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# MODELLING THE DRYING OF A PARALLELEPIPEDIC OIL SHALE PARTICLE

P. S. S. Porto<sup>1</sup> and A. C. L. Lisbôa<sup>2</sup>

School of Chemical Engineering, State University of Campinas, 13083-970 – Campinas, SP, Brazil. E-mail: porto@feq.unicamp.br1 E-mail: lisboa@feq.unicamp.br<sup>2</sup>

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**Abstract** - A numerical model is proposed to describe the process of drying a parallelepipedic oil shale particle. Assuming Fick's law, the diffusion equation for the shape of the particle was used. The objective of the study was to develop a computer program in Fortran to estimate the moisture content of an oil shale particle undergoing drying as a function of time and position. The average moisture content was also obtained. The model takes into account the migration of water by diffusion within the solid and its loss at the interface. The model results were compared to experimental data from an apparatus which measured the mass loss of a particle. The apparatus comprised an electronic balance attached by a thin wire to the particle placed inside an incubator.

Keywords: Oil shale; Drying; Mathematical modelling.

# **INTRODUCTION**

Oil shale is a sedimentary rock that contains organic matter scattered throughout a mineral matrix. Thermal decomposition, also known as retorting, pyrolysis or devolatilization, converts the solid organic material of the shale into liquid and gaseous fractions and a solid carbonaceous residue. Oil shale moisture consumes the available heat for pyrolysis. Little is known about the kinetics of moisture removal from oil shale.

The particles used were obtained from the Petrosix process feedstock. Petrosix is the retorting process developed by Petrobras, a Brazilian oil company, which has been operating for almost fifteen years. The retorting process particles, which measure between 1.2 cm and 7.5 cm, have an approximately parallelepipedic shape. This work complements previous work on modelling the pyrolysis of Petrosix particles by Lovo Jr. (2003),

\*To whom correspondence should be addressed

which also addressed parallelepipedic particles. In their work on Petrosix, Rajagopal et al. (1986) concluded that "the thermodynamic efficiency of the retort and the overall process will be improved by predrying and/or preheating the shale rock".

The aim of this work was to develop a threedimensional numerical model to estimate the moisture content of an oil shale particle undergoing drying as function of time and position and the average moisture content of the particle.

## MODEL DEVELOPMENT

The mathematical model is based on the diffusional model. A three-dimensional model is proposed to describe the drying of parallelepipedic oil shale particles. The model is based on the following assumptions:

• the moisture content is uniformly distributed throughout the particle.

- the diffusion coefficient is constant.
- the environmental conditions are constant. The diffusion model is given by Equation 1.

$$\frac{\partial X}{\partial t} = D \left( \frac{\partial^2 X}{\partial x^2} + \frac{\partial^2 X}{\partial y^2} + \frac{\partial^2 X}{\partial z^2} \right)$$
(1)

with the following boundary and initial conditions, for a Cartesian coordinate system with its origin placed at the particle center are:

x axis

$$x = 0$$
  $\frac{\partial X}{\partial x} = 0$ ,  $x = L_x - k \frac{\partial X}{\partial x} = k_C (X - X_E)$ ,  
 $t = 0$   $X = X_0$ 

y axis

$$y = 0$$
  $\frac{\partial X}{\partial y} = 0$ ,  $y = L_y - k \frac{\partial X}{\partial y} = k_C (X - X_E)$ ,

 $t = 0 X = X_0$ 

z axis

x axis

$$z=0$$
  $\frac{\partial X}{\partial z}=0$ ,  $z=L_z -k\frac{\partial X}{\partial z}=k_C (X-X_E)$ ,

 $t = 0 X = X_0$ 

The solution of the three-dimensional Equation 1 is obtained as the product of one-dimensional problems. In dimensionless variables (defined below):

$$\begin{split} & \psi(\xi_{x},\xi_{y},\xi_{z},\tau_{x},\tau_{y},\tau_{z}) = \\ & = \psi_{x}(\xi_{x},\tau_{x}) \psi_{y}(\xi_{y},\tau_{y}) \psi_{z}(\xi_{z},\tau_{z}) \end{split}$$

in which  $\psi_i$ , i = x, y, z are obtained by solving Equation 2:

$$\frac{\partial \psi_i}{\partial \tau_i} = D \left( \frac{\partial^2 \psi_i}{\partial \xi_i^2} \right)$$
(2)

with the following boundary and initial conditions:

$$\xi_{x} = 0 \quad \frac{\partial \psi_{x}}{\partial x} = 0, \qquad \qquad \xi_{x} = 1 \quad \frac{\partial \psi_{x}}{\partial x} = -Bi_{x} \quad \psi_{x},$$

$$\tau_{\rm x} = 0 \ \psi_{\rm x} = 1$$

y axis

$$\xi_{y} = 0 \quad \frac{\partial \psi_{y}}{\partial y} = 0, \qquad \qquad \xi_{y} = 1 \quad \frac{\partial \psi_{y}}{\partial y} = -Bi_{y} \quad \psi_{y}.$$

 $\tau_y = 0 \ \psi_y = 1$ 

z axis

$$\xi_z = 0 \quad \frac{\partial \psi_z}{\partial z} = 0, \qquad \qquad \xi_z = 1 \quad \frac{\partial \psi_z}{\partial z} = -Bi_z \quad \psi_z,$$

 $\tau_z=0 \ \psi_z=1$ 

The dimensionless variables are given below:

y axis

$$\xi_{y} = \frac{y}{L_{y}}, \qquad \tau_{y} = \frac{D}{L_{y}^{2}}t, \qquad \psi_{y} = \frac{X - X_{E}}{X_{0} - X_{E}}$$

x axis

$$\xi_x = \frac{x}{L_x}$$
,  $\tau_x = \frac{D}{L_x^2}t$ ,  $\psi_x = \frac{X - X_E}{X_0 - X_E}$ 

z axis

$$\xi_{z} = \frac{z}{L_{z}}, \qquad \tau_{z} = \frac{D}{L_{z}^{2}}t, \qquad \psi_{z} = \frac{X - X_{E}}{X_{0} - X_{E}}$$

in which  $X_E$  is the moisture content at equilibrium with the environmental conditions and  $X_0$  is the initial moisture content:

The solution of Equation 2 is given as an infinite series (Crank, 1975):

$$\psi_{i} = \sum_{n=1}^{\infty} \frac{4 \operatorname{sen} \lambda_{n}}{(2\lambda_{n} + \operatorname{sen} 2\lambda_{n})} \cos(\lambda_{n}\xi_{i}) e^{-\lambda_{n}^{2}\tau_{i}}$$
(3)

in which each  $\lambda_n$  is a root of Equation 4:

$$\tan \lambda_n + \frac{Bi_i}{\lambda_n} = 0 \tag{4}$$

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Bi<sub>i</sub> is defined for each direction, as shown below.

$$\operatorname{Bi}_{x} = \frac{\operatorname{k_{C}}\operatorname{L}_{x}}{\operatorname{D}}, \qquad \operatorname{Bi}_{y} = \frac{\operatorname{k_{C}}\operatorname{L}_{y}}{\operatorname{D}}, \qquad \operatorname{Bi}_{z} = \frac{\operatorname{k_{C}}\operatorname{L}_{z}}{\operatorname{D}}$$

The average moisture content of the parallelepipedic particle  $(\bar{\Psi})$  may be obtained by averaging the moisture values at points on a tridimensional grid established within the limits  $L_x$ ,  $L_y$ ,  $L_z$ .

#### **EXPERIMENTAL PROCEDURE**

### Samples

Oil shale particles were supplied by Petrobras from its mine in the city of São Mateus do Sul. Samples had approximately parallelepipedic shapes. The particle sizes were 0.06 m x 0.05 x 0.03 m for the run at 60 °C and 0.08 m x 0.04 m x 0.02 m for the run at 90 °C.

#### **Experimental Apparatus**

An incubator was used to dry the particle as shown in the schematic setup (Figure 1). The particle was placed in the incubator, hanging by a wire, which was attached to the bottom of an electronic balance. The particle was partially shielded by a metal wall to avoid the effect of air circulation. Mass loss was monitored continuously. The drying temperature was monitored by two thermocouples, one near the sample and the other in the incubator chamber. Before starting the drying, the particle was weighed. The experimental results on mass loss were collected until constant weight was obtained. Drying conditions were maintained constant throughout the runs. Mass loss during drying was recorded.



Figure 1: Schematic setup.

# **RESULTS AND DISCUSSION**

The experiments were carried out at 60 °C and 90 °C. The drying of the oil shale particles was described by plotting the moisture content versus time, as shown in Figure 2. The rate of moisture removal increased when the air temperature increased from 60 °C to 90 °C. The drying run at 60 °C ended before equilibrium was reached (more than 3 days).

The drying rate curve was calculated by numerical differentiation of the moisture content with respect to time. Data were plotted against moisture content, as shown in Figure 3. This curve indicates that the drying occurs in the falling rate period, with no constant rate period.

Table 1 compares the experimental and theoretical results for the runs at 60 °C and 90 °C. The theoretical results were obtained by averaging local values at points on a 11x11x11 grid. Figures 4 and 5 show numerical and experimental curves at 60 °C and 90 °C, respectively. The agreement between experimental and predicted results is satisfactory.

Table 2 shows values for the diffusion coefficient for oil shale, obtained from the literature.

The effective diffusivity of water vapor within the shale and the mass coefficient of water vapor from

slab surface to the incubator had to be assumed in the model. The diffusion coefficient value for the Brazilian oil shale was assumed to be  $8 \times 10^{-8}$  m<sup>2</sup>/s, within the range found in the literature (Table 2). The mass transfer coefficient values assumed to calculate the Biot number were  $12 \times 10^{-8}$  m/s (run at

60° C) and  $25 \times 10^{-8}$  m/s (run at 90 °C). The Biot numbers calculated were in the same range as those found by Lane et al. (1988).

Table 3 indicates that increasing the grid points from 11x11x11 to 31x31x31 had little influence on the results.



Figure 2: Dimensionless moisture content curves

Figure 3: Drying rate curves.

| Table 1: Dimensionless moisture obtained experimentally and | d |
|---|---|
| by the mathematical model at 60 °C and 90 °C.               |   |

|          | T = 0  | 50 °C    |        |          | T = 9  | 90 °C    |        |
|----------|--------|----------|--------|----------|--------|----------|--------|
| Experi   | mental | Mo       | del    | Experi   | mental | Mo       | del    |
| time (s) | У      |
| 0        | 1.0000 | 0        | 1.0000 | 0        | 1.0000 | 0        | 1.0000 |
| 3240     | 0.9248 | 3240     | 0.9541 | 6000     | 0.7143 | 6000     | 0.7701 |
| 6600     | 0.8764 | 6600     | 0.9064 | 13200    | 0.5245 | 13200    | 0.5658 |
| 9900     | 0.8391 | 9900     | 0.8597 | 20400    | 0.3978 | 20400    | 0.4152 |
| 12900    | 0.8068 | 12900    | 0.8171 | 25200    | 0.3401 | 25200    | 0.3378 |
| 16200    | 0.7739 | 16200    | 0.7704 | 31800    | 0.2738 | 31800    | 0.2543 |
| 19800    | 0.7405 | 19800    | 0.7193 | 38400    | 0.2236 | 38400    | 0.1915 |
| 21600    | 0.7277 | 21600    | 0.7003 | 45000    | 0.1721 | 45000    | 0.1442 |
| 25200    | 0.6993 | 25200    | 0.6638 | 51600    | 0.1326 | 51600    | 0.1086 |
| 28800    | 0.6759 | 28800    | 0.6273 | 58200    | 0.1013 | 58200    | 0.0818 |
| 32400    | 0.6487 | 32400    | 0.5909 | 64200    | 0.0704 | 64200    | 0.0632 |
| 84720    | 0.3318 | 84720    | 0.3104 | 70200    | 0.0428 | 70200    | 0.0488 |
| 172800   | 0.0919 | 172800   | 0.0890 | 76200    | 0.0272 | 76200    | 0.0377 |
| 256320   | 0.0000 | 256320   | 0.0062 | 82200    | 0.0099 | 82200    | 0.0291 |
| -        | -      | -        | -      | 88200    | 0.0078 | 88200    | 0.0225 |
| -        | -      | -        | -      | 94200    | 0.0008 | 94200    | 0.0174 |
| -        | -      | -        | -      | 100800   | 0.0000 | 100800   | 0.0131 |

Table 2: Diffusion coefficient values found in the literature.

| Туре             | T, °C     | $D, m^2/s$             | References               |
|------------------|-----------|------------------------|--------------------------|
| Stuart oil shale | 100 - 250 | 2.1 x 10 <sup>-7</sup> | Lane et al., (1988)      |
| Stuart oil shale | 22        | 7.3 x 10 <sup>-8</sup> | Duffy and Haynes, (1992) |
| Condor oil shale | 22        | 9.5 x 10 <sup>-8</sup> | Duffy and Haynes, (1992) |







**Figure 5:** Theoretical and experimental dimensionless moisture content at 90 °C.

| Experimental |        | Theoretical |          |          |  |
|--------------|--------|-------------|----------|----------|--|
|              |        | 11x11x11    | 21x21x21 | 31x31x31 |  |
| time (s)     | У      | У           | У        | У        |  |
| 0            | 1.0000 | 1.0000      | 1.0000   | 1.0000   |  |
| 6000         | 0.7143 | 0.7710      | 0.7717   | 0.7719   |  |
| 13200        | 0.5245 | 0.5658      | 0.5663   | 0.5664   |  |
| 20400        | 0.3977 | 0.4152      | 0.4155   | 0.4157   |  |
| 25200        | 0.3401 | 0.3378      | 0.3381   | 0.3382   |  |
| 31800        | 0.2738 | 0.2543      | 0.2546   | 0.2546   |  |
| 38400        | 0.2235 | 0.1915      | 0.1917   | 0.1917   |  |
| 45000        | 0.1721 | 0.1442      | 0.1443   | 0.1444   |  |
| 51600        | 0.1326 | 0.1086      | 0.1087   | 0.1087   |  |
| 58200        | 0.1013 | 0.0818      | 0.0819   | 0.0819   |  |
| 64200        | 0.0704 | 0.0632      | 0.0633   | 0.0633   |  |
| 70200        | 0.0428 | 0.0488      | 0.0489   | 0.0489   |  |
| 76200        | 0.0272 | 0.0377      | 0.0378   | 0.0378   |  |
| 82200        | 0.0099 | 0.0291      | 0.0292   | 0.0292   |  |
| 88200        | 0.0078 | 0.0225      | 0.0225   | 0.0225   |  |
| 94200        | 0.0008 | 0.0174      | 0.0174   | 0.0174   |  |
| 100800       | 0.0000 | 0.0131      | 0.0131   | 0.0131   |  |

Table 3: Effect of number of grid points on theoretical results, run at 90 °C.

# CONCLUSIONS

The mathematical model developed enables prediction of the distribution of moisture within parallelepipedic oil shale particles undergoing drying. Results indicate that the predicted and measured moisture content profiles agree satisfactorily. Further studies are necessary to obtain accurate values for the effective diffusion coefficient of moisture within the oil shale particle. The model may be integrated with a general pyrolysis model.

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### NOMENCLATURE

| Bi | Biot number           | (-)  |
|----|-----------------------|------|
| D  | effective diffusivity | m²/s |

Е

| $L_x$ , $L_y$ , $L_z$ | particle half-thickness   | m                       |
|-----------------------|---------------------------|-------------------------|
| k                     | thermal conductivity      | W/m K                   |
| k <sub>C</sub>        | mass transfer coefficient | m/s                     |
| t                     | time                      | S                       |
| Т                     | temperature               | °C                      |
| Х                     | moisture content          | kg H <sub>2</sub> 0/ kg |
|                       |                           | dry solid               |
| x,y,z                 | space coordinates         | m                       |

#### **Greek Symbols**

| τξ   | dimensionless time<br>dimensionless<br>coordinates | (-)<br>(-) |
|--|--|------------|
| $\psi = \frac{X - X_E}{X_0 - X_E}$                       | dimensionless moisture                             | (-)        |
| $\overline{\Psi} = \frac{\overline{X} - X_E}{X_0 - X_E}$ | average dimensionless moisture                     | (-)        |

#### **Subscripts**

| 0 initial condition | (- | ) |
|---------------------|----|---|
|---------------------|----|---|

equilibrium condition (-)

## REFERENCES

- Crank, J., The Mathematics of Diffusion. Great Britain, 2<sup>nd</sup> ed. Claredon Press, Oxford (1975).
- Duffy, B.L. and Haynes, B.S., Transport Mechanisms in Oil Shale Drying and Pyrolysis. Energy and
- Fuel, 6, pp. 831-835 (1992).
- Lane, D., Ramjas, S. and Haynes, B.S., Drying Kinectics of Stuart Oil Shale, Fuel, 67, pp. 1321-1326 (1988).
- Lovo Jr., P., Transferência de Calor e Perda de Massa no Processo de Pirólise de Xisto em Leito Móvel, M.Sc. Thesis, State University of Campinas, (in Portuguese) (2003).
- Rajagopal, K., Mundstock, R. and Casavechia, L.C., Thermodynamic Analysis of a Shale Oil Production Plant for Energy Conservation, Energy Progress, 6, No. 1, pp. 33-36 (1986).
- Tamimi, A. and Uysal, B.Z., Drying Characteristics of Oil Shale, Energy, 17, No. 3, pp. 303-308 (1992).