Is it possible to accommodate massive photons in the framework of a gauge-invariant electrodynamics?

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The construction of an alternative electromagnetic theory that preserves Lorentz and gauge symmetries, is considered. We start off by building up Maxwell electrodynamics in (3+1)D from the assumption that the associated Lagrangian is a gauge-invariant functional that depends on the electron and photon fields and their first derivatives only. In this scenario, as well-known, it is not possible to set up a Lorentz invariant gauge theory containing a massive photon. We show nevertheless that there exist two radically different electrodynamics, namely, the Chern-Simons and the Podolsky formulations, in which this problem can be overcome. The former is only valid in odd space-time dimensions, while the latter requires the presence of higher-order derivatives of the gauge field in the Lagrangian. This theory, usually known as Podolsky electrodynamics, is simultaneously gauge and Lorentz invariant; in addition, it contains a massive photon. Therefore, a massive photon, unlike the popular belief, can be adequately accommodated within the context of a gauge-invariant electrodynamics.

Keywords: Podolsky Electrodynamic; Massive Photons; Gauge-invariant Electrodynamics.

1. INTRODUCTION

Maxwell electrodynamics, or its quantum version, i.e., QED, is widely recognized as the adequate theory for the description of the electromagnetic phenomena, because of the astonishing agreement between theory and experiment. However, it only enjoyed this high status after some of its intrinsic problems were solved. Among them, the most remarkable one is certainly the presence of divergences or infinities, even at the classical level [1].

This aspect of the Maxwell electromagnetic theory naturally emerges when the self-energy of an elementary (charged) particle, like the electron, for example, is considered. An object of this sort has no internal structure, which means that it must be regarded (classically) as a geometric point. Its Coulomb energy, given by

$$E_{Coul}\ \alpha \int_0^\infty E^2 dV \ , \tag{1}$$

where E is the electron electric field, like the associated self-energy, diverges.

Objects having finite extension, on the other hand, such as composite particles, must be described by internal degrees of freedom since in this case the aforementioned problem, at least in principle, does not occur. Hadronic particles, for instance, belong to this category since their static properties, like mass, are finite and, in principle, obtained through the quark dynamics.

In spite of the mentioned success of the electromagnetic theory, it remain some intriguing questions that cannot be completely answered by a simple comparison between experiment and theory. One of most remarkable, among others, is the question of the massless character of the photon. From a theoretical point of view the existence of massive photons is perfectly compatible with the general principles of elementary particle physics. This possibility cannot be discarded either from an experimental viewpoint. Indeed, despite the fact that a very small value for the photon mass has not been found experimentally up to now, this does not allows to conclude that its mass must be identically zero. In fact, the more accurate experiments currently avaliable can only set up upper bounds on the photon mass. Incidentally, the recently recommended limit published by *Particle Data Group* is $m_\gamma \leq 2\times 10^{-25} GeV$ [2]. On the other hand, using the uncertainty principle, we obtain an upper limit on the photon rest mass equal to $10^{-34} GeV$, which is found by assuming that the universe is 10^{10} years old [3]. Nonetheless, the relevant question, from a theoretical point of view, is that a nonvanishing value for the photon mass is incompatible with Maxwell electrodynamics.

So, we can ask ourselves whether or not it would be possible to construct a gauge-invariant electrodynamics, such as Maxwell one, but in which a massive photon could be accommodated. At first sight, it seems that the Proca theory [4], described by the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu, \tag{2}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, fulfills the aforementioned requirements. Lagrangian (2) leads to massive dispersion relations for the gauge boson, implying in a Yukawa potential in the static case. Since this potential has a finite range, the electron self-energy is finite [1]. Besides, Proca electrodynamics is Lorentz invariant. However, gauge invariance is lost, which is certainly undesirable since, as a consequence, this model would be in disagreement with the predictions of the Standard Model $SU(3)\times SU(2)\times U(1)$ [5].

Other alternative models, such as the Chern-Simons [6] and the Podolsky [7] ones, can be constructed in the same vein.

In the Chern-Simons electrodynamics a coupling between the gauge field and the field strength is introduced into the Lagrangian through the Levi-Civita tensor. This coupling yelds a massive dispersion relation for the gauge field. As a result of this mechanism, a massive photon is generated. Nevertheless, the mentioned mechanism explicitly breaks the Lorentz invariance in four dimensions, unless a 2-form gauge field is also introduced that mixes up with the Maxwell po-

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tential. In odd dimensions, however, this model is simultaneously Lorentz and gauge invariant.

Podolsky electrodynamics, on the other hand, seems more interesting in comparison to the above cited models since it can accomaddate a massive photon without violating the Lorentz and gauge symmetries in (3+1)D.

There are other interesting aspects of Podolsky theory that deserves to be exploited. For instance, within its context magnetic monopoles and massive photons can coexist without conflict. That is not the case as far as the Proca model [8] is concerned.

The aim of this paper is precisely to discuss the issue of the photon mass in the framework of some outstanding electromagnetic theories. To start off, Maxwell theory is considered in section II. In particular, it is shown in this section that this theory can be built up via simple and general assumptions; it is also demonstrated that Lorentz and gauge invariance constrain the photon mass to be equal to zero. In section III, we discuss the Chern-Simons theory and prove that in odd dimensions the photon can acquire mass without breaking the Lorentz and gauge symmetries. In section IV the Podolsky electromagnetic theory is analyzed. We show that within the context of this model, massive photons are allowed while the Lorentz and gauge symmetries are preserved. It is worth mentioning that the approach to the Podolsky model we have taken in this paper may be regarded as an alternative method to those employed by A. Accioly [9] and H. Torres-Silva [10].

2. MAXWELL ELECTRODYNAMICS

We shall construct Maxwell electrodynamics based on the following three assumptions:

(i)Lorentz invariance holds.

(ii)There exists a Lagrangian $\mathcal{L}$ for the theory which is a functional of the electron and photon fields, as well as of their first derivatives, namely,

$$\mathcal{L} = \mathcal{L}\left(\psi, \partial_\mu \psi; A_\mu, \partial_\mu A_\nu\right). \quad (3)$$

(iii)$\mathcal{L}$ is invariant under a local gauge transformation.

In this spirit, we consider the following gauge transformations with respect, respectively, to the bosonic field

$$A_\mu \;\rightarrow\; A'_\mu = A_\mu + \partial_\mu \beta\left(x\right), \quad \delta A_\mu = \partial_\mu \beta, \quad (4)$$

and the matter field

$$\psi \;\rightarrow\; \psi' = \exp\left(ie\beta\right)\psi, \quad \delta\psi = ie\beta\psi. \quad (5)$$

In the above equations β is a local gauge parameter, $\beta = \beta\left(x\right)$.

The requirement of the invariance of the Lagrangian with respect to these transformations, $\delta_{gauge}\mathcal{L} = 0$, yields

$$\frac{\partial \mathcal{L}}{\partial \psi}\left(ie\psi\right)\beta + \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \psi\right)}\left(ie\psi\right)\partial_\mu \beta + \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \psi\right)}\left(ie\partial_\mu \psi\right)\beta + \frac{\partial \mathcal{L}}{\partial A_\mu}\left(\partial_\mu \beta\right) + \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu A_\nu\right)}\left(\partial_\mu \partial_\nu \beta\right) = 0. \quad (6)$$

Now, since β is an arbitrary parameter, we promptly obtain

$$\frac{\partial \mathcal{L}}{\partial \psi}\left(ie\psi\right) + \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \psi\right)}\left(ie\partial_\mu \psi\right) = 0. \quad (7)$$

Using the Euler-Lagrange equations for the ψ field in the above expression, we then find

$$\partial_\mu\left[ie\frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \psi\right)}\psi\right] = 0. \quad (8)$$

This result clearly shows there exists a Noetherian vector current associated to the gauge symmetry

$$j^\mu \equiv \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \psi\right)}\left(ie\psi\right), \quad (9)$$

which is conserved $\left(\partial_\mu j^\mu = 0\right)$.

On the other hand, to first-order in β derivatives, we have

$$\left[\frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \psi\right)}ie\psi\right] + \frac{\partial \mathcal{L}}{\partial A_\mu} = 0, \quad (10)$$

which can be written as

$$\frac{\partial \mathcal{L}}{\partial A_\mu} = -j^\mu. \quad (11)$$

This relation tells us how the gauge field must be coupled to a conserved current in the Lagrangian.

Finally, the second-order derivative terms in the gauge parameter yield the condition

$$\left(\partial_\mu \partial_\nu \beta\right)\frac{\partial \mathcal{L}}{\partial\left(\partial_\mu A_\nu\right)} = 0, \quad (12)$$

which implies that the symmetric part of the derivative term in the Lagrangian must be null, i.e,

$$\frac{\partial \mathcal{L}}{\partial\left[\partial_{(\mu} A_{\nu)}\right]} = 0. \quad (13)$$

Thus, we can write

$$\frac{\partial \mathcal{L}}{\partial\left[\partial_{[\mu} A_{\nu]}\right]} = H_{\mu\nu}, \quad (14)$$

where $H_{\mu\nu}$ is a totally antisymmetric rank-two tensor. Here

$$\partial_{[\mu} A_{\nu]} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu, \quad (15)$$

$$\partial_{(\mu} A_{\nu)} \equiv \partial_\mu A_\nu + \partial_\nu A_\mu. \quad (16)$$

Consequently, the bosonic sector of the Lagrangian is given by

$$\mathcal{L} = a\partial_{[\mu} A_{\nu]} H^{\mu\nu} + b\, j_\mu A^\mu. \quad (17)$$

The first term in Eq. (17) is related to the vector field only, and must be bilinear in A_μ. As a consequence, one of the Lorentz indices of $H_{\mu\nu}$ must necessarily be associated to the gauge field. The simplest choice for the kinetic term, which is quadratic in $\partial_\mu A_\nu$, is

$$H_{\mu\nu} = \partial_{[\mu}A_{\nu]}, \quad (18)$$

i.e., the tensor $H_{\mu\nu}$ can be identified with the usual electromagnetic field strength $F_{\mu\nu}$. Taking this into account, the corresponding Lagrangian can be written in the general form

$$\mathcal{L} = aF_{\mu\nu}F^{\mu\nu} + bj^\mu A_\mu, \quad (19)$$

where a and b are arbitrary constants. By analyzing the equations of motion related to (17), it is trivial to see that a convenient choice for these constants is $a = -\frac{1}{4}$ and $b = 1$, which allows us to write

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + j^\mu A_\mu, \quad (20)$$

which is nothing but Maxwell Lagrangian.

The field A_μ in (18) is massless. This raises the interesting question: Could we have chosen the tensor $H_{\mu\nu}$ such that it contained a gauge-invariant mass term related to A^μ, besides the massless term? Since in the selection of the early $H_{\mu\nu}$ we have excluded the possibility that A^μ could be massive, this is a pertinent question. Let us then discuss this possibility.

The kinetic part of the gauge field in the Lagrangian, as commented above, must have the general form

$$\mathcal{L} \;\alpha\; \partial_{[\mu}A_{\nu]}H^{\mu\nu}. \quad (21)$$

In other words, $H_{\mu\nu}$ must be a function of A_μ and its first derivatives only. Therefore, $H^{\mu\nu}$ can be written in the alternative form

$$H^{\mu\nu} = F^{\mu\nu} + h^{\mu\nu}, \quad (22)$$

where, obviously, $h^{\mu\nu}$ is an antisymmetric tensor. Accordingly,

$$h^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta}\,(?)_\alpha A_\beta, \quad (23)$$

where $\varepsilon^{\mu\nu\alpha\beta}$ is the Levi-Civita tensor and the quantity $(?)_\alpha$ is a Lorentz vector to be determined. There are two possibilities to be considered. The first one is to assume that the mentioned quantity is a constant vector, which implies that it would play the role of a fundamental quantity of nature. In this case, the aforementioned constant vector would single out a special direction in space-time leading, as a consequence, to a breaking of the Lorentz symmetry. The remaining choice is $(?)_\alpha = \partial_\alpha$, which would imply that the searched quantity should be proportional to the electromagnetic field-strength, $h^{\mu\nu} \;\alpha\; F^{\mu\nu}$. Thus, we come to the conclusion that the gauge field is massless due to the two very general assumptions considered in the construction of the Lagrangian, in addition to the Lorentz invariance.

3. CHERN-SIMONS ELECTRODYNAMICS

In the preceding section we concluded that a Lagrangian which is a functional of the electron and photon fields, as well as of their first derivatives and, besides, is invariant under local gauge transformations and consistent with the Lorentz symmetry, confers a massless character to the vector field. Our proof, however, relied upon the fact that the space-time was endowed with (3 + 1) dimensions. Yet, it is possible to show that in odd dimensional space-times the form of the antisymmetric tensor $H_{\mu\nu}$ need not be proportional to $F_{\mu\nu}$ only. That is the case of the so-called Chern-Simons electrodynamics. In order to obtain the Lagrangian corresponding to this theory we suppose that the same assumptions utilized in the construction of the Maxwell theory still hold. As long as the quantity $H_{\mu\nu}$ is concerned, we consider another alternative: the space-time has (2+1)dimensions (in particular). In such a case we have to construct an antisymmetric tensor ($h^{\mu\nu}$). This quantity can now be expressed as follows

$$h^{\mu\nu} = \varepsilon^{\mu\nu\alpha}A_\alpha. \quad (24)$$

A Lorentz invariant term can be then constructed by contracting this term with the usual electromagnetic tensor $F^{\mu\nu}$. This means that the Lagrangian can be written in the form

$$\mathcal{L} \;\alpha\; aF_{\mu\nu}F^{\mu\nu} + b\varepsilon^{\mu\nu\alpha}F_{\mu\nu}A_\alpha, \quad (25)$$

where a and b are arbitrary constants. Here b has dimension of mass.

The above result may be extended to any odd dimension, because we can always construct the Chern-Simons term through the contraction of a field with n Lorentz indices with its field-strength containing $n+1$ indices. The Levi-Civita tensor, on the other hand, will have $2n+1$ indices. For instance, in a 5-dimensional space time, we have

$$\textit{Chern-Simons term} = \varepsilon^{\mu\nu\lambda\alpha\beta}B_{\mu\nu}H_{\lambda\alpha\beta}, \quad (26)$$

with

$$H_{\lambda\alpha\beta} = \partial_\lambda B_{\alpha\beta} + \partial_\alpha B_{\beta\lambda} + \partial_\beta B_{\lambda\alpha}. \quad (27)$$

We remark that we have only considered gauge 1−forms to build the Chern-Simons term; nevertheless, it is also possible to use a gauge 2−form (the so called "*BF* " term $\left(\varepsilon^{\mu\nu\kappa\lambda}B_{\mu\nu}F_{\kappa\lambda}\right)$) to accomplish this goal. However, in order to avoid the introduction of new degrees of freedom [11], we have opted in this paper to work in the Chern-Simons scenario.

4. PODOLSKY ELECTRODYNAMICS

In the preceding sections, we have found that in $(1+3)D$ the vector gauge field is massless as a consequence of the very general assumptions made in order to build the associated Lagrangian. That is not the case whenever odd dimensional space-times are concerned. Indeed, in these space-times a mass term for the vector field is allowed. We are now ready to focus on the issue theme of this work, i.e., the question of whether or not massive photons can be accommodated in the context of a gauge-invariant electromagnetic theory in $(3+1)D$. To do that, we shall relax one of the assumptions made in the construction of the preceding electrodynamics, namely, the one that forbids the presence of higher derivatives of the gauge field in the Lagrangian. As a result, the gauge sector will be altered while the matter contribution remains unchanged. To be more explicit, let us suppose that the Lagrangian is as follows

$$\mathcal{L} = \mathcal{L}\left(\psi, \partial_\mu \psi; A_\mu, \partial_\nu A_\mu, \partial_\lambda \partial_\mu A_\nu\right). \tag{28}$$

Imposing now that (26) is invariant with respect to the transformations (4) and (5) yields

$$\frac{\partial \mathcal{L}}{\partial \psi}(ie\beta\psi) + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\left[ie(\partial_\mu \beta)\psi\right] + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}(ie\beta\partial_\mu \psi) + \frac{\partial \mathcal{L}}{\partial A_\mu}(\partial_\mu \beta) + \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)}(\partial_\mu \partial_\nu \beta) + \frac{\partial \mathcal{L}}{\partial(\partial_\lambda \partial_\mu A_\nu)}(\partial_\lambda \partial_\mu \partial_\nu \beta) = 0. \tag{29}$$

Noting, as it was expected, that the lower-order terms in the gauge parameter β have not changed, we come to the conclusion that the conditions (9), (11) and (14) will not be altered. The term with third order derivatives, on the other hand, tells us that

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu A_\lambda)}(\partial_\lambda \partial_\mu \partial_\nu \beta) = 0. \tag{30}$$

A possible solution to (27), is

$$\left(\partial^\lambda F^{\mu\nu}\right) G_{\lambda\mu\nu},$$

where the quantity G cannot be symmetric with respect to all its indices due to the Lorentz invariance. Actually, $G_{\mu\nu\lambda}$ must be antisymmetric in the last two indices, i.e., $G_{\mu\nu\lambda} = G_{\mu[\nu\lambda]}$ so that we may identify $\frac{\partial \mathcal{L}}{\partial(\partial_\lambda \partial_{[\mu} A_{\nu]})}$ with $G_{\lambda[\mu\nu]}$. As a consequence,

$$\frac{\partial \mathcal{L}}{\partial\left(\partial_\lambda \partial_{[\mu} A_{\nu]}\right)} = G_{\lambda[\mu\nu]}, \tag{31}$$

where

$$G_{\lambda[\mu\nu]} = G_{\lambda\mu\nu} - G_{\lambda\nu\mu}.$$

Therefore, the corresponding Lagrangian must have the general form

$$\mathcal{L} = aF^{\mu\nu}F_{\mu\nu} + b\left(\partial^\lambda F^{\mu\nu}\right) G_{\lambda[\mu\nu]} + c\, j^\mu A_\mu. \tag{32}$$

The functional above is a function of A_μ, $\partial_\mu A_\nu$ and $\partial_\mu \partial_\nu A_\lambda$. Now, since the second derivatives, $\partial_\mu \partial_\nu$, commute, the antisymmetric part of $G_{\lambda[\mu\nu]}$ must be constructed with first derivatives of the field A_μ only. Since the term $\left(\partial^\lambda F^{\mu\nu}\right) G_{\lambda[\mu\nu]}$ must be quadratic in the gauge field, the remaining index of $G_{\lambda[\mu\nu]}$ will be identified with the first derivative of the antisymmetric part of the aforementioned tensor. This means that the quantity $G_{\lambda[\mu\nu]}$ is nothing but the derivative of the usual field-strength tensor. Hence, the Lagrangian is given by

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{b^2}{4}\left(\partial^\lambda F^{\mu\nu}\right)\partial_\lambda F_{\mu\nu} + j^\mu A_\mu, \tag{33}$$

where judicious values for the arbitrary constants were chosen. The above Lagrangian is known as the Podolsky Lagrangian. Here b is a constant with dimension of $(mass)^{-1}$.

Now, in order not to conflict with well-established results of QED, the parameter b must be very small, which implies that the massive photon, unlike what is claimed in the literature, is a heavy photon. Indeed, recently, Accioly and Scatena [12] found that its mass is $\sim 42 GeV$, which is of the same order of magnitude as the mass of the $W\,(Z)$ boson [13]. This is an interesting coincidence.

To conclude, we call attention to the fact that Podolsky theory plays a fundamental role in the discussion about the issue of the compatibility between magnetic monopoles and massive photons.

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[1] L. D. Landau, E. M. Lifshits, The classical theory of fields. Cambridge, Mass.: Addison-Wesley, 1951;
J. D. Jackson, Classical Electrodynamics (Wiley, New York, 1999), 3rd. ed.

[2] K. Hagiawara *et al.*, Phys. Rev. D **66**, 010001 (2002).

[3] L. C. Tu, J. Luo and G. T. Gillies, Rep. Prog. Phys. **68** (2005) 77.

[4] A. Proca, Compt. Rend. **190** (1930) 1377.

[5] Theory of elementary particles, T.P. Cheng and L.F. Lee, Oxford University Press, 1982.

[6] S. Chern and J. Simons, Annals. Math. **99** (1974) 48;
S. Carrol, G. Field, and R. Jackiw, Phys. Rev. D **41**, 1231 (1990).

[7] B. Podolsky, Phys. Rev. **62** (1942) 66;
B. Podolsky, C. Kikuchi, Phys. Rev. **65** (1944) 228;
B. Podolsky, P. Schwed, Rev. Mod. Phys. **20** (1948) 40.

[8] A. Yu. Ignatiev and G. C. Joshi, Phys. Rev. D **53** (1996) 984.

[9] A. Accioly and H. Mukay, Braz. J. Phys. **28** (1998) 35.

[10] H. Torres-Silva, Rev. chil. ing. **16** (2008) 65.

[11] M.Henneaux, V.E.R. Lemes, C.A.G. Sasaki, S.P. Sorella, O.S. Ventura and L.C.Q. Vilar, Phys. Lett. **B 410** (1997) 195.

[12] A. Accioly and E. Scatema, Mod. Phys. Lett. **A 25** (2010) 1115.

[13] C. Amsler *et al.* (PDG), Phys. Lett. **B 667** (2008)1.