

# GUP and Higher Dimensional Reissner-Nordström Black Hole Radiation

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Based on the generalized uncertainty principle (GUP), in which the quantum gravitational effects are taken into account, the corrected Beckenstein-Hawking entropy of the higher dimensional Reissner-Nordström black hole, up to the square order of Planck length, is calculated. Using the corrected entropy, the black hole radiation probability is calculated in the tunneling formalism, which is corrected up to the same order of the Planck length and shows a more probable quantum tunneling.

Keywords: Generalized uncertainty principle, Higher dimensional R-N black hole, Black hole radiation

## 1. INTRODUCTION

Since the original analysis of black hole radiation was done [1], several derivations of Hawking radiation were subsequently presented in the literature [2]. None of them, however, corresponds directly to one of the heuristic pictures that visualizes the source of radiation as tunneling. In this method [3, 4], the particles are allowed to follow the classically forbidden trajectories, by starting just behind the horizon onward to infinity. The particles then travel back in time, since the horizon is locally to the future of the external region. Thus the classical one particle action becomes complex and so the tunneling amplitude is governed by the imaginary part of this action for the outgoing particle. However, the action for the ingoing particle must be real, since classically a particle can fall behind the horizon. This is an important point of calculations of black hole tunneling. The essence of tunneling based calculations is, thus, the computation of the imaginary part of the action for the process of s-wave emission across the horizon, which in turn is related to the Boltzmann factor for the emission at the Hawking temperature. There are two different methods to calculate the imaginary part of the action: one is by Parikh-Wilczek [3] radial null geodesic method and another is the Hamilton-Jacobi method which was first used by Srinivasan *et al.* [4]. Later, many people [5] used the radial null geodesic method to find out the Hawking temperature for different space-time metrics. Recently, tunneling of a Dirac particle through the event horizon was also studied [6]. All of these computations are, however, confined to the semiclassical approximation only. The issue of quantum gravity corrections is generally not discussed. In [7] it is found that the corrections to the temperature and entropy by including the effects of back reaction knowing the modified surface gravity of the black hole due to one loop back reaction for the Schwarzschild case by radial null geodesic method. As an extension, in [8] also applied this method for a noncommutative Schwarzschild metric. Recently, a

problem in this approach has been discussed in [9] which corresponds to a factor two ambiguity in the original Hawking temperature. From a pure theoretical point of view one can expect that the properties of black holes might also have played an important role in understanding the nature of gravity in higher dimensions. This expectation has triggered the study of black holes in higher dimensional gravity theories as well as in string theory [10]. Whoever the emergence of the TeV-scale gravity provides a motivation [11] for the black hole experiments in the future accelerator such as the CERN Large Hadron Collider. Thus, it is important to investigate the effect of the extra dimensions in the various properties of black holes. Even if a full description of quantum gravity is not yet available, there are some general features that seem to go hand in hand with all promising candidates for such a theory. One of them is the need for a higher dimensional space-time, one other the existence of a minimal length scale. The scale at which the running couplings unify and quantum gravity is likely to occur is called the Planck scale. At this scale the quantum effects of gravitation get as important as those of the electroweak and strong interactions. In this article through a suitable adaptation of higher dimensional and the minimal length scale, we show that the quantum tunneling probability is modified when quantum gravitational effects are properly taken into account, with respect to the Planck scale. We obtain the radiation tunneling of a higher dimensional Reissner-Nordström black hole, using the corrected Beckenstein-Hawking entropy obtained from the GUP, in which the gravitational effects are taken into account.

## 2. THE CORRECTED BECKENSTEIN-HAWKING ENTROPY

A natural candidate for charged black holes of higher dimensional is that of Reissner-Nordström d-dimensional solution of Einstein field equation,

$$ds^2 = f(r)c^2 dt^2 - f^{-1}(r)dr^2 - r^2 d\Omega_{d-2}, \quad (2.1)$$

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where

$$f(r) = 1 - \frac{\mu}{r^{(d-3)}} + \frac{\theta^2}{r^{2(d-3)}}.$$

The parameter  $\mu$  is related to mass  $M$  of the black hole

$$\mu = \frac{8\pi G_d}{(d-2)\mathcal{A}_{d-2}}M,$$

where  $G_d$  is the d-dimensional Newton constant.  $\mathcal{A}_{d-2}$  is the of the unit  $(d-2)$ -sphere given by

$$\mathcal{A}_{d-2} = \frac{2\pi^{(d-1)/2}}{\Gamma(\frac{d-1}{2})}.$$

The electric charge of the black hole is given by

$$Q^2 = \frac{(d-2)(d-3)}{8\pi G_d}\theta^2.$$

There is an outermost horizon situated at

$$r_h = \left[ \mu + (\mu^2 - \theta^2)^{\frac{1}{2}} \right]^{\frac{1}{d-3}}, \quad \mu^2 > \theta^2.$$

Let us consider the black hole as a d-dimensional cube of size equal to twice its radius  $r_h$ , the uncertainty in the position of a Hawking particle, during the emission, is

$$\begin{aligned} \Delta x &= 2r_h = 2 \left[ \mu + (\mu^2 - \theta^2)^{\frac{1}{2}} \right]^{\frac{1}{d-3}} \\ &= \left[ \frac{Q^2 M}{a^2(d-3)} \left( 1 + \sqrt{1 - \frac{a^2}{M^2}} \right) \right]^{\frac{1}{d-3}}, \end{aligned} \quad (2.2)$$

where  $a^2 = \frac{(d-2)}{(d-3)} \frac{Q^2 \mathcal{A}_{d-2}}{8\pi G_d}$ .

Using the usual uncertainty principle, uncertainty in the energy of the Hawking particles is

$$\Delta E \approx c\Delta p \approx \frac{\hbar c}{\Delta x} = \hbar c \left[ \frac{Q^2 M}{a^2(d-3)} \left( 1 + \sqrt{1 - \frac{a^2}{M^2}} \right) \right]^{\frac{-1}{d-3}}. \quad (2.3)$$

It is easy to obtain the temperature of black hole in d-dimensional space-time. The Hawking temperature is related to the horizon radius by

$$T = \frac{1}{4\pi r_h} = \frac{1}{2\pi \Delta x}. \quad (2.4)$$

The Bekenstein-Hawking entropy is usually derived from the Hawking temperature. The entropy  $S$  may be found from the well known thermodynamics relation,

$$T = \frac{dE}{dS} \approx \frac{dM}{dS}, \quad (2.5)$$

where  $M$  means energy and  $T$  means temperature. From (2.2), (2.4) and (2.5) we obtain

$$\begin{aligned} S &= A_{rea} \left[ \frac{a^2(d-3)}{Q^2} \right]^{\frac{1}{d-3}} \left[ M \left( 1 + \sqrt{1 - \frac{a^2}{M^2}} \right) \right]^{\frac{-2}{d-3}} \\ &\times \int dM \left[ M \left( 1 + \sqrt{1 - \frac{a^2}{M^2}} \right) \right]^{\frac{1}{d-3}} + const., \end{aligned} \quad (2.6)$$

where  $A_{rea} = 4\pi r_h^2$  is the surface area of the black hole horizon.

The evaporation of black hole would leave very distinctive imprints on the detectors and temperature of such black hole could be calculated. To study the quantum gravity effects on the Hawking temperature, one can take into account the gravitational effects through the GUP. Recently GUP has been the subject of much interesting works and a lot of papers have appeared in which the usual uncertainty is modified at the framework of microphysics as [12]:

$$\Delta x \geq \frac{\hbar}{\Delta p} + L_P^2 \frac{\Delta p}{\hbar}, \quad (2.7)$$

where  $L_P$  is the Planck length. The term  $L_P^2 \frac{\Delta p}{\hbar}$  in Eq.(2.7) shows the gravitational effects to usual uncertainty principle. Let us consider a quantum black hole, an attempt to measure the radius of the black hole, more precisely that is, to make  $r_h$  small thus resulting an increase in  $\Delta p$ , but according to Eq.(2.7) for detection of small distances by going to very high momenta, the behavior of the *Heisenberg microscope* changes and a lower bound on the black hole radius  $r_h$  could be obtained. Setting  $2r_h$  as  $\Delta x_i$  and inverting Eq.(2.7) we obtain

$$\frac{r_h}{L_P^2} \left( 1 - \sqrt{1 - \frac{L_P^2}{r_h^2}} \right) \leq \frac{\Delta p}{\hbar} \leq \frac{r_h}{L_P^2} \left( 1 + \sqrt{1 - \frac{L_P^2}{r_h^2}} \right). \quad (2.8)$$

From Eq.(2.8) one can write

$$\frac{\Delta p}{\hbar} = \frac{r_h}{L_P^2} \left( 1 - \sqrt{1 - \frac{L_P^2}{r_h^2}} \right) = \frac{1}{2r_h} + O(L_P^4). \quad (2.9)$$

Substitution in Eq.(2.7) leads to

$$\Delta x' = \Delta x \left( 1 + \frac{L_P^2}{4r_h^2} \right). \quad (2.10)$$

Using Eqs.(2.4), and (2.10) we obtain the Hawking temperature of d-dimensional black hole,

$$T' = \frac{1}{2\pi \Delta x'} \left( 1 + \frac{L_P^2}{4r_h^2} \right)^{-1} \simeq T \left( 1 - \frac{L_P^2}{4r_h^2} \right). \quad (2.11)$$

The corrected entropy  $S'$  may be found from the thermodynamics relation (2.5),

$$S'(M) = S(M) + L_P^2 F(M), \quad (2.12)$$

where  $S(M)$  is given in Eq.(2.6) and

$$\begin{aligned} F(M) &= \frac{1}{4} A_{rea} \left[ \frac{a^2(d-3)}{Q^2} \right]^{\frac{3}{d-3}} \left[ M \left( 1 + \sqrt{1 - \frac{a^2}{M^2}} \right) \right]^{\frac{-2}{d-3}} \\ &\times \int dM \left[ M \left( 1 + \sqrt{1 - \frac{a^2}{M^2}} \right) \right]^{\frac{-1}{d-3}}. \end{aligned} \quad (2.13)$$

Eq.(2.12) is the corrected entropy of a higher dimensional charged black hole whose temperature is modified based on the GUP.

One can show that the radiated energy, through Hawking radiation is the same as that given in Eq.(2.3).

### 3. BLACK HOLE RADIATION VIA QUANTUM TUNNELING

Classical black holes are perfect absorbers, they accrete their (irreducible) mass and no fraction of it can escape as there are no classical allowed trajectories crossing the horizon on the way out. It is interesting to note how the inclusion of quantum effects allows, for particles in the Reissner-Nordström geometry, to propagate through classically forbidden regions. This suggests that it should be possible to describe the black hole emission process, in a semiclassical fashion, as quantum tunneling. In the WKB approximation the tunneling probability is a function of the imaginary part of the action

$$\Gamma \sim e^{-2I_m(I)}, \quad (3.1)$$

where  $I_m$  is the imaginary part and  $I$  is the classical action of trajectory. Eq.(3.1) can be written as [13]

$$\Gamma \sim \frac{e^{S_f}}{e^{S_i}} = e^{\Delta S}, \quad (3.2)$$

in which  $\Delta S$  is the difference between final and initial values of the black hole entropy.

The corrected Beckenstein-Hawking entropy in which the gravitational effects are taken in to account is given by Eq.(2.12), so that

$$\Delta S' = \Delta S + L_p^2 \Delta F, \quad (3.3)$$

where

$$\Delta S = S(M-E) - S(M), \quad \Delta F = F(M-E) - F(M).$$

Substituting (3.3) in (3.2) we obtained

$$\Gamma' \sim \Gamma e^{L_p^2 \Delta F}, \quad (3.4)$$

which shows the corrected tunneling probability up to the square order of Planck length. Appearance of an exponential coefficient in the corrected tunneling probability in Eq.(3.4) predicts a generalized quantum tunneling through the horizon of the Reissner-Nordström black hole, which obtains from the quantum gravitational effects on the black hole radiation.

### 4. CONCLUSION

Through the GUP, in which the gravitational effects up to the square order of the Planck length are taken in to account, we were able to calculate the corrected Beckenstein-Hawking entropy of Reissner-Nordström black hole in higher dimensional space-times. Using this corrected Beckenstein-Hawking entropy, we have calculated the quantum tunneling probability of the higher dimensional charged black holes radiation, which contains a correction up to the same order in the Planck length. The mathematical consequence of these calculations is a more probable quantum tunneling through the horizon of the black hole, which comes from the quantum gravitational consideration in the GUP.

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