s,b)$. $G^{(i)}(s,b)$ and ψ (with k =10.775) are given by Eqs. (4) and (9), respectively. The value of $\xi(s)$ is obtained from Eq. (13). This completes the specification of model and we have computed the overall multiplicity distributions at energies above mentioned. We show in Fig. 1 the result of computation of Φ at the pp-ISR energy ($\sqrt{s}$=44.5 GeV). We can see that the curve obtained assuming up to three parton-parton collisions (or equivalently, the formation of up to three initial strings) shows reasonable agreement with the data [10]. This is not surprising since the multiplicity distributions observed at the CERN-ISR obey KNO scaling and the theoretical curve predicted from the *FSM* at 52.6 GeV, in Ref. [3], also showed good agreement with the data assuming the formation of up to three initial strings. However, at 200 and 900 GeV, Figs. 2 and 3 , a better agreement with data [11] are obtained if we assume the formation of up to four initial strings. We recall that the good agreement with the data was obtained at 546 GeV and also assuming the formation of up to four initial strings [3]. Thus, the results suggests that one more parton-parton collision is active in $p\bar{p}$ interactions over the range from 200 to 900 GeV than the ISR energies (30.4 - 62.2 GeV). In addition, we

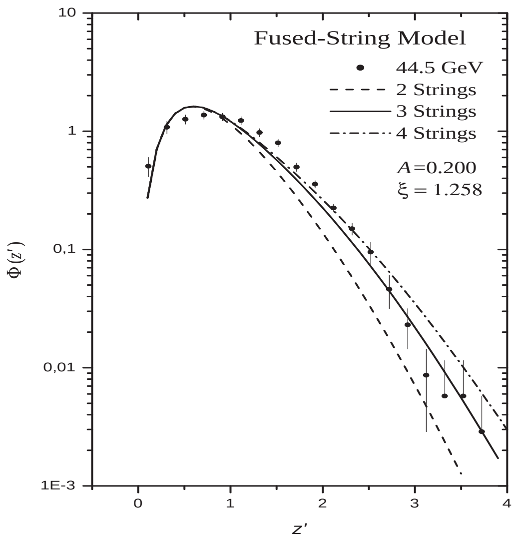

FIG. 1: Overall scaled multiplicity distribution data for pp at ISR energy [10], compared to theoretical prediction using the Fused-String Model, Eqs. (14) and (13).

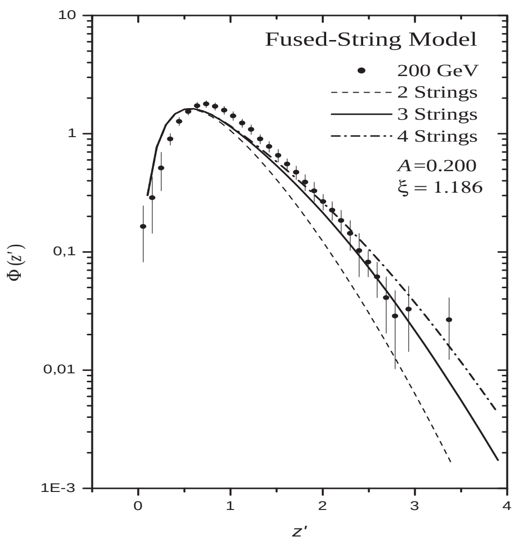

FIG. 2: Overall scaled multiplicity distribution data for $p\bar{p}$ at $\sqrt{s}$=200 GeV [11], compared to theoretical prediction using the Fused-String Model, Eqs. (14) and (13).

also have calculated the i-string formation probability ($q^{(i)}$), as a function of $\sqrt{s}$, that the model predicts and obtained the results shows in Fig. 4. This quantity is defined in Ref. [3] and for completeness we record here:

$$q^{(i)}(s) = \frac{\sigma_{in}^{(i)}(s)}{\sigma_{in}(s)} = \frac{\int d^2b\, G^{(i)}(s,b)}{\int d^2b\, G_{in}(s,b)}. \quad (15)$$

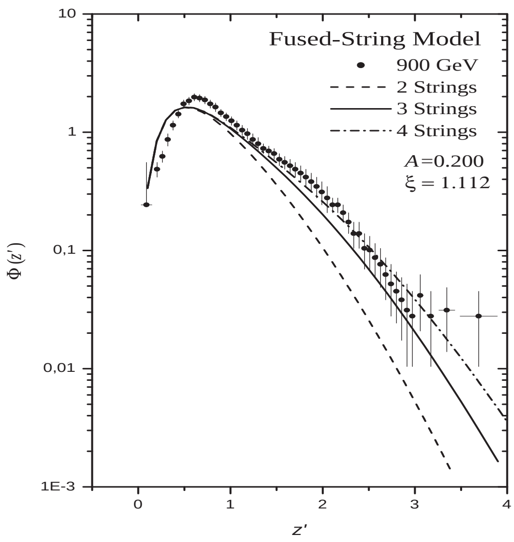

FIG. 3: Overall scaled multiplicity distribution data for $p\overline{p}$ at $\sqrt{s}$=900 GeV [11], compared to theoretical prediction using the Fused-String Model, Eqs. (14) and (13).

The results demonstrate that the probability of formation of one ($q^{(1)}$) and two ($q^{(2)}$) strings are approximately constants in a wide interval of energy (10-5000 GeV). In counterpart, the probability of formation of three ($q^{(3)}$) and four ($q^{(4)}$) strings increases as $\sqrt{s}$ also increases. As each initial string created is associated with a parton-parton collision, it may indicate that the gluons and sea quarks are increasingly important in determining the characteristics of the interactions as the energy is increased. This result agrees with the specific result from an analysis of experimental data done in Ref. [8], in which the author showed that the multiple parton-parton collisions become increasingly important as the collision energy increases. We have expressed Φ in terms of modified the scaling variable $z' = n - N_o / < n - N_o >$ with N_o=0.9 representing the average number of leading particles [5].

3.2. Range of impact parameters for multiple collisions of partons

Since our framework is based on the concept of impact parameter, we would like to investigate for which range of impact parameters we need to consider the superposition of a

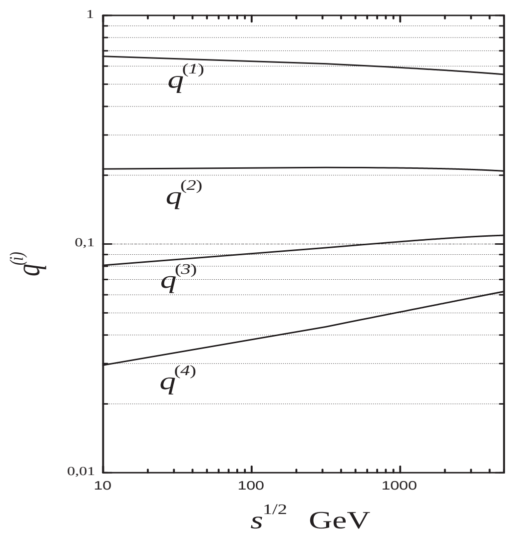

FIG. 4: Energy dependence of i string formation probability, $q^{(i)}$, for several values of i, Eq. (15).

specific number of parton-parton collisions in $pp/p\overline{p}$ interactions. Equations (14) and (13) are the basic equations of model. In the Eq. (14) the weight function is the overlap function for formation of i strings ($G^{(i)}$). Thus, to a first approximation, we can expect that the integrals in (14) vanishes when $G^{(i)} \to 0$. It enables us to estimate the range of impact parameters in which may occur the superposition of 2,3,4,... strings, produced by pairs of colliding partons. For this purpose we show, in Figs 5, 6 and 7, plots of $G^{(i)}$ versus b at energies 52.6, 546 and 900 GeV, respectively, as determined from Eq. (4). Based on this results we present, in Table 1, the estimated range of impact parameters in which i initial pairs of colliding partons are active and the corresponding energies. The interval of impact parameters are essencially the same at 546 and 900 GeV.

3.3. Average multiplicity as a function of impact parameter

Now we proceed to study the average number of hadrons created in inelastic collisions taking place at b and $\sqrt{s}$, namely the average multiplicity, $< n(s,b) >^{(i)}$. Unfortunately, $< n(s,b) >^{(i)}$ can not be directly measured. However, being the impact parameter b an essential variable in a geometrical description of hadronic collisions, we apply the model to compute this quantity as a function of b at energies 52.6 and 546 GeV. Let us recall that, in the *FSM* model, the fractional energy that is deposited for particle production is proportional to $\Omega(s,b)$, given by Eq. (10), and the average multiplicity is parametrized in a general power law form, Eq. (11). Thus, combining Eqs. (10) and (11) we have

$$< n(s,b) >^{(i)} = \gamma[i\beta(s)\Omega(s,b)]^A, \quad (16)$$

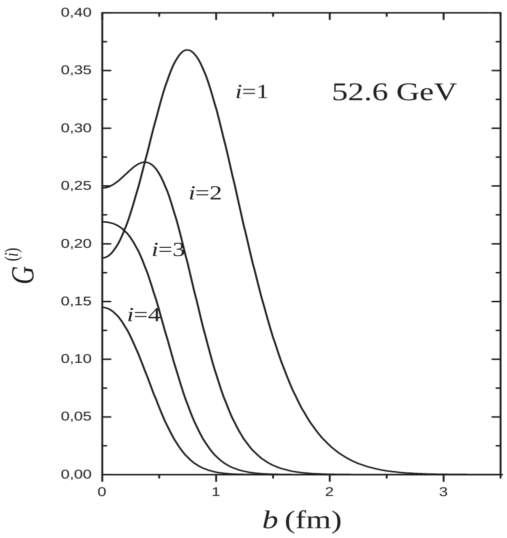

FIG. 5: Overlap function for formation of i strings at $\sqrt{s}$=52.6 GeV and for 4 values of i, Eq. (4).

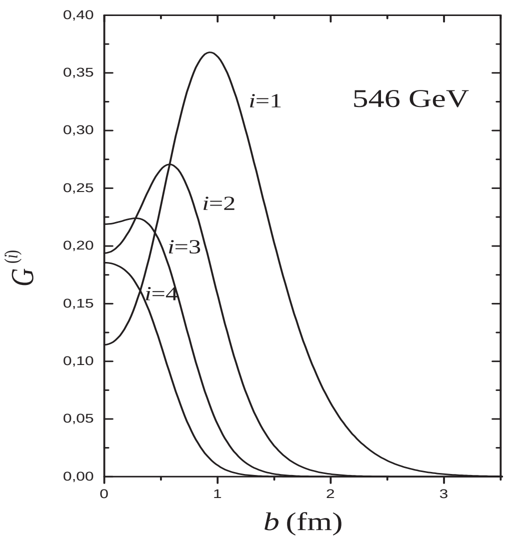

FIG. 6: Overlap function for formation of i strings at $\sqrt{s}$=546 GeV and for 4 values of i, Eq. (4).

where $\Omega(s,b)$ comes from analysis of the elastic scattering data, A=0.2 and γ=3.36 from fits to e^-e^+ data, as discussed in Section 2, i represents the number of initial strings created in the collision, being inferred from multiplicity distribution data. However, we can not calculate the $< n(s,b) >^{(i)}$ until the value of $\beta(s)$ is known. To estimate $\beta(s)$ we note that the

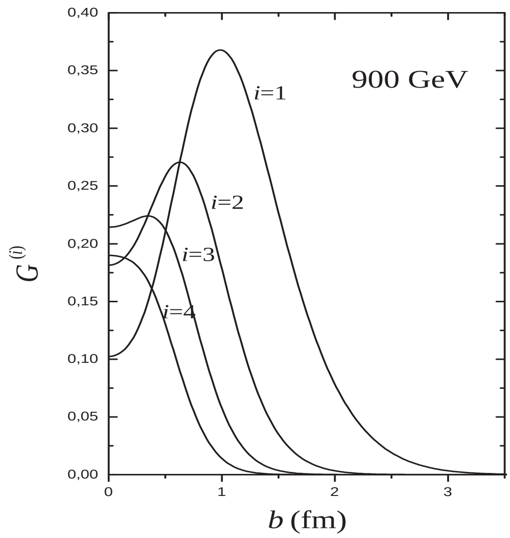

FIG. 7: Overlap function for formation of i strings at $\sqrt{s}$=900 GeV and for 4 values of i, Eq. (4).

parameter $\xi(s)$, introduced in Eq. (12), is related with $\beta(s)$ by

$$\xi(s) = \frac{\gamma[\beta(s)]^{2A}}{< N(s) >}, \tag{17}$$

where $< N(s) >$ is the average multiplicity at $\sqrt{s}$. By using the values of $< N(s) >$ imputed from experiments ($< N(52.6) >$=11.55 [10] and $< N(546) >$=27.5 [12]) and observing that $\xi(s)$ is obtained from Eq. (13), we have estimated the values of $\beta(52.6\ )$=38.27 and $\beta(546\ )$=263.82. With the results above we have computed the $< n(s,b) >^{(i)}$ that the model predicts at ISR and Collider energies. The results are displayed as the curves in Fig. 8. Since $< n(s,b) >^{(i)}$ is proportional to the eikonal $\Omega(s,b)$, we can see that $< n(s,b) >^{(i)}$ is greatest when the hadron is the most black and the least when the hadron is the most transparent. This is a physically realistic description of $< n(s,b) >^{(i)}$ since that more particles tend to be produced in smaller b collisions, where the overlap of the hadronic matter distributions is greater. In Fig. 8 we also can see that both curves are presenting inflection points around 1 *fm*. The behavior of the average multiplicity in both energies seems to be compatible with the interpretation gave in Ref. [13], in which the authors found, by model-independent fit to elastic pp differential cross section, that the profile function at $\sqrt{s}$=52.8 GeV presents a change of curvature around 1 *fm*. It was interpreted as due two different dynamical contributions, one associated with a dense central region and another with an evanescent peripheral region.

TABLE I: Range of impact parameters (fm) in which may occur the superposition of i strings, each one produced by a pair of colliding partons, and the corresponding energies.

i	52.6 GeV	546 GeV	900 GeV
1	$b < 2.7$	$b < 3.2$	$b < 3.3$
2	$b < 1.8$	$b < 2.2$	$b < 2.3$
3	$b < 1.4$	$b < 1.6$	$b < 1.7$
4	$b < 1.0$	$b < 1.3$	$b < 1.3$

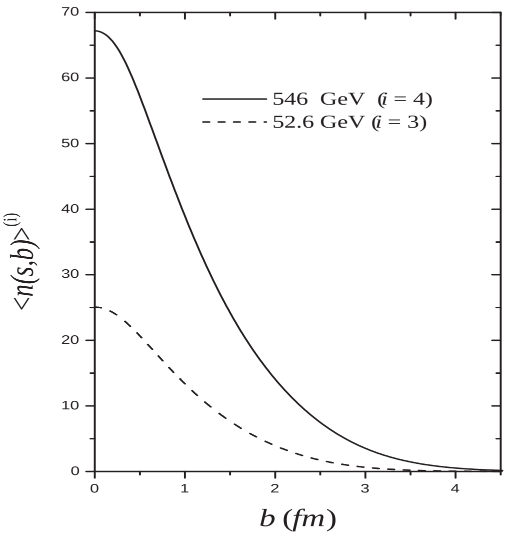

FIG. 8: Average multiplicity as function of impact parameter, obtained from the model calculation, Eq.(16). Both curves present a change of curvature around 1 fm.

4. DISCUSSION AND CONCLUDING REMARKS

We have studied the effects produced by multiple collisions of partons in hadronic interactions at ISR (44.5 - 52.6 GeV) and Collider (200 - 900 GeV) energies by using a phenomenological procedure referred to as *FSM*. It is assumed that in each parton-parton collision an object (string) similar to the one in e^-e^+ annihilations is created. Based on results from Ref. [9], which are suggestive of a universal mechanism of particle production in strongly-interacting systems, we have considered the experimental data available on e^-e^+ collisions as possible source of information concerning elementary hadronic interactions. The strings originated due multiple parton-parton collisions are fused to from just one larger final chain, ψ, which decays producing particles and follows a gamma distribution at each value of impact parameter. However, the dynamical origin for the strings fusion process remains to be understood at a deeper level. Our results has been analyzed together with those obtained in Ref. [3]. The model predictions for overall multiplicity distributions revealed that three pairs of colliding partons are active at ISR energies and four over the range from 200 to 900 GeV. This result indicates that the increase in the number of parton parton collision leads to the violation of KNO scaling. We also have computed the i strings formation probability (Fig.4). The results obtained demonstrate that the probability for three and four parton-parton collisions increases as the energy is increased. It may indicate that the gluons and sea quarks are increasingly important in determining the characteristics of the interactions as the energy is increased. We have presented a criterion and estimated the range of impact parameters, of the incident hadronic system, for multiple collisions of partons at energies 52.2, 546 and 900 GeV. The average number of particles produced depends upon the hadron opacity at each impact parameter. Despite some simplifications made in our description, the results seems to be consistent with the multiplicity distributions data in a wide interval of energy (44.5 - 900 GeV) and therefore has a chance of continuing to work at higher energies also.

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[1] S. M. Troshiin and N. E. Tyurin, J. Phys. G: Nucl. Part. Phys. **29**, 1061 (2003).

[2] I. M. Dremin, JETP Letters **84** no. 5, 235 (2006).

[3] P. C. Beggio and Y. Hama, Braz. J. Phys, **37**, 1164 (2007).

[4] I. M. Dremin and J. W. Gary, Phys. Rep. **349**, 301 (2001).

[5] P. C. Beggio, M. J. Menon, and P. Valin, Phys. Rev. D **61**, 034015 (2000).

[6] S. Barshay and J. Goldberg, Phys. Lett. B **196**, 566 (1987).

[7] R. Henzi and P. Valin, Phys. Lett. B **160**, 167 (1985).

[8] W. D. Walker, Phys. Rev. D **69**, 034007 (2004).

[9] M. Basile *et al.*, Phys. Lett. B **92**, 367 (1980); B. B. Back *et al.*, nucl-ex/**0301017 v1** (2003).

[10] ABCDWH Collaboration; A. Breakstone *et al.*, Phys. Rev. D **30**, 528 (1984).

[11] UA5 Collaboration; R. E. Ansorge et al., Z. Phys. C **43**, 357 (1989).

[12] UA5 Collaboration, G. J. Alner *et al.*, Phys. Rep. **154**, 247 (1987).

[13] G. L. P. Silva, M. M. Menon, and R. F. Avila, Int. J. Mod. Phys. E **16**, 2923 (2007).