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A GENERAL FRAMEWORK FOR SIMULTANEOUS CYCLIC SCHEDULING AND OPERATIONAL OPTIMIZATION OF MULTIPRODUCT CONTINUOUS PLANTS

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Abstract - This paper addresses the problem of integrating in a single model operational optimization and cyclic scheduling of continuous plants. Considered are multiproduct, multistage plants with finite intermediate storage capacity (FIS). A combined optimization approach introduces synergic effects for more effective scheduling and plant operation (Alle and Pinto, 2001a,b). The representation proposed for this problem results in an MINLP model with a nonconvex feasible region and nonconvex objective function. In order to deal with nonconvexity, a spatial branch-and-bound global optimization algorithm is applied. Results show that the global approach is effectively able to yield more profitable solutions than those obtained by local optimization methods.
Keywords: scheduling, optimization of process conditions, continuous plants, mathematical programming, global optimization.

INTRODUCTION

In multiproduct continuous plants, scheduling involves several trade-offs between length of production cycle, inventory levels, changeover times and costs (Pinto and Grossmann, 1994). The introduction of variable processing rates brings additional interactions into the scheduling model. The faster a unit runs, the lower its yield due to the shortening of residence times. Moreover, operational costs may increase. On the other hand, the unit would be free in a shorter period of time. Alle and Pinto (2001a,b) presented the TSPFLOP model, which incorporates variable processing rates and yields for simultaneous scheduling and operational optimization of these plants. Results showed that a combined optimization approach may better capture the complexity of the trade-offs involved because some operational variables are additional degrees of freedom in the scheduling model. However, TSPFLOP does not guarantee conditions for global optimality because its objective function and feasible region are nonconvex. As a matter of fact, a locally optimal schedule may differ to a great extent from a globally optimal one because they may be in completely different regions of the solution space. As a consequence, solutions from TSPFLOP may be subject to significant improvement. In order to avoid suboptimal schedules, global optimization methods are required to solve the TSPFLOP model. A review of the most important methods in global optimization may be found in Pardalos et al. (2000). Floudas (2000) presents an overview of recent applications of global optimization methods in the areas of process design and control.

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The aims of this work are the following: (1) to introduce a general formulation for the simultaneous problem of scheduling and operational optimization of cyclic continuous multiproduct plants with finite storage and (2) to make use of a spatial branch-and-bound, based on Horst and Tuy algorithm (1993) general formulation, as described by Smith and Pantelides (1999), to solve the simultaneous problem.

PROBLEM DESCRIPTION

Given is a number of specified products (i=1…N) that are to be produced in a continuous plant consisting of several stages that are interconnected by intermediate inventory tanks for each product (Fig. 1). Each stage consists of one production line, which is interconnected with a fixed topology in order to perform a set of operations (reactions, separations etc.). Transition times that arise between the processing of two successive products are sequence dependent. Constant demand rates in the form of lower bounds that are to be satisfied are also given. Intermediate capacities are limited. Final capacity is not limiting and may be neglected. Moreover, stages may have their processing rates changed within a range. The processing yield in a stage may depend on its processing rate.

The following are the assumptions for modeling the problem (Alle and Pinto, 2001a):

A1) Every product must be processed in the same sequence at all stages (i.e., flowshop plant);

A2) Intermediate inventory depends on the maximum level of material accumulated, as in Buzacott and Ozkarahan (1983).

A3) Inventory cost of final products depends on the average amount of material to be stored, as in Sahinidis and Grossmann (1991).

A4) Stages must operate continuously within one cycle, i.e., waiting times are not allowed between operations once production has started.

A5) The product yield in every stage is an exponential decaying function of the processing rate, as in Alle and Pinto (2001a,b).

The problem then consists of determining the following items for a cyclic scheduling:
(a) sequence of products, (b) length of cycle time, (c) length of production times, (d) amounts to be produced, (e) levels of intermediate storage and (f) processing rates for every product in stages. The objective is the maximization of profitability (profit per unit of time), which includes income from the sale of products, inventory, transition, raw material and operational costs.

MODEL FOR SIMULTANEOUS SCHEDULING AND OPERATIONAL OPTIMIZATION

Model TSPFLOP (Alle and Pinto, 2001a,b) was presented for the case in which only the processing rates and yields of the first stage were allowed to vary. The proposed model is extended to cover a more general case where every stage may have its rate adjusted.

Binary variables z_{ij} are used to determine product sequence:

z_{ij} : 1 if product i precedes product j; otherwise 0.

As the plant is a flowshop, every product j must be preceded by the same product i at all stages. Only one product succeeds and precedes the other, as shown in (1).

$$\sum_i z_{ij} = 1 \quad \forall j, \quad \sum_i z_{ij} = 1 \quad \forall i \tag{1}$$

A transition time, τ_{ijm}, and a transition cost, Ctr_{ijm}, are incurred every time a unit changes from the production of product i to that of another product, j. The overall transition cost for a product i in a cycle Ct is given by (2).

$$Ct_i = \sum_j z_{ij} \sum_m Ctr_{ijm} \quad \forall i \tag{2}$$

The total amount of product i produced at stage m, W_{im} (ton), during one subcycle is the product of the processing rate, γ_{im} (ton.h^{-1}), and processing time, Tp_{im} (h), as follows:

$$W_{im} = \gamma_{im} Tp_{im} \quad \forall i, m \tag{3}$$

The amount produced at stage m must be completely consumed at stage m+1 in order to avoid accumulation of material within cycles.

$$W_{im} = \alpha_{im+1} W_{im+1} \quad \forall i, m = 1...M-1 \tag{4}$$

The mass balance coefficient, α_{im}, is the inverse of process yield of product i at stage m. It is assumed to depend on the processing rate.

$$\alpha_{im} = \exp\left(\gamma_{im} / b_{im}\right) \quad \forall i, m \tag{5}$$

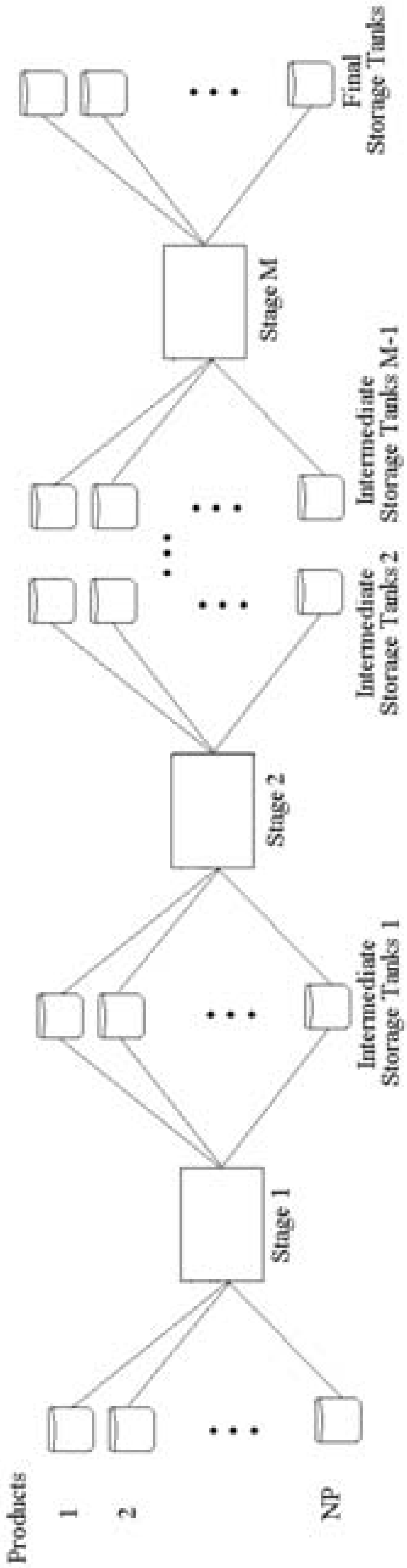

Figure 1: Multiproduct, cyclic continuous plants with intermediate storage.

Processing rates are allowed to vary within limits imposed by the operational range of the units:

$$\gamma_{im}^{lo} \le \gamma_{im} \le \gamma_{im}^{up} \qquad \forall i, m \tag{6}$$

Equation (5) states that α_{im} increases (thus yield decreases) exponentially as processing rate increases. The amount of raw material F_i (ton) consumed to produce final product i is given by (7).

$$F_i = \alpha_{i1} W_{i1} \qquad \forall i \tag{7}$$

The total demand for final products must be satisfied at the end of the cycle:

$$W_{iM} \ge d_i Tc \qquad \forall i \tag{8}$$

The maximum level of intermediate inventory $Imax_{im}$ (ton) is modeled as in Alle and Pinto (2001 a,b) with the aid of binary variables y_{im} , defined as follows:

y_{im} : 1 if processing of product i finishes at stage m before starting processing at stage m+1; otherwise, 0.

The continuous variable Inv_{im} (ton) is defined as the difference between W_{im} and the maximum inventory level, $Imax_{im}$. Therefore,

$$Imax_{im} = W_{im} - Inv_{im} \qquad \forall i, m = 1 \ldots M-1 \tag{9}$$

$$0 \le Inv_{im} \le y_{im} W_{im}^{up} \qquad \forall i, m = 1 \ldots M-1 \tag{10}$$

$$0 \le Inv_{im} - \delta_{im}(Ts_{im} + Tp_{im} - Ts_{im+1}) \le \le (1 - y_{im}) W_{im}^{up} \qquad \forall i, m = 1 \ldots M-1 \tag{11}$$

where auxiliary variable δ_{im} is defined as follows:

$$\delta_{im} \le \gamma_{im} \qquad \forall i, m = 1 \ldots M-1 \tag{12}$$

$$\delta_{im} \le \alpha_{im+1} \gamma_{im+1} \qquad \forall i, m = 1 \ldots M-1 \tag{13}$$

As the plant has finite intermediate storage capacity (FIS), there is a limit on the storage capacity of the intermediate tanks:

$$Imax_{im} \le Imax_{im}^{up} \qquad \forall i, m = 1 \ldots M-1 \tag{14}$$

Constraint (15) states that product j starts in stage m at Ts_{jm}, immediately after the processing of the preceding product i ($Ts_{jm}+Tp_{im}$) plus the corresponding transition time, τ_{ijm}.

$$-(1 - z_{ij}) U_m^T \le \le Ts_{jm} - (Ts_{im} + Tp_{im} + z_{ij}\tau_{ijm}) \le \le (1 - z_{ij}) U_m^T \quad \forall i, j > 1, m \tag{15}$$

As the plant is a flowshop, the processing of product i at stage m must start (end) before the start (end) of processing of the same product at stage m+1:

$$Ts_{im} \ge Ts_{im+1}$$

and

$$Ts_{im} + Tp_{im} \ge Ts_{im+1} + Tp_{im+1} \qquad \forall i, m = 1 \ldots M-1 \tag{16}$$

As the schedule is cyclic, product 1 is arbitrarily chosen as the first to enter the production line:

$$Ts_{11} = \sum_i z_{i1} \tau_{i11} \tag{17}$$

At any stage, the sum of the total occupation time plus the transition times for all products must not exceed the cycle time, Tc.

$$Tc \ge \sum_i Tp_{im} + \sum_i \sum_j z_{ij} \tau_{ijm} \qquad \forall m \tag{18}$$

Every unit has a processing or operational cost for every product i, OC_{im}, that is assumed to be proportional to the processing rate, γ_{im}; the total amount processed, $\alpha_{im}W_{im}$; and a cost coefficient, Co_{im}:

$$OC_{im} = co_{im} \gamma_{im} \alpha_{im} W_{im} \qquad \forall i, m \tag{19}$$

The objective function is given by the difference between revenues due to sale of final products and costs (transition, raw material, operation, and intermediate and final inventory).

$$\text{max profitability} = \frac{1}{Tc}\left(\sum_i p_i W_{iM} - Ct_i - c_i F_i - \sum_m co_{im}\gamma_{im}\alpha_{im}W_{im} - \sum_m^{M-1} Cinv_{im} Imax_{im}\right)$$
$$-\frac{1}{2}\sum_i Cinvf_i W_{iM}\left(1-\frac{Tp_{iM}}{Tc}\right) \qquad (20)$$

Note that a continuous time domain representation is used. For more details on the model, please refer to Alle and Pinto (2001b).

GLOBAL OPTIMIZATION ALGORITHM

The proposed model is reformulated in order to eliminate nonconvex terms and to contain only linear (convex) constraints and nonlinear term definitions, as proposed by Smith and Pantelides (1999). For instance, the nonconvex objective function (20) is reformulated as

$$\text{max profitability} = FR1 - 0.5\sum_i Cinvf_i W_{iM} \qquad (21)$$

where

$$FR1 = \frac{A1}{Tc}, \quad A1 = \sum_i (p_i W_{iM} - \sum_j z_{ij}Ctr_{ij} - c_i F_i - \sum_m co_{im} BL1_{im} - \sum_m^{M-1} Cinv_{im} Imax_{im} - 0.5 Cinvf_i BL2_{iM})$$

$$BL1_{im} = \gamma_{im} BL3_{im}, \qquad BL3_{im} = \alpha_{im} W_{im}, \qquad BL2_{iM} = W_{im} Tp_{iM}$$

New variable FR1 replaces a fractional term, whereas $BL1_{im}$, $BL2_{im}$ and $BL3_{iM}$ replace bilinear terms; A1 replaces the sum that is the numerator of the fractional term. All constraints that contain nonlinear terms are submitted to similar transformation, except constraint (5). Actually, this equation may be relaxed to

$$\alpha_{im} \geq \exp(\gamma_{im} / b_{im}) \qquad \forall i, m \qquad (22)$$

without changing the global optimum. The reason is that inequality (22) must be active in the global optimum since the smaller the α_{im} (i.e, the greater the yield) for a given γ_{im}, the greater the plant profitability. Since the OA/ER/AP algorithm (Viswanathan and Grossmann, 1990) used here is based on equality relaxation, constraint (5) does not need to be replaced by a linear constraint and a nonlinear term definition because it defines a convex region when relaxed.

After the replacements, the global optimization algorithm shown in Fig. 2 is applied. It is a spatial branch-and-bound, based on Horst and Tuy algorithm (1993) general formulation, extended by Quesada and Grossmann (1995) and Ryoo and Sahinidis (1995, 1996), as described by Smith and Pantelides (1999). The algorithm makes use of the nonconvex MINLP reformulated model to generate lower bounds for the max problem and of a MINLP convex relaxation subproblem to find upper bounds. The convex relaxation is obtained through substitution of the nonlinear term definitions (fractional and bilinear terms) by new variables that are constrained by linear over- and under-estimators such as those from McCormick (1976), shown in Table 1.

Table 1: McCormick over- and underestimators for bilinear and fractional terms.

Bilinear term $BL_{im} \equiv B_{im}L_{im}$	Underestimators	$BL_{im} \geq B_{im}^{lo}L_{im} + B_{im}L_{im}^{lo} - B_{im}^{lo}L_{im}^{lo}$
		$BL_{im} \geq B_{im}^{up}L_{im} + B_{im}L_{im}^{up} - B_{im}^{up}L_{im}^{up}$
	Overestimators	$BL_{im} \leq B_{im}^{lo}L_{im} + B_{im}L_{im}^{up} - B_{im}^{lo}L_{im}^{up}$
		$BL_{im} \leq B_{im}^{up}L_{im} + B_{im}L_{im}^{lo} - B_{im}^{up}L_{im}^{lo}$
Fractional term $FR_{im} \equiv \frac{F_{im}}{R_{im}}$	Underestimators	$F_{im} \geq FR_{im}^{lo}R_{im} + FR_{im}R_{im}^{lo} - FR_{im}^{lo}R_{im}^{lo}$
		$F_{im} \geq FR_{im}^{up}R_{im} + FR_{im}R_{iM}^{up} - FR_{iM}^{up}R_{iM}^{up}$
	Overestimators	$F_{im} \leq FR_{im}^{lo}R_{iM} + FR_{im}R_{im}^{up} - FR_{im}^{lo}R_{im}^{up}$
		$F_{im} \leq FR_{im}^{up}R_{im} + FR_{im}R_{im}^{lo} - FR_{im}^{up}R_{im}^{lo}$

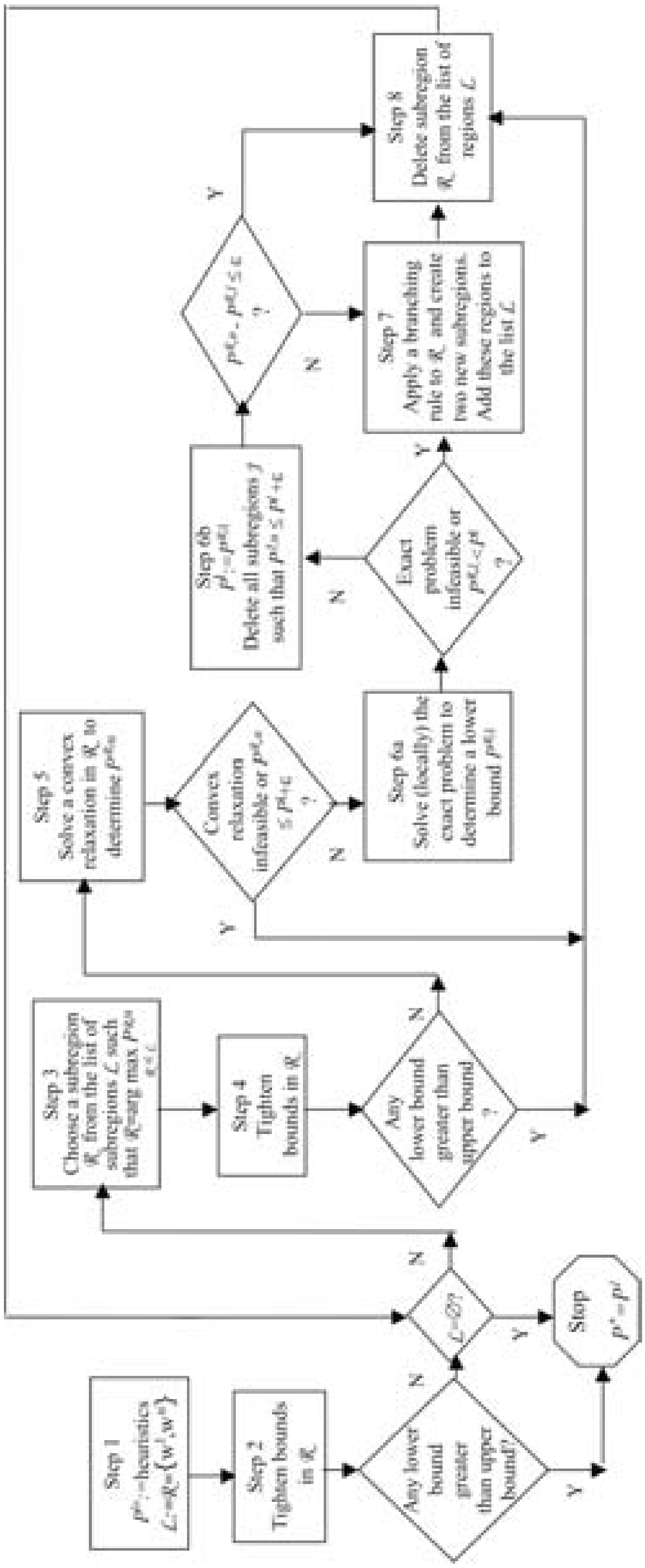

Figure 2: Flow diagram for the global optimization algorithm.

RESULTS

The spatial branch-and-bound algorithm was implemented in GAMS (Brooke et al., 1998). The MINLP solver used for both the nonconvex problem and the convex relaxation subproblem is DICOPT++ based on the OA/ER/AP method (Viswanathan and Grossmann, 1990). CONOPT2 (Drud, 1992) and XPRESS 12.5 solver (Dash Associates, 1999) were the solvers for the NLP subproblems and MILP master problems, respectively. The ε (global optimality gap) adopted is 1 %.

Table 2 shows the relative difference, ΔP^*, between global and local optimal profitability for different problems. Global optimization yields solutions as good as or better than the straightforward local optimization procedure at the expense of a much larger computational effort, attributed mainly to steps 2 and 4 of the algorithm (Smith and Pantelides, 1999).

Table 3 contains data on a plant with two stages that processes three products. Fig. 3 shows the difference between global and local optimal schedules for this plant. The profitability increases 3.6% from the local to the global optimum. Upper bounds of Tc and processing rates are all active in both solutions. Note the large difference between the processing times for products B and C in each schedule as well as the different inventory control profiles.

Note, however, that global optimization performance highly depends on the quality of the convex relaxation. The closer the relaxed model to the exact one, the better the algorithm performs. As seen in Table 4, tight bounds for cycle time Tc are essential for a good relaxation. In fact, Tc bounds most of the variables in the model. As a consequence, the algorithm performance is very sensitive to Tc bounds.

Table 2: Results for local and global optimization.

Problem size		ΔP^*	Local opt.	Global opt.	
Products	Stages		CPU(s)	CPU (s)	Iterations
3	2	3.6%	0.4	32.5	9
4	2	0%	1.7	25.9	1
5	3	0%	5.0	101.2	1

Table 3: Plant data for the example of the 3-product-2-stage plant.

Pr.	P_i	d_i	Co_{i1}	Co_{i2}		$Cf_i = 30\ \$/\text{ton}$	$Cinv_{im} = 10\ \$/\text{ton}$
	($/ton)	(ton/d)	(ton/d)	(ton/d)		$Cinvf_{im} = 0.1\ \$/\text{ton/h}$	$Imax_{im}^{up} = 10.0$ ton
A	290	0.05	28	25		β_{i1}=10.0 ton/h	β_{i2} =1000.0 ton/h
B	320	0.10	20	25		γ_{im}^{lo}=1.1 ton/h	γ_{im}^{up} =1.25 ton/h
C	330	0.25	25	30		$Tc^{lo} = 0$	$Tc^{up} = 800$ h

	Transition times, τ_{ijm} (h)						Transition costs, $\sum_m Ctr_{ijm}$ ($)		
	Stage 1			Stage 2			Stages 1+2		
Pr.	A	B	C	A	B	C	A	B	C
A	0	3	8	0	3	4	0	46000	26000
B	10	0	3	7	0	0	25000	0	35000
C	3	6	0	3	10	0	37000	17000	0

Table 4: Dependence of algorithm performance on Tc bounds (3 product/2 stage example).

Tc^{up}	Tc^{lo}	Initial relaxation gap	Iterations
800	0	4.8 %	9
1100	0	9.8%	11
1400	0	16.4%	>200

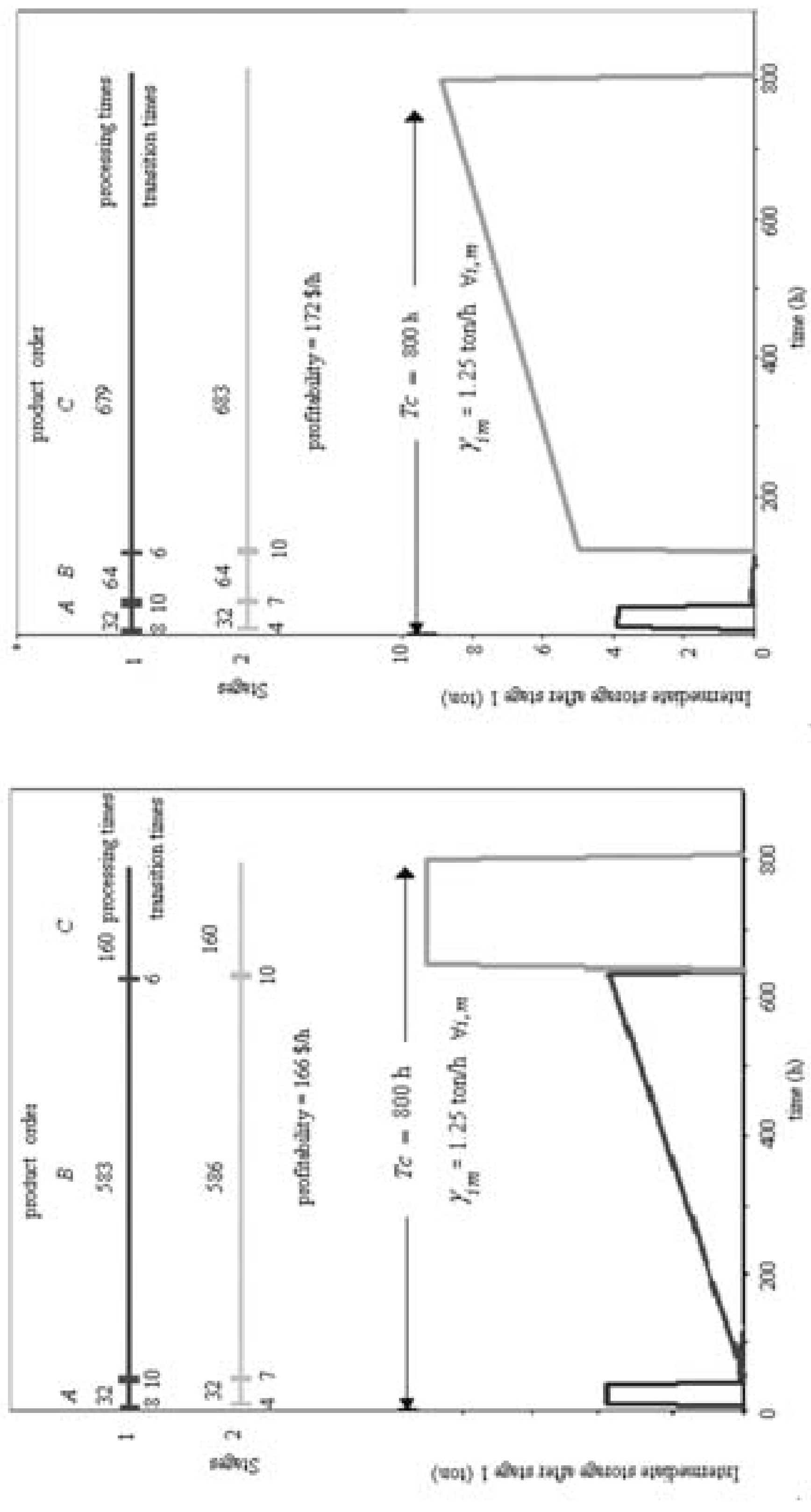

Figure 3: Local (left) and global (right) optimal schedules for a continuous multiproduct plant.

CONCLUSIONS

A general framework model for global optimization of the simultaneous problems of scheduling and operating conditions for continuous multiproduct plants was developed. The model extends Alle and Pinto's (2001a,b) formulation to a more general case, where operating conditions (processing rates and yields) are allowed to vary at every plant stage. A spatial branch-and-bound algorithm was successfully applied to achieve globally optimal solutions. Results showed that the difference between a local and a global optimal schedule means a completely different way of planning production.

NOMENCLATURE

Sets

Products	$i, j = 1,..., N$
Stages	$m = 1,..., M$

Binary Variables

x_{im}	denotes whether the production of i in stages m and m+1, occurs simultaneously
z_{ij}	denotes whether product i is immediately preceded by j

Continuous Variables

I_{im}	intermediate inventory level of product i in stage m
Inv_{im}	difference between amount produced and maximum inventory level of product i between stages m and m+1
Tc	cycle time
Tp_{im}	processing time of product i in stage m
Ts_{im}	start time of product i in stage m
W_{im}	amount of product i produced in stage m
F_i	amount of raw material consumed by product i

Parameters

$Cinvf_i$	cost coefficient for inventory of final product i
$Cinv_{im}$	cost coefficient for inventory of product i in stage m
Co_{im}	operating cost coefficient for processing product i at stage m
Ctr_{ijm}	cost of transition between product i and product j at unit m
d_i, p_i	minimum demand rate and price of product i
$Imax_{im}$	maximum inventory capacity for product i after stage m
U^I_{im} U^T_{im}	upper bounds of processing time and inventory of product i in stage m
ΔP^*	$\left(\dfrac{\text{Profitability}^*_{\text{global}} - \text{Profitability}^*_{\text{local}}}{\text{Profitability}^*_{\text{global}}}\right)\times 100\%$
τ_{ijm}	transition time from product i to product j in stage m
γ_{im} α_{im}	processing rate and mass balance coefficient of product i in stage m

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