



A mathematical model to optimize the volumetric capacity of trucks utilized in the transport of food products

Modelo matemático para otimização da capacidade volumétrica de caminhões para transporte de produtos alimentícios

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Abstract: Road transport accounts for 61% of the cargo transport in Brazil. Researches that focus on the optimization of truck capacity are important because they may help reduce the number of trucks on the roads, improving flow and safety. The objective of this paper is to conduct a logistical study using a mathematical model based on the Three-dimensional Bin Packing Problem to optimize the arrangement of boxes inside the vehicles, leading to a better use of the vehicle's volumetric capacity to transport food. The proposed model aims to make better use of the vehicle's volumetric capacity, considering a box arrangement in three dimensions, support area, delivery sequence of boxes to avoid blocking, rotation of boxes in the X-Y plane, and rectangular boxes. The solution achieved by the model showed significant gains compared with the manual planning done by the company, reducing the number of vehicles and creating economic gains.

Keywords: Three-dimensional Bin Packing Problem; Food product logistics; Cargo transport.

Resumo: Estudos que visem otimizar a utilização da capacidade dos caminhões são importantes no Brasil, país em que o transporte rodoviário representa 61,1% da movimentação de cargas, pois contribuem para a redução da quantidade de caminhões nas estradas, melhorando a fluidez e a segurança. O objetivo deste trabalho é promover um estudo logístico por meio de um modelo matemático baseado no Three Dimensional Bin Packing Problem para arrumação da carga dentro dos veículos utilizados no transporte de produtos alimentícios de forma a otimizar a sua capacidade volumétrica. O modelo proposto visa um melhor aproveitamento da capacidade volumétrica dos veículos considerando a disposição em três dimensões das cargas, área de suporte, sequência de entrega das cargas, para evitar bloqueios, rotação das caixas no plano X-Y e caixas de geometria paralelepípedica. A solução alcançada pelo modelo apresentou ganhos importantes em relação ao planejamento manual realizado pela empresa, reduzindo a utilização de caminhões e gerando ganhos econômicos.

Palavras-chave: Three Dimensional Bin Packing Problem; Logística de produtos alimentícios; Transporte de carga.

1 Introduction

Road transport is very important to the Brazilian economy and it is the most important way to transport goods in Brazil. It represents 61.1% of all cargo transported in 2013 (CNT, 2013). Researches that aims to optimize the vehicles' capacity utilization in order to reduce the number of trucks on the roads are important because they tend to contribute to a reduction in accidents and to decrease road wear.

This paper analyzes the distribution of products manufactured by a Brazilian company of chocolates

that is settled in the State of Espírito Santo since 1929 and sells its products to all the States of Brazil. Its logistics is based in large shipments for its own distribution centers (CDs) or for third parties' CDs. The trips to the CDs happen along the main highways of Brazil. Then, there is no need making routes for the trucks, since, in most cases, the route is unique. The company itself is responsible for choosing the truck and also planning the truck's routes that are used to carry their products. However, the company

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outsources all trucks that are used in the transport, and the carrier gets paid by the number of trucks used for each destiny. Data from the year of 2013 shows that an average of 700 trips per month was made and about 80% of the trucks used were refrigerated trailers.

The load of chocolates is packed in boxes that allow stacking up to the limit of the vehicle load compartment height. Since the vehicle load compartment is all closed, the boxes are supported by the compartment side, offering no risk to the load stability. For the long-distance travel considered in this paper, the boxes of one product sold to one client are all grouped together and this grouping is called picking. In this paper, the pickings will simply be called boxes. The boxes in the vehicle cargo compartment can be rotated by 90° in the X-Y plane parallel to the floor. It is not allowed to rotate on the Z axes, leaving the box upside down, even if it is supported on one of its sides, because its structure could not support the weight and it provides a risk for cargo damage.

Since the freight cost payment is made per trip, i.e., by truck used, planning tools that allow a more effective analysis of how to arrange the load inside the vehicle in order to make better use of the available space are very important. With these tools, it is expected to reach a reduction in the number of trucks and the quantity of trips required in these logistics operations that will, probably, lead to a logistic cost reduction. The studied company does not have a computational tool to optimize the volumetric capacity of the trucks. Currently, the vehicle occupancy decision is performed manually by company analysts based on their practical experience.

To plan the cargo stowage in three dimensions, it was proposed in the scientific literature the following related problems: Three Dimensional Container Loading Problem (3D-CLP) (Wäscher et al., 2007), Three Dimensional Bin Packing Problem (3D-BPP) (Wäscher et al., 2007) Three-dimensional Loading Capacitated Vehicle Routing Problem (3L-CVRP) (Junqueira et al., 2013).

The objective of this paper is to make a logistical study to maximize the use of the volumetric capacity of the trucks used to deliver food products. For this, a mathematical model based on the 3D-BPP problem is proposed to optimize the arrangement of the cargo inside the trucks. The proposed model considers the arrangement in three dimensions of the loads, support area, delivery sequence and rotation of the boxes in the X-Y plane for loading rectangular boxes, searching for a better use of the volumetric capacity of the trucks.

This paper is organized as follows: in Section 2, the loading problem in three dimensions in vehicles is explained and it is also done a review of papers that deal with 3D-CLP, 3D-BPP and 3L-CVRP; Section 3 presents the proposed mathematical model; Section 4

presents the results and an analysis; and Section 5 presents the conclusions.

2 Three dimensional loading problem

It was proposed in the literature three classes of problems to deal with the planning of the loading problem in three dimensions: Three Dimensional Bin Packing Problem (3D-BPP) (Wäscher et al., 2007), Three Dimensional Container Loading Problem (3D-CLP) (Wäscher et al., 2007) and Three-dimensional Loading Capacitated Vehicle Routing Problem (3L-CVRP) (Junqueira et al., 2013). The 3D-BPP considers that the items to be stowed in the container are rectangular boxes that must be set at right angles in a minimum number of identical size rectangular containers (Wäscher et al., 2007).

The 3D-CLP considers that the items to be stowed are weakly heterogeneous boxes that must be stowed in a given container so the unused space of the container must be minimized (Wäscher et al., 2007). The 3L-CVRP comes to integrate the Vehicle Routing Problem (VRP) and the 3D-BPP problem with the objective of optimizing the routes to meet the demand by bringing together in a single solution capacity constraints and restrictions of shipment and transport of cargo. Typically, in the 3L-CVRP, routing is seen as the starting point. After the route is defined, then the loading organization is analyzed (Junqueira et al., 2013). Constraints usually found in 3L-CVRP are: loading only rectangular boxes; multiple containers; multiple sizes of boxes; volumetric capacity of the vehicle; boxes must be positioned in a specific place; boxes from one client must be included in one specific vehicle; the box must be stowed orthogonal to the sides of the vehicle; the boxes can or cannot be rotated or the boxes can be rotated only in 90°; fragile boxes cannot be stacked under no fragile boxes; a minimum support area must be respected; and LIFO (Last in First out) sequence policy also called sequential loading (Junqueira et al., 2013). However, in practical problems, not all constraints must be placed on the problem.

George & Robinson (1980) presented a heuristic called Wall Building to the 3D-BPP. Chen et al. (1995) proposed a mathematical model to solve the stowage of rectangular boxes with different dimensions in containers. The model's objective is to reduce the empty spaces inside the container, providing a better use of available space and providing an efficient organization of the load. Although Chen et al. (1995) paper title refers to the CLP, the authors, in fact, propose a 3D-BPP model. The proposed model was the first which admitted orthogonal rotations.

Martello et al. (2000) developed a Branch-and-Bound algorithm for the 3D-orthogonal BPP. Lodi et al. (2002)

proposed a Tabu Search metaheuristics for 3D-BPP, which makes the packaging in layers. Pisinger (2002) refined the approach Wall Building, proposing a new heuristic with the objective of dividing the problem into smaller sub-problems. Faroe et al. (2003) presented a Guided Local Search metaheuristic. Den Boef et al. (2005) made a characterization of the algorithm proposed by Martello et al. (2000), showing that such an algorithm is not able to generate all the orthogonal packing. Packings, however, have the property of being a robot packaging, i.e., packaging is obtained by placing the items starting from the lower left corner, and in such a way that each item is in front of, to the right, or above each item placed previously. Zhang et al. (2007) proposed a heuristic combining Simulated Annealing with Personification that was used to improve the result of the Personification heuristic.

Martello et al. (2007) presented an extension of the heuristic proposed by Martello et al. (2000) through the insertion of a new packaging procedure for a single bin. Crainic et al. (2008) used heuristics based on the concept of extreme point to 3D-BPP. Wang et al. (2008) proposed a heuristic which uses a decomposition method of dynamic space. Peng et al. (2009) proposed a hybrid Simulated Annealing to solve 3D-CLP. Crainic et al. (2009) used a Tabu Search of two levels to solve 3D-BPP: the first level aims to reduce the number of bins and the second level aims to optimize the packaging of the bins. Wu et al. (2010) adapted the model of Chen et al. (1995) for the problem of loading a single container with variable height and proposed a solution for 3D-BPP based on Genetic Algorithms. Ceschia & Schaerf (2013) proposed a Local Search metaheuristic to solve the variant of 3D-PLC that includes delivery of boxes to multiple destinations (multi-drop). Bortfeldt and Wäscher (2013) did a review of 3D-CLP and characterized the various constraints used in previous studies of container loading problems.

About the 3L-CVRP, Gendreau et al. (2006) proposed a Tabu Search metaheuristic. Tarantilis et al. (2009) used a Guided Tabu Search (GTS) metaheuristic. Fuellerer et al. (2010) proposed an Ant Colony Optimization (ACO) metaheuristic. Tao and Wang (2010) presented a Tabu Search that calls a heuristic to solve the subproblem of packing. Araujo et al. (2011) proposed an approach that combines the use of constructive heuristics for loading configuration and Tabu Search for elaboration of the routes. Bortfeldt (2012) proposed a hybrid approach that combines Tabu Search, to solve the routing path and a tree search algorithm for the loading problem. Zhu et al. (2012) proposed a two-stage Tabu Search, which incorporate two packaging heuristics. Ruan et al. (2013) proposed an approach that combines Honey Bee Mating Optimization (HMMO) to treat the routing

problem and six loading heuristics. Junqueira et al. (2013) presented a Mixed Integer Linear Programming (MILP) mathematical model for the 3L-CVRP, aiming to find lower cost delivery routes. Iori & Martello (2010, 2013) did a review of the literature on integrated Routing and loading problems. Koloch & Kaminski (2010) proposed two approaches to solve the 3L-CVRP called nested and a joint approach based on a Memetic Genetic Algorithm for the routing problem and a Wall Building heuristic for the packing problem.

About the variants of the 3L-CVRP, Attanasio et al. (2007) proposed a MILP model for the periodic 3L-CVRP. Moura (2008) solved the 3L-CVRP with time window using Genetic Algorithm with multiobjective function which deals with the number of vehicles, the total distance traveled and the used volume. Moura & Oliveira (2009) also addressed the 3L-CVRP with time window, proposing constructive heuristics. Bortfeldt & Homberger (2013) proposed a two-stage heuristic for the problem proposed by Moura & Oliveira (2009). The first step optimizes the packaging, while the second deals with aspects of routing. Junqueira et al. (2011) presented a MILP model to deal with a problem similar to the one studied in this paper. They seek to organize loads with multiple destinations within the cargo compartment of a truck, respecting the delivery sequence to avoid blockages of the loads to be unloaded.

3 Proposed mathematical model

The proposed mathematical model in this paper is based on the model of Chen et al. (1995) for the 3D-BPP. However, the proposed model differs from the model of Chen et al. (1995) in several points. First, the objective function has one extra part that represents the sum of the coordinates of the boxes over the vehicle sides. The intention is to minimize the unused space in such a way that the final arrangement is more compact, seeking for a better use of the space in the cargo compartment, providing greater security to the load and preventing boxes to be left without any support.

Another problem found in Chen et al. (1995) can be seen in the arrangement of Figure 1 for a scenario with six boxes where two boxes have no support area. This support area could be the floor of the cargo compartment or the top face of another box. Also the arrangement shown in Figure 1 does not take into consideration the order of unloading the vehicle, which, in practice, leads to a mandatory relocation of boxes each time a new client is visited. For the representation of Figure 1 and the analysis of the model's results, it was defined that the boxes of the same color belong to the same customer and the unloading sequence goes from the clearer boxes, the white ones, to the darker boxes, the black ones.

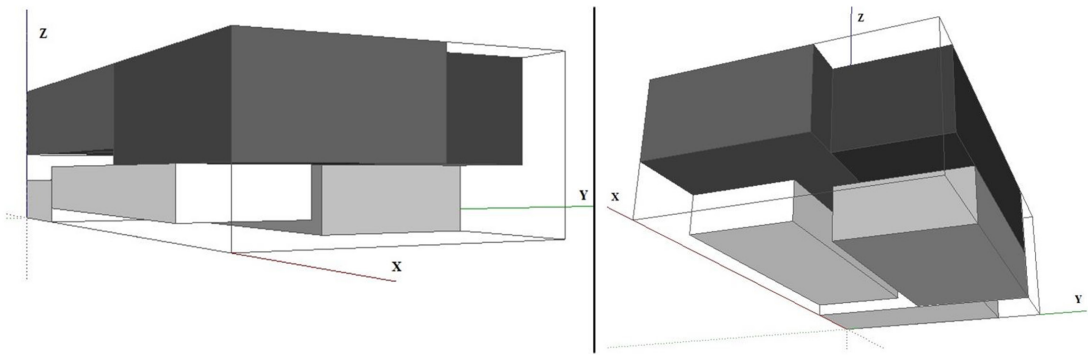


Figure 1. Solution from original model solution of Chen et al. (1995).

In this scenario, the darker (black) boxes blocks the removal of the white and grey boxes, the firsts to be unloaded.

The proposed model solves problems with a heterogeneous fleet; in other words, the fleet can be composed of trucks that have different dimensions of the cargo compartment, length, width and height. The model also solves problems when the boxes that will be stowed in the truck have different dimensions, length, width and height. The model assumes that the dimensions of the vehicle, the dimensions of the boxes to be transported and the sequence of the delivery points of the route are all known.

In addition, the proposed model satisfies the following constraints: 1) the volumetric capacity

of each vehicle must not be exceeded; 2) It is not allowed to overlap boxes; 3) the position of the boxes must always be orthogonal to the sides of the compartment; 4) LIFO policy must be respected; and 5) load stability, guaranteed by the support area during transport, must be ensured. The constraints mentioned above make the model closer to the reality of the analyzed company and also makes the model applicable to a wide range of transport companies that transport rectangular boxes. In addition, the model allows boxes to be rotated 90° in X-Y axis. The sets, parameters, decision variables, objective function and constraints of the proposed model are listed next.

Sets

- C Set of boxes i , ranging from 1 a nm ;
 D Set of vehicles j , ranging from 1 a nm ;

Parameters

- nm Number of boxes to be loaded;
 nm Number of available vehicles;
 M A big number used to the model's logic;
 m A small number used to the model's logic;
 as Support area to be considered in the model, can assume values between 0 and 1, where 0 provides 0% of support area and grows gradually to 1 that guarantees 100% of support area for the box;
 o_i Indicates the delivery sequence of box i . The box with the lowest value will be visited first and the box with the highest value will be the last to be visited;
 p_i, q_i, r_i Length, width and height of box i ;
 L_j, W_j, H_j Length, width and height of the compartment of vehicle j ;
 φ_j Cost of each m^3 of the vehicle's j compartment;
 ω Weight factor for the second part of the objective function.

Decision Variables

x_i, y_i, z_i	Coordinates of the lower left front corner of box i ;
lx_i, ly_i, lz_i	Define if the length of box i is parallel to the X, Y or Z axis. For example, lx_i is equal to 1 if the length of box i is parallel to X axis, otherwise, lx_i is equal to 0;
wx_i, wy_i, wz_i	Define if the width of box i is parallel to the X, Y or Z axis. For example, wx_i is equal to 1 if the width of box i is parallel to X axis, otherwise, wx_i is equal to 0;
hx_i, hy_i, hz_i	Define if the height of box i is parallel to the X, Y or Z axis. For example, hx_i is equal to 1 if the height of box i is parallel to X axis, otherwise, hx_i is equal to 0;
s_{ij}	Binary variable which indicates if box i is placed in the vehicle j . It is equal to 1 if box i has been placed in the vehicle j and 0 otherwise;
n_j	Binary variable which indicates if vehicle j was used. It is equal to 1 if vehicle j was used and 0 otherwise;
δ_{ikj}	Binary variable which indicates if box i and box k are placed in the vehicle j . It is equal to 0 if box i and box k are placed in the vehicle j and 1 otherwise;
$a_{ik}, b_{ik}, c_{ik}, d_{ik}, e_{ik}, f_{ik}$	Binary variables that indicate the relative position between two boxes. Variable a_{ik} is equal to 1 if the box i is to the left of box k . Similarly, variables $b_{ik}, c_{ik}, d_{ik}, e_{ik}, f_{ik}$ indicate whether the box i is to the right, behind, ahead, below or above box k , respectively. These variables are necessary only when $i \neq k$.

Objective Function

$$\text{Minimizar} \left(\left(\sum_{j \in D} \varphi_j (L_j W_j H_j n_j - \sum_{i \in C} p_i q_i r_i s_{ij}) \right) + \omega \left(\sum_{i \in C} z_i + \sum_{i \in C} x_i + \sum_{i \in C} y_i \right) \right) \tag{1}$$

Constraints

$$x_i + p_i lx_i + q_i (lz_i - wy_i + hz_i) + r_i (1 - lx_i - lz_i + wy_i - hz_i) - \delta_{ikj} M \leq x_k + (1 - a_{ik}) M \quad \forall i, k \in C : i \neq k \tag{2}$$

$$x_k + p_k lx_k + q_k (lz_k - wy_k + hz_k) + r_k (1 - lx_k - lz_k + wy_k - hz_k) - \delta_{ikj} M \leq x_i + (1 - b_{ik}) M \quad \forall i, k \in C : i \neq k \tag{3}$$

$$y_i + q_i wy_i + p_i (1 - lx_i - lz_i) + r_i (lx_i + lz_i - wy_i) - \delta_{ikj} M \leq y_k + (1 - c_{ik}) M \quad \forall i, k \in C : i \neq k \tag{4}$$

$$y_k + q_k wy_k + p_k (1 - lx_k - lz_k) + r_k (lx_k + lz_k - wy_k) - \delta_{ikj} M \leq y_i + (1 - d_{ik}) M \quad \forall i, k \in C : i \neq k \tag{5}$$

$$z_i + r_i hz_i + q_i (1 - lz_i - hz_i) + p_i lz_i - \delta_{ikj} M \leq z_k + (1 - e_{ik}) M \quad \forall i, k \in C : i \neq k \tag{6}$$

$$z_k + r_k hz_k + q_k (1 - lz_k - hz_k) + p_k lz_k - \delta_{ikj} M \leq z_i + (1 - f_{ik}) M \quad \forall i, k \in C : i \neq k \tag{7}$$

$$a_{ik} + b_{ik} + c_{ik} + d_{ik} + e_{ik} + f_{ik} \geq 1 - \delta_{ikj} \quad \forall i, k \in C : i \neq k; j \in D \tag{8}$$

$$\sum_{j \in D} s_{ij} = 1 \quad \forall i \in C \tag{9}$$

$$\sum_{i \in C} s_{ij} \leq M n_j \quad \forall j \in D \tag{10}$$

$$x_i + p_i lx_i + q_i (lz_i - wy_i + hz_i) + r_i (1 - lx_i - lz_i + wy_i - hz_i) \leq L_j + (1 - s_{ij}) M \quad \forall i \in C, j \in D \tag{11}$$

$$y_i + q_i wy_i + p_i (1 - lx_i - lz_i) + r_i (lx_i + lz_i - wy_i) \leq W_j + (1 - s_{ij}) M \quad \forall i \in C, j \in D \tag{12}$$

$$z_i + r_i h z_i + q_i (1 - l z_i - h z_i) + p_i l z_i \leq H_j + (1 - s_{ij}) M \quad \forall i \in C, j \in D \quad (13)$$

$$l x_i + l y_i + l z_i = 1 \quad \forall i \in C \quad (14)$$

$$w x_i + w y_i + w z_i = 1 \quad \forall i \in C \quad (15)$$

$$h x_i + h y_i + h z_i = 1 \quad \forall i \in C \quad (16)$$

$$l x_i + w x_i + h x_i = 1 \quad \forall i \in C \quad (17)$$

$$l y_i + w y_i + h y_i = 1 \quad \forall i \in C \quad (18)$$

$$l z_i + w z_i + h z_i = 1 \quad \forall i \in C \quad (19)$$

$$2 - s_{ij} - s_{kj} \geq m \delta_{ikj} \quad \forall i, k \in C, j \in D \quad (20)$$

$$2 - s_{ij} - s_{kj} \leq M \delta_{ikj} \quad \forall i, k \in C, j \in D \quad (21)$$

$$x_i, y_i, z_i \geq 0 \quad \forall i \in C \quad (22)$$

$$l x_i, l y_i, l z_i, w x_i, w y_i, w z_i, h x_i, h y_i, h z_i, n_i \in \{0, 1\} \quad \forall i \in C \quad (23)$$

$$s_{ij} \in \{0, 1\} \quad \forall i \in C, j \in D \quad (24)$$

$$a_{ik}, b_{ik}, c_{ik}, d_{ik}, e_{ik}, f_{ik} \in \{0, 1\} \quad \forall i, k \in C \quad (25)$$

$$\delta_{ikj} \in \{0, 1\} \quad \forall i, k \in C, j \in D \quad (26)$$

The objective function, Equation 1, is divided in two parts. The first part aims to reduce the unoccupied volume in all vehicle compartments. The unoccupied volume of the vehicle compartment is calculated as the total volume of the vehicle compartment minus the total volume of the boxes placed in the compartment. The unoccupied volume of the compartment is multiplied by the cost of each m³ of the compartment of vehicle j . Then, this parameter is calculated in preprocessing as $\varphi_j = \beta_j / \gamma_j$. Where β_j is the cost of the total volume of the compartment of the vehicle and γ_j is the volumetric capacity of the vehicle's compartment.

The second part aims to minimize the sum of the coordinates x , y and z , seeking to reach a more compact arrangement of the boxes. Moreover, this part avoids stowing boxes without touching the compartment floor or without touching the top face of other boxes below and by its side. This part of the objective function makes the boxes arrangement more realistic than the one made by the model of Chen et al. (1995) that leaves boxes without any support beneath them. This second part of the objective function is multiplied by the parameter ω which is a

small amount to give less weight to this part in the objective function. It was adopted $\omega = 0.001$.

Constraints (2) to (7) ensure that if two boxes are inside the same vehicle, box i cannot overlap box k in any side, or above, or below it. Constraints (8) ensure that if boxes i and box k are in the same vehicle, they must have at least one relative position between each other. Thus, one or more variables $b_{ik}, c_{ik}, d_{ik}, e_{ik}, f_{ik}$ must be equal or greater than zero. The left side of Constraints (8) is equal or greater than 1 if the two boxes are in the same vehicle and 0 otherwise.

Constraints (9) ensure that each box must be placed inside only one vehicle. Constraints (10) ensure that if any box is placed in a vehicle, then this vehicle is considered used. Constraints (11) to (13) ensure that all boxes placed in a vehicle can fit in the physical dimensions of the vehicle. Constraints (14) to (19) ensure that the dimensions of a box (length, width and height) must be parallel to one side of the vehicle compartment. Constraints (20) and (21) together define that variable δ_{ikj} assumes value 0 if the boxes i and k are inside the same vehicle j , and 1 otherwise. Constraints (22) define that all boxes coordinates must be equal or greater than zero. Constraints (23) to (26) define the binary variables of the model.

Constraints (27) ensure that each box has its sides parallel to the sides of the vehicle compartment. Constraints (28) to (30) ensure the reciprocal positioning of boxes, i.e., if box i is to the left of the

box j , necessarily, box j must be to the right of box i . Constraints (31) are inserted to ensure that all boxes will be rotated only in the X-Y plane and then all the boxes will have their heights parallel to the axis Z.

$$lx_i + ly_i + lz_i + wx_i + wy_i + wz_i + hx_i + hy_i + hz_i = 3 \quad \forall i \in C \tag{27}$$

$$a_{ik} = b_{ki} \quad \forall i, k \in C : i \neq k \tag{28}$$

$$c_{ik} = d_{ki} \quad \forall i, k \in C : i \neq k \tag{29}$$

$$e_{ik} = f_{ki} \quad \forall i, k \in C : i \neq k \tag{30}$$

$$h_{zi} = 1 \quad \forall i \in C \tag{31}$$

It may be noted from Figure 2 that the solution of the mathematical model reached after the insertion of the Constraints (27) to (31) was satisfactory, but it still does not fully respect the support area issues.

bottom is smaller than the area of the box at the top, thereby the box on top is in an unstable situation due to lack of support area. Moreover, it is not yet respected the LIFO policy. Constraints (32) to (35) were proposed to ensure the LIFO policy and, thus, avoid blockages.

Although in Figure 2 all the boxes are placed on top of another box, the area of the box at the

$$x_i + (1 - c_{ik})M \geq x_k \quad \forall i, k \in C : i \neq k; o_i < o_k \tag{32}$$

$$x_i + p_i lx_i + q_i (1 - lx_i) + (1 - c_{ik})M \geq x_k + p_k lx_k + q_k (1 - lx_k) \quad \forall i, k \in C : i \neq k; ; o_i < o_k \tag{33}$$

$$z_i + (1 - e_{ik})M \geq z_k \quad \forall i, k \in C : i \neq k ; o_i < o_k \tag{34}$$

$$z_i + r_i + (1 - e_{ik})M \geq z_k + r_k \quad \forall i, k \in C : i \neq k; o_i < o_k \tag{35}$$

Constraints (32) and (33) ensure that the vehicle unloading operation, along the X axis, must respect the LIFO policy, which means that the last boxes loaded on the vehicle will be the first to be unloaded. This ensures that customers are served according to

the established sequence on the route without the need to shift boxes to reach the box to be unloaded in every delivery. Similarly, Constraints (34) and (35) ensure the LIFO order of unloading for the Z axis. This prevents other boxes, that are going to be

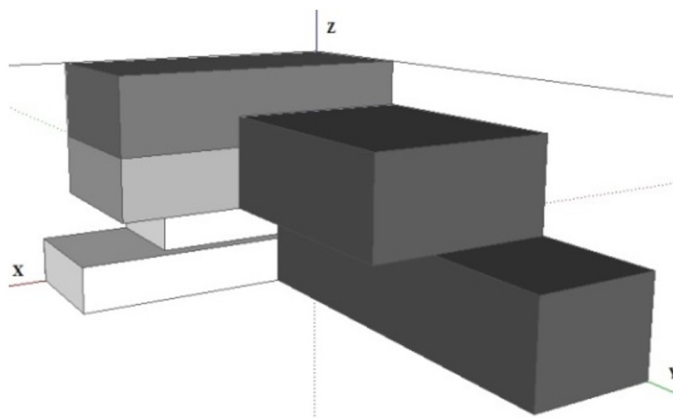


Figure 2. Solution without the restrictions of LIFO and Support Area.

delivered later, be placed on top of the ones that are going to be delivered first. With the inclusion of these constraints, the mathematical model ensures the LIFO policy, as shown in Figure 3.

However, up to this point, the model does not ensure the support area for the boxes. Then, Constraints (36) to (39) are added to the mathematical model to ensure that the boxes will have a support area.

$$x_i + p_i l x_i + q_i (1 - l x_i) + (1 - e_{ik}) M \geq x_k \quad \forall i, k \in C : i \neq k \tag{36}$$

$$x_i + p_i l x_i + q_i (1 - l x_i) - x_k + (1 - e_{ik}) M \geq (p_k l x_k + q_k (1 - l x_k)) as \quad \forall i, k \in C : i \neq k \tag{37}$$

$$y_i + p_i l y_i + q_i (1 - l y_i) + (1 - e_{ik}) M \geq y_k \quad \forall i, k \in C : i \neq k \tag{38}$$

$$y_i + p_i l y_i + q_i (1 - l y_i) - y_k + (1 - e_{ik}) M \geq (p_k l y_k + q_k (1 - l y_k)) as \quad \forall i, k \in C : i \neq k \tag{39}$$

Constraints (36) to (39) guarantee the existence of the support area so that the boxes are supported over each other safely. Variable e_{ik} is equal to 1 when box i is below box k and then constraints (36) to (39) are activated. A separate check was made on the X axis, constraints (36) and (37), and the Y axis, constraints (38) and (39), to ensure that the box on top has a corresponding acceptable percentage of the support area. This percentage is established by the company and defined in the model as the parameter as .

Figure 4 is used to explain Constraints (36) and (37). There is a box i with length p_i parallel to the X axis and so $l x_i = 1 e l y_i = 1$. There is also a box k with length p_k not parallel to the X axis and so $l x_k = 0 e l y_k = 1$.

Based on Figure 4 and Constraints (36), it is known that box i has its length parallel to the X axis and then $l x_i = 1$. So the term $q_i (1 - l x_i)$ is equal to 0 and the term $p_i l x_i$ is equal to p_i . Thus, it will be taken that $x_i + p_i \geq x_k$, i.e., the start position of the box k in the X axis is in between the initial position of box i and the end position of box i , $(x_i + p_i)$. Thus, the reduction of the Constraints (36) will lead to $x_i + p_i \geq x_k$, i.e., the start position of the box k in the X axis is in between the initial and the final position of box i , $(x_i + p_i)$. If box

i is not parallel to the X axis, therefore, parallel to the Y axis, then $l x_i = 0$ making the term $q_i (1 - l x_i) = q_i$ and the term $p_i l x_i = 0$. Thus, the reduction of the Constraints (36) will lead to $x_i + q_i \geq x_k$, i.e., the start position of the box k in the X axis is in between the initial and the final position of box i . In this case, it was considered the width of the box i because it is the dimension that is parallel to the X axis, $(x_i + q_i)$.

Based on Figure 4 and Constraints (37), box i has the length parallel to the X axis, then $l x_i = 1$ that leads to $q_i (1 - l x_i) = 0$ and $p_i l x_i = p_i$. Box k is parallel with its width to the X axis, then $l x_k = 0$ that leads to $q_k (1 - l x_k) = q_k$ and $p_k l x_k = 0$. Thus, the reduction of Constraints (37) will lead to $x_i + p_i - x_k \geq q_k as$. So, it follows that the width of the box k that is supported over the box i is calculated as the initial position of the box i plus its length, $(x_i + p_i)$, minus the initial position of the box k , x_k . The width that is supported must be greater or equal to box's k width multiplied by the parameter as that represents the percentage of support area defined by the company to give stability to the box.

Another situation that can occur is box k be placed with its length parallel to the X axis and so $l x_k = 1$ leading to $q_k (1 - l x_k) = 0$ and $p_k l x_k = p_k$. Thus, the reduction of

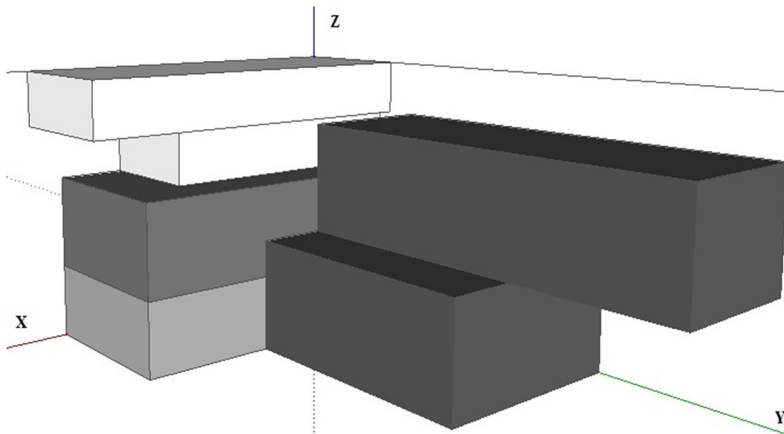


Figure 3. Solution with LIFO policy.

Constraints (37) will lead to $x_i + p_i - x_k \geq p_k as$. So, it follows that the length of box k that is supported over the box i is calculated as the initial position of the box i plus its length, $(x_i + p_i)$, minus the initial position of box k . The length that is supported must be greater or equal to box k multiplied by the parameter as . Constraints (38) and (39) are explained using the same logic used to explain the constraints (36) and (37). Constraints (38) and (39) consider the Y axis.

Thus, the model features a more complete mathematical formulation, guaranteeing a delivery sequence in which there is no need to organize the boxes during the route and also guaranteeing a minimum support area for the boxes. This can be seen comparing the 3D arrangement of Figure 5 with the 3D arrangement of Figure 3.

As mentioned before, the white boxes must be removed first, then the grey ones and, in the end, the black ones. Figure 4b shows that the arrangement of the boxes obtained by the model respects this unloading sequence and also all boxes have the needed support area. After complete description of the model, in the next section will present the results achieved and the analyzes.

4 Results and analysis

It was used the solver CPLEX 12.6 to test the proposed mathematical model presented in Section 3. CPLEX was run in a computer with an Intel i7 processor with 8 GB of RAM. The real world scenarios definition to be tested was based on real data from the trips that left the Brazilian State of Espírito Santo

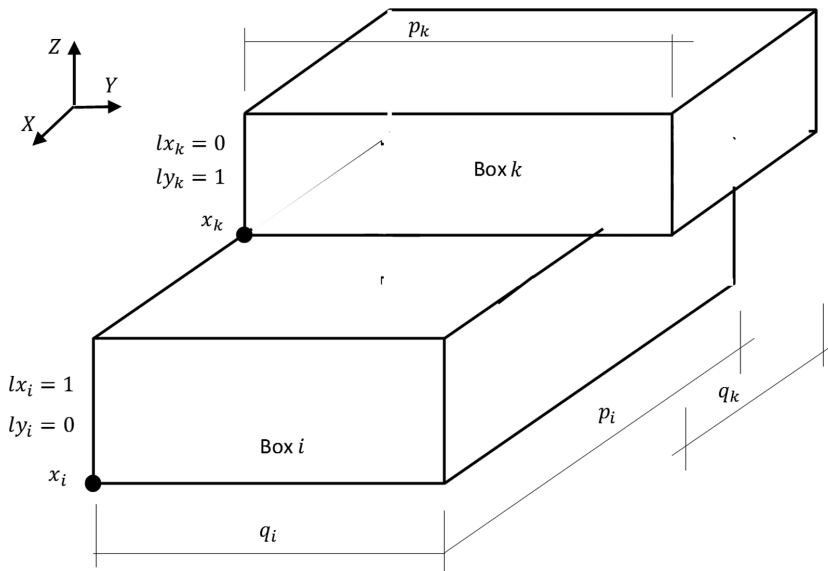


Figure 4. Schematic drawing of two boxes inside a vehicle.

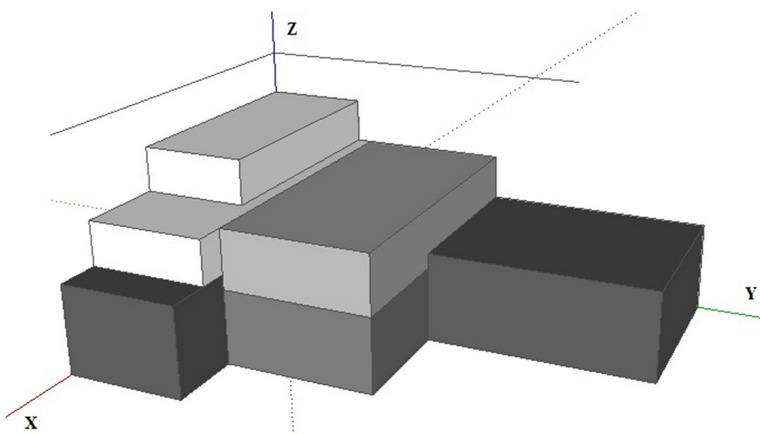


Figure 5. Solution with LIFO and Support Area.

during the period of January to September 2013. During this period, the company made 5,578 trips and this paper analyzed 3,613 of these trips in which was transported non-palletized boxes. Within this universe, 2,995 trips are for transport of seasonal products. An analysis was made to identify how many destinations each trip had. It was detected that 169 trips were made to more than one destination, i.e., for more than one city. From these 169 trips, it was identified 67 different destinations.

After that, it was applied a percentage of 12% over the total number of destinations identified to set the number of scenarios to be evaluated. Thus, it was determined eight real-world scenarios to be tested. Among the destinations identified, it was noticed that on a trip there were deliveries, in an average, for two to four cities. As there are different destinations in each trip, it is necessary to stow the cargo in such a way that the first boxes to be delivered on the trip must be stowed closer to the door so that they can be removed without having to move other boxes. As seen before, this constraint is known as LIFO policy.

Thus, it was decided to test the mathematical model with six scenarios with deliveries for two cities,

one scenario with deliveries for three cities and one scenario with deliveries for four cities. These tests were taken in order to verify the behavior of the mathematical model in each of these three situations. For the three situations mentioned, were chosen destinations with better historic data from the trips. It was compared the solution achieved by the manual process of the company with the solution found by the mathematical model for the eight real-world scenarios. The comparison between both was made by the space not occupied in the vehicle compartment, as this is the value that the model aims to minimize.

Table 1 presents the information from the real shipments for a period of 30 days for each selected destination. Column 1 represents the number of the scenario; columns 2, 3 and 4 represent the States, the cities and the number of clients served; columns 5 to 7 represent the travel number, the travel date and the travel type; columns 8 to 12 represent the characteristics of the vehicles that are, respectively: type, volumetric space (m³), length, width and height of the cargo compartment of the vehicle. Column 13 represents the number of boxes considered to be arranged in the vehicles, and as mentioned before, represent the grouping of

Table 1. Information from real-world scenarios.

Scenario	Destination			Trip			Vehicle				Load			
	State	City	Number of Clients	Number	Date	Type	Type	m ³	Length (m)	Width (m)	Height (m)	N° Box / Pickings	Not occupied Space (m ³)	
1	PE	Garanhuns Recife	2	1	03/sept	Transfer	Trailer	90	10	3	3	4	51	39
				2	13/sept	Transfer	Trailer	90	10	3	3	4	50	40
				3	19/sept	Transfer	Trailer	90	10	3	3	4	42	48
2	SE AL	Aracajú Maceió	2	1	03/sept	Transfer	Trailer	90	10	3	3	4	63	27
				2	13/sept	Transfer	Trailer	90	10	3	3	4	64	26
				3	19/sept	Transfer	Trailer	90	10	3	3	4	58	32
3	TO PA	Palmas Benevides	2	1	02/sept	Transfer	Trailer	90	10	3	3	4	35	55
				2	12/sept	Transfer	Trailer	90	10	3	3	4	36	54
				3	19/sept	Transfer	Trailer	90	10	3	3	4	58	32
4	PR SC	Pato Branco Chapecó	2	1	02/may	Transfer	Trailer	90	10	3	3	4	42	48
				2	17/may	Transfer	Trailer	90	10	3	3	6	44	46
5	BA	Feira de Santana Juazeiro	2	1	13/sept	Transfer	Trailer	90	10	3	3	6	51	39
				2	20/sept	Transfer	Trailer	90	10	3	3	4	50	40
6	ES	Guarapari Cachoeiro	2	1	02/aug	Sale	Truck	12	6	2	1	4	4	8
				2	18/aug	Sale	Truck	12	6	2	1	4	6	6
7	BA	Vitória da Conq. Lauro de Freitas	3	1	15/sept	Transfer/Sale	Trailer	90	10	3	3	6	57	33
				2	20/sept	Transfer/Sale	Trailer	90	10	3	3	6	50	40
8	PA	Benevides Castanhal Ananindeua Paragominas	4	1	05/feb	Transfer	Trailer	90	10	3	3	8	71	19
				2	06/feb	Transfer	Trailer	90	10	3	3	8	60	30
				3	28/feb	Transfer	Trailer	90	10	3	3	8	71	19

several small boxes from one client/destination, i.e., one picking. Column 14 represents the volumetric space occupied by boxes/picking for a specific trip. Finally, column 15 represents the volumetric space not occupied in the vehicle in one trip. It is observed that for each scenario, there may be more than one destination and more than one trip to fulfill the sale over the 30 days examined.

With the data of Table 1, the proposed model was run in the CPLEX using the boxes of all trips that make up each scenario. After that, the results were analyzed by: 1) checking if the LIFO policy was respected, 2) if there was enough support area for the boxes, 3) the volumetric space gains achieved by the model and 4) if all constraints were respected. Then, it was verified if the model achieved a better use of vehicles, reducing the number of trips required in the analysis period and also reducing the volumetric space not occupied.

Table 2 presents the results achieved by the model for the eight scenarios described in Table 1. Column 1 represents the number of the scenario, column 2 shows the total runtime of the CPLEX, column 3 represents the execution time that CPLEX took to find the optimal solution, column 4 represents the GAP when the CPLEX did not find an optimal solution, column 5 shows the FO value that is dimensionless, column 6 and 7 represent, respectively, the upper bound and lower bound found by CPLEX when it has not been possible to find an optimal solution and column 8 shows how many trips were necessary to carry the whole shipment to the destination and column 9 represents the number of clients served.

In Table 2, it can be seen that CPLEX found an optimal solution for Scenarios 1 to 7, GAP equal to 0.0%. CPLEX was not able to find an optimal solution for Scenario 8. Even after 4.0 hours, CPLEX found a GAP of 36.8%. Then, it was considered the upper bound as the solution for the model and it was compared it with the company's results. The objective function (FO) resulting from the model represents the sum of the costs of not occupied volume of

the vehicle compartment of all vehicles added to the coordinates of the bottom, front, left corner of all the boxes, as shown in Section 3, Equation 1. However, it was analyzed the gains in reduction of the not occupied volumetric space obtained by the model. Then, to calculate the not occupied volumetric space in the vehicle it is proposed the equation

$$\theta = \sum_{j \in D} (L_j W_j H_j n_j) - \sum_{i \in C} (p_i q_i r_i s_{ij})$$

Table 3 presents an analysis about the not occupied volume of the vehicle found by the company's manual process, by the mathematical model and also a comparison between both. Column 1 shows the number of the scenario and column 2 shows the number of each trip in each scenario. Column 3 shows the volumetric capacity of each vehicle. Columns 4 and 5 represent, respectively, the not occupied volume achieved by the company using the manual procedure and the percentage of not occupied volume over the total volume. Columns 6 and 7 represent, respectively, the not occupied volume achieved by the proposed mathematical model and the percentage of not occupied volume over the total volume. Columns 8 and 9 represent, respectively, the gain obtained by the model when compared to the company's solution.

The most significant gains were achieved in the scenarios where there was a reduction in the amount of trips required. This happened because the model was able to use better the volumetric capacity of the vehicle. This can be seen in Scenarios 1, 3 and 6. In these scenarios, the model solution made all the deliveries with one trip less than the company's solution. In Scenario 1 and 3 it was observed that two of the trips had a strong reduction in the not occupied space, reducing the number of trips and, consequently, reducing the number of necessary vehicles. This situation leads to a reduction in freight cost. In Scenario 6, the boxes, which, in practice, has been distributed between two vehicles, was allocated in a single vehicle by the model. So, again, there is

Table 2. Information about CPLEX execution.

Scenario	Total Execution Time (s)	Time to Find Optimal Solution (s)	GAP (%)	OF	UB	LB	N° Trips
1	1056.0	5.8	0.0	9222.1	-	-	2
2	99.0	9.4	0.0	19125.3	-	-	3
3	261.1	47.9	0.0	11475.4	-	-	2
4	93.0	4.1	0.0	21150.2	-	-	2
5	21.7	3.3	0.0	17775.2	-	-	2
6	52.6	1.8	0.0	450.3	-	-	1
7	953.0	8.2	0.0	16425.3	-	-	2
8	14400.0	5040.0	36.8	-	23,817.8	15,052.9	3

a reduction in terms of the number of vehicles and, consequently, a reduction in freight cost.

It can be noticed in Scenarios 2, 4, 5, 7 and 8 that the model solution is the same of the company's solution. The difference was the distribution of the boxes among the available vehicles, maintaining the same not occupied volume of the vehicles. However, the model always ensures the LIFO policy and also the minimum support area that is not always guaranteed by the current company's methodology. Nevertheless, this does not change the freight cost value, because it is paid for truck trip.

Considering that in Scenario 4 the solution found by the model presented a high percentage of not occupied volume, 66.7%, it was decided to test the scenario with a heterogeneous fleet, although this is not the reality of the company today. In this way, were created three new scenarios, Scenario 4A, 4B and 4C. In all three scenarios were added two trucks with 54.0 m³ in the fleet and was kept the two trucks with 90.0 m³ from Scenario 4. For Scenario 4A, this new truck has the freight cost, per m³, 30.0% higher than the trucks with 90.0 m³. In Scenario 4B, this truck has the freight cost, per m³, 30.0% lower than the trucks with 90.0 m³. In Scenario 4C, this truck has the same freight cost, per m³, of the trucks with 90.0 m³. The results for these three new scenarios can be seen in Table 4.

Results from Table 4 show that when a heterogeneous fleet is available, the model also decides about what type of truck that reduces the not occupied volume. So, the model may help the company to decide what type of truck is better for its operations. It is interesting to note that in Scenario 4A, where the truck with 54.0 m³ has the freight cost more expensive than the 90.0 m³, the mathematical model still chose the truck with 54.0 m³. This happened because if the model had chosen the truck with 90.0 m³, the maximum volume that could be stowed in the truck was 72.0 m³ and then the model would have to choose another truck with 54.0 m³ to carry the remaining 8.0 m³. Thus, the truck with 90.0 m³ would have a not occupied space of 18.0 m³ representing a cost of R\$ 4,590.00 and the truck with 54.0 m³ would have a not occupied space of 46.0 m³ representing a cost of R\$ 8,188.00, which gives a total cost of R\$ 12,778.0. The model chose two trucks with 54.0 m³, being loaded with 42.0 m³ and 44.0 m³, respectively, which gives a total of not occupied volume of 22.0 m³ at a total cost of R\$ 3,916.0, i.e. a much better solution. Results from Table 4 show that with heterogeneous fleet the model can also help the company choose the best fleet.

For the analysis of the financial gains of Scenarios 1, 3 and 6, it was decided to compare the cost of a trip between the company's solution and the mathematical model solution. Thus, it was

Table 3. Comparison between the company's solution and the model solution.

Scenario	Trip	Vehicle Volume (m ³)	Company		Mathematical Model		Difference Model x Company	
			Not Occupied Volume (m ³)	Not Occupied (%)	Not Occupied Volume (m ³)	Not Occupied (%)	Not Occupied Volume (m ³)	Not Occupied (%)
1	1	90.0	39.0	43.3	18.0	20.0	21.0	23.3
	2	90.0	40.0	44.4	20.0	22.2	20.0	22.2
	3	90.0	48.0	53.3	90.0	100.0	-42.0	100.0
2	1	90.0	27.0	30.0	27.0	30.0	0.0	0.0
	2	90.0	26.0	28.9	36.0	40.0	-10.0	-11.1
	3	90.0	32.0	35.6	22.0	24.4	10.0	11.1
3	1	90.0	55.0	61.1	90.0	100.0	-35.0	100.0
	2	90.0	54.0	60.0	24.0	26.7	30.0	33.3
	3	90.0	32.0	35.6	27.0	30.0	5.0	5.6
4	1	90.0	48.0	53.3	34.0	37.8	14.0	15.6
	2	90.0	46.0	51.1	60.0	66.7	-14.0	-15.6
5	1	90.0	39.0	43.3	47.0	52.2	-8.0	-8.9
	2	90.0	40.0	44.4	32.0	35.6	8.0	8.9
6	1	12.0	8.0	66.7	2.0	16.7	6.0	50.0
	2	12.0	6.0	50.0	12.0	100.0	-6.0	100.0
7	1	90.0	33.0	36.7	29.0	32.2	4.0	4.4
	2	90.0	40.0	44.4	44.0	48.9	-4.0	-4.4
8	1	90.0	19.0	21.1	22.0	24.4	-3.0	-3.3
	2	90.0	30.0	33.3	27.0	30.0	3.0	3.3
	3	90.0	19.0	21.1	19.0	21.1	0.0	0.0

Table 4. Results for heterogeneous fleet.

		Not Occupied Space Analysis - Heterogeneous Fleet								
Scenario	Trip	Company			Mathematical Model				Reduction Model x Company	
		Vehicle Volume (m ³)	Not Occupied Volume (m ³)	Not occupied (%)	Vehicle Volume (m ³)	Not Occupied Volume (m ³)	Not occupied (%)	Freight Cost (R\$)	Not Occupied Volume (m ³)	Not occupied (%)
4A	1	90.0	48.0	53.3	54.0	10.0	18.5	5,590.2	38.0	34.8
	2	90.0	46.0	51.1	54.0	12.0	22.2		34.0	28.9
4B	1	90.0	39.0	43.3	54.0	10.0	18.5	3,897.2	29.0	24.8
	2	90.0	40.0	44.4	54.0	12.0	22.2		28.0	22.2
4C	1	90.0	48.0	53.3	54.0	10.0	18.5	3,987.2	38.0	34.8
	2	90.0	46.0	51.1	54.0	12.0	22.2		34.0	28.9

Table 5. Financial gains for the company if it had implemented the model solution.

Scenario	Freight Cost per Vehicle (R\$)	Number of trips		Total Cost Freight (R\$)		Financial Gains Model x Company		
		Company	Model	Company	Model	R\$	%	
1	7,757.0	3	2	23,272.0	15,515.0	7,757.0	33	
3	11,240.0	3	2	33,721.0	22,481.0	11,240.0	33	
6	1,596.0	2	1	3,191.0	1,596.0	1,595.0	50	
Monthly reduction (R\$)							20,593.0	
Estimated annual gains (R\$)							247,116.0	

found the freight cost for each destination of each scenario and then it was multiplied by the number of trips of both solutions. These results can be seen in Table 5 and if the solutions achieved by the model were implemented, they would represent a monthly reduction of R\$ 20,593.0 and an estimated annual gain of R\$ 247,116.0. This represents a reduction of approximately 8% of the cost of shipping to the States of Tocantins and Pará, a 3% reduction for the State of Pernambuco and a 2% reduction for the State of Espírito Santo.

So, these results demonstrate that the proposed mathematical model brings significant financial gains for the company, reducing the cost of its logistics distribution and, consequently, the cost of the final product, which would make the company more competitive to achieve a greater market share. It is important to say the mathematical model also defines the routes and the arrangement inside the truck, respecting the LIFO policy and the minimum support area, what may bring a time reduction in the unloading operations and may avoid damage to the cargo.

5 Conclusions

This paper did a logistical study of the food products transportation using the proposed mathematical model that is based on the Three Dimensional Bin Packing

Problem to optimize the stowage of the boxes inside the vehicles. The solutions presented showed that the model found better solutions than the current manual process of the company. In the case of the studied company, the route definition proved to be unnecessary, since the distribution trips are done normally for a few cities along the main roads of Brazil.

Good solutions were found by the model, considering that in the studied scenarios with the implementation of the solution proposed by the model, there would be a financial monthly reduction of R\$ 20,593.0 and an estimated annual gain of R\$ 247,116.0. The achieved gains would represent a reduction of approximately 8% of the cost of shipping to the States of Tocantins and Pará, a 3% reduction for the State of Pernambuco and a 2% reduction for the State of Espírito Santo. These values can be substantially increased if all the trips of the company are analyzed. Besides all of the financial gains, the model develops routes that respect the LIFO policy and the support area, providing more quality and efficiency to the company.

In this way, the model proved to be an important tool in reducing logistics costs, which have direct impact on the cost of the product and, consequently, on the competitiveness of the company. As suggestions for future work, it would be interesting the proposition of a metaheuristic for the same problem dealing with bigger instances.

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