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A BILEVEL DECOMPOSITION TECHNIQUE FOR THE OPTIMAL PLANNING OF OFFSHORE PLATFORMS

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Abstract - There is a great incentive for developing systematic approaches that effectively identify strategies for planning oilfield complexes. This paper proposes an MILP that relies on a reformulation of the model developed by Tsarbopoulou (UCL M.S. Dissertation, London, 2000). Moreover, a bilevel decomposition technique is applied to the MILP. A master problem determines the assignment of platforms to wells and a planning subproblem calculates the timing for fixed assignments. Furthermore, a heuristic search procedure that relies on the distance between platforms and wells is applied in order to reduce the search region. Results show that the decomposition approach using heuristic generates optimal solutions for instances of up to 500 wells and 25 platforms in 10 discrete time periods that otherwise could not be solved with a full-scale approach. One important feature regarding these instances is that they correspond to problems of real-world dimension.
Keywords: Oilfield exploration; Integer programming; Discrete time; decomposition methods; Optimization.

INTRODUCTION

There is a great incentive for developing systematic approaches that effectively identify strategies for planning and designing oilfield complexes, due to the economic impact of the underlying decisions. On the other hand, the development and application of optimization techniques in problems that involve oilfield exploration represents a challenging and complex problem.

The literature presents models and solution techniques for solving problems in the design and planning of infrastructure in oilfields. This problem has been initially presented in the literature by Devine and Lesso (1972) that proposed an optimization model for the development of offshore oilfields.

According to Van den Heever and Grossmann (2000), in the past decisions that concerned platform capacities, scheduling of perforations and production yields had been frequently made separately. Moreover, certain assumptions were made in order to reduce the required computational effort. Another approach was to assume a fixed perforation schedule and then to determine the production yield from an LP (Linear Programming) model. A third approach was to determine the perforation schedule for a fixed production yield from an LP and subsequently round the non integer solution to integer values or even to solve directly the MILP in the simpler cases.

Frair (1973) proposed independent models for calculating the number of production platforms, their capacities and the scheduling of well perforation. However, this approach has lead to infeasible or sub-optimal decisions since these two levels of decision were not considered in an integrated model.

Iyer et al. (1998) proposed a multiperiod MILP for the planning and scheduling of investment and operation in offshore oilfields. The formulation incorporates the nonlinear behavior of the reservoirs,

*To whom correspondence should be addressed pressure constraints in the well surface and equipment constraints. The formulation presents a general objective function that optimizes a given economic indicator, such as the Net Present Value (NPV). A sequential decomposition technique is proposed to solve the problem that relies on the aggregation of time periods followed by successive disaggregating steps.

Iyer and Grossmann (1998) proposed a decomposition algorithm originally designed for process network optimization that solves a design problem in the reduced space of binary variables to determine the assignment of wells to platforms. The planning model is then solved for fixed values determined in the design subproblem.

Tsarbopoulou (2000) proposed an MILP model for the optimization of the exploration of oil and gas in a petroleum platform. The proposed model is based on binary variables to determine the existence of a given platform and the potential connection between wells and platforms.

Ortíz-Gomez et al. (2002) developed multiperiod optimization models for the production planning of wells in an oil reservoir. The major decisions include the calculation of oil production profiles and operation/shut in times of the wells in each time period and the authors assume nonlinear time-behavior for the well flowing pressure while calculating the oil production. Recently, Goel and Grossmann (2004) considered the optimal investment and operational planning of gas field developments under uncertainty in gas reserves. The authors showed that the proposed approach yields solutions with significantly higher expected net present value than that of solutions obtained using a deterministic approach.

This paper proposes a reformulation of the MILP model of Tsarbopoulou (2000) that relies on a smaller number of binary variables that requires a smaller computational effort. Moreover, a bilevel decomposition technique proposed by Iyer and Grossmann (1998) is applied to the reformulated model that is composed of assignment and planning sub-problems. The master problem determines the assignment of platforms to wells and the planning sub problem that calculates the timing for fixed assignments. With the decrease in the number of binary variables and with the application of the decomposition technique, it becomes possible to solve problems of realistic dimension. Furthermore, a heuristic-based constraint that limits the search region was developed and its impact on the optimal solution of the problem and computational time is studied.

The paper is structured as follows. In the following section, the problem of planning the offshore oilfield infrastructure is defined. In section 3, the problem formulation is reproduced as in Tsarbopoulou (2000). Section 4 presents the proposed reformulated model (model MR) that contains a smaller number of binary variables. In section 5, the decomposition algorithm proposed by Iyer and Grossmann (1998) is presented and applied to the reformulated model (model MD). Model MD is then modified to include the heuristic procedures to reduce the search region in section 6. Examples are given in sections 4 to 6 to validate the models and a detailed sensitivity analysis of the main parameters is performed. Finally, conclusions are drawn on the approach proposed in this work.

PROBLEM DEFINITION

This problem is concerned with the optimal planning of offshore oilfield infrastructure. An offshore oilfield consists of J wells that contain oil and gas. A set of I platforms are required to extract these substances from one or more wells. The planning decisions are related to the assignment of platforms to wells in addition to the timing of extraction and production. Figure 1 shows the oilfield infrastructure as well as its elements.

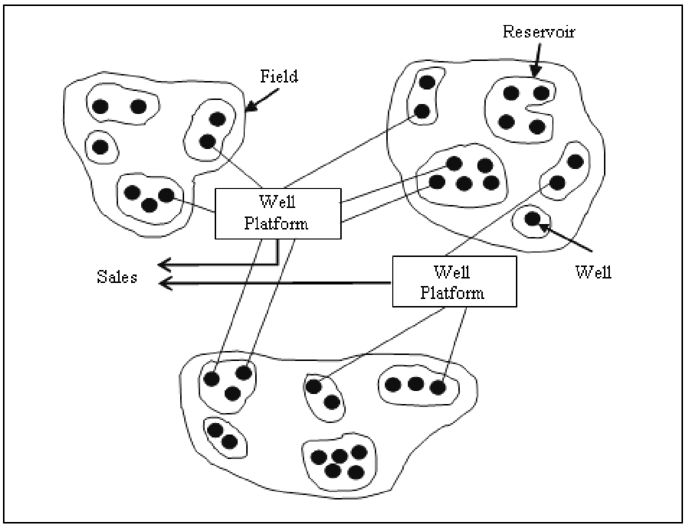


Figure 1: Problem representation (Van den Heever and Grossmann, 2000)

Figure 1 represents the real-world problem in which several oilfields, reservoirs, wells and platforms are considered. The scope of this paper is the infrastructure planning of a single oilfield that contains a set of wells. Reservoirs are not explicitly considered and in principle all wells can be connected to all platforms. The cases of multiple fields and reservoirs will be considered in future work.

MATHEMATICAL MODEL

The planning of infrastructure in offshore oilfields includes discrete and continuous decisions along the project lifetime. Discrete variables represent the installation of platforms and wells in each period. Continuous variables are concerned with oil and gas production. Based on these considerations, the model that represents the infrastructure is a Mixed Integer Programming (MIP) problem.

In the case of the planning of infrastructure of petroleum fields, MINLP models have been avoided in favor of MILP or even LP models, because of the inherent difficulties of treating nonlinear constraints and in the latter case because of the combinatorial explosion that results from discrete decisions.

Despite the fact that many authors propose MINLP models that in principle are more suitable to represent the system behavior, in this paper we rely on a linear model. The main motivation is to generate models that are simpler and better structured to solve and therefore larger instances can be handled.

Model Assumptions

The following are the main assumptions of the proposed model:

(A1) Only two substances are removed, which are oil and gas.
(A2) The productivity index (PI) is constant for each well throughout the planning horizon.
(A3) Whenever oil is removed from a reservoir, its pressure decreases linearly.
(A4) The pressure is the same in each well at any given time period.
(A5) There is no pressure loss along the pipelines that connect the wells and the platforms.
(A6) A linear model represents the gas-to-oil rate.
(A7) The initial amounts of each substance are known for each well.
(A8) The production limit for each substance is known along the planning horizon.
(A9) The area of the field is known and it is divided into a rectangular grid and it is possible to allocate a platform in the center of each rectangle.
(A10) The wells are randomly distributed in the field.
(A12) The planning time horizon is discretized in intervals of equal length.
(A13) Production costs and yields for all substances are known for each time period.
(A14) Interest and inflation rates are constant along the planning horizon.
(A15) Investment costs are represented by fixed parameters and are not subject to depreciation.

There are pertinent assumptions and those that are considered a relaxation for the model. Assumptions A1, A4, A7, A8 and A13 are pertinent and the others can be relaxed and are discussed in the sequence.

Iyer et al. (1998) state that PI, a measure of the daily amount of fluids which an oil well can produce per unit of reservoir pressure, depends on the permeability-thickness product which is obtained from a geological map of each reservoir. Values for PI are obtained from random sampling from a normal distribution for a given mean and standard deviation for each reservoir. According to this paper, PI is assumed to be constant throughout the planning horizon, as presented in the second assumption.

Van den Heever and Grossmann (2000) presented a model in which the non linear behavior of the reservoirs is incorporated directly in the formulation. Based on their assumptions, the reservoirs contain a substantial volume of gas so that a single linear constraint would be imprecise if the pressure varied over a long interval. Moreover, the gas-to-oil rate is treated as a nonlinear function of the oil removed from the well. Note that Iyer et al. (1998) already considered nonlinear behavior of the reservoir through a piecewise linear interpolation, including binary variables and Van den Heever and Grossmann (2000) included the nonlinear model directly, reducing the number of binary variables. Therefore, the third and sixth assumptions in the present model represent simplifications over previous ones in order to improve computational efficiency.

The area of the field is known and platform allocation is determined at the project level. We adopt the assumption to allocate a platform in the center of the rectangle has the objective of discretizing the area into a finite number of locations.

Potential location of the wells is known. In the absence of real data, their location is randomly distributed in the field. Information regarding taxes and inflation rates are time, dependent specific period and country dependent and can be easily adapted to the problem instance.

Drilling and connection costs are considered fixed parameters. Iyer and Grossmann (1998) considered that drilling and production costs have fixed and variable components that depend on the capacity of the platforms. The authors consider that the maximum capacity is equal to the largest value obtained in all time periods. In the model developed by Tsarbopoulou (2000) platform capacity was not considered and therefore variable costs are not included.

In summary, assumptions A2, A3, A5, A6 and A15 are those that could be treated in a more realistic way by introducing non linearities to the model.

Common methods of capital budgeting include net present value (NPV), i.e. the present value of cash inflow is subtracted from the present value of cash outflows. NPV compares the value of a currency unit today versus the value of that same currency unit in the future after taking inflation and return into account. If the NPV of a prospective project is positive then it should be accepted, otherwise the project probably should be rejected because cash flows are negative. In this context, the objective is to maximize the NPV.

The MILP model originally proposed by Tsarbopoulou (2000) maximizes NPV and it is reproduced below (also denoted as Model MO).

The objective function in Equation 1 includes the revenues of oil and gas defined as GAS and OIL, reduced by the drilling and connection costs, DR and CON, respectively.

$$\text{Max NPV=GAS+OIL-DR-CON} \tag{1}$$

s.t.

$$OIL = \sum_j \sum_t \left[F_{o,j,t} \times (APO_t - PCO) \times D_t \right] \tag{2}$$

$$GAS = \sum_j \sum_t \left[F_{g,j,t} \times (APG_t - PCG) \times D_t \right] \tag{3}$$

Equations 2 and 3 are related to the revenues from oil and gas that depend on annual oil and gas prices at time period t, APO_t and APG_t, respectively. These prices are subtracted from their production costs, PCO and PCG. Moreover, revenues depend on depreciation D_t. The general equation for depreciation D_t is the following:

$$D_t = \left(\frac{1 + INFLATION}{1 + INTEREST} \right)^{t-1} \qquad \forall t \tag{4}$$

Equations 5 and 6 are based on assumption A15 and are related to the drilling and connection costs, respectively.

$$DR = \sum_i (100 M_i + 10 \times \sum_j X_{i,j}) \times 10000 \tag{5}$$

$$CON = \sum_i \sum_j COST_{i,j} \times X_{i,j} \tag{6}$$

The cost depends directly on the assignment of the well to the platform where connection cost $COST_{i,j}$ between wells and platforms is the same as the one mentioned by Devine and Lesso (1972):

$$COST_{i,j} = 122.6 - 21.43 \times WD_j + 2.39 \times WD_j^2 + 12.24 \times H_{i,j} + 5 \times \left(\frac{H_{ij}}{WD_j} - 1.5\right)^2 \qquad \forall i,j \tag{7}$$

where WD_j is the depth of well j and $H_{i,j}$ is the horizontal distance between well j to be connected to platform i.

The horizontal distance shown in Equation 7 is a function of platform and well co-ordinates. Its corresponding equation may be written as:

$$H_{i,j} = \left((PX_i - WX_j)^2 + (PY_i - WY_j)^2 \right)^{\frac{1}{2}} \qquad \forall i,j \tag{8}$$

Process conditions are assumed to have linear behavior according to assumptions A3 and A6 and are represented as follows.

$$CUM_{s,t} = CUM_{s,t-1} + \sum_j F_{s,j,t} \qquad \forall s,t \tag{9}$$

$$P_t = 100 - 8.0 \times 10^{-6} \times CUM_{o,t} \qquad \forall t \tag{10}$$

Equation 9 states that the cumulative production of each substance (oil/gas) is the same as the cumulative production in the previous time period increased by an amount equal to the flow from all wells at the present time. Equation 10 states that the initial pressure of the reservoir is 100 bar and that it decreases linearly with accumulated production (in barrels).

$$FMAX_{o,j,t} = PI_j \times P_t \qquad \forall j,t \tag{11}$$

$$FMAX_{g,j,t} = PI_j \times (60 - 2.6 \times 10^{-6} \times CUM_{o,t}) \qquad \forall j,t \tag{12}$$

$$F_{s,j,t} \le FMAX_{s,j,t} \qquad \forall s,j,t \tag{13}$$

$$\sum_t F_{s,j,t} \le INVAL_{s,j} \qquad \forall s,j \tag{14}$$

Equations 11 and 12 are related to the maximum flow of production of the oil and gas in barrels, respectively. Equation 13 states that the flow of each substance from each well should not exceed the maximum production limits. Equation 14 enforces that the flow of all substances throughout the time horizon should not exceed their initial amounts.

$$a_{j,t} = a_{j,t-1} + \sum_i x_{i,j,t} \qquad \forall j,t \qquad (15)$$

Equation 15 states that a well is opened only once and remains open throughout the whole time horizon. Note also that $a_{j,0} = 0$ for every well j; in other words, the well will eventually be made available during the planning horizon.

$$F_{o,j,t} \le FOMAX \times a_{j,t} \qquad \forall j,t \qquad (16)$$

$$F_{g,j,t} \le FGMAX \times a_{j,t} \qquad \forall j,t \qquad (17)$$

$$F_{o,j,t} \ge FOMIN \times a_{j,t} \qquad \forall j,t \qquad (18)$$

$$F_{g,j,t} \ge FGMIN \times a_{j,t} \qquad \forall j,t \qquad (19)$$

Constraints 16 to 19 state that the oil and gas flow should not exceed upper and lower bounds. Furthermore, note that Equations 16 and 17 set the oil and gas flow rates to zero in case a well is not made available.

Logical Constraints 20 to 24 relate the decision variables from the model.

$$X_{i,j} = \sum_t x_{i,j,t} \qquad \forall i,j \qquad (20)$$

$$\sum_t Y_{i,t} \le M_i \qquad \forall i \qquad (21)$$

$$x_{i,j,t} \le \sum_t Y_{i,t} \qquad \forall i,j,t \qquad (22)$$

$$\sum_i \sum_t x_{i,j,t} \le 1 \qquad \forall j \qquad (23)$$

$$\sum_t x_{i,j,t} \le M_i \qquad \forall i,j \qquad (24)$$

Equation 20 states that a well is connected to a platform only if it has been connected to the same platform at one time period during the whole time horizon. Equation 21 enforces that every platform is installed at most once within the whole time horizon. Equation 22 states that if a well is connected to a platform during the whole time period, the corresponding platform has to be installed. Equation 23 enforces that a well is connected to a platform at most once. Equation 24 states that a well is connected to a platform only if the same platform was allocated.

REFORMULATED MODEL

Problem MR corresponds to a reformulation of the model proposed by Tsarbopoulou (2000) and presented in the previous section. The main difference between both models relies on the representation of the binary decision variables.

Model MR

Tsarbopoulou (2000) considered five sets of binary variables. The first set assigns wells to platforms ($X_{i,j}$), the second and third represent the selection and the timing of platforms (M_i and $Y_{i,t}$), fourth the availability of wells ($a_{j,t}$) and the last one relates wells to platforms at every time period ($x_{i,j,t}$). The reformulated model (MR) contains only the last three sets of variables, which is sufficient to model the discrete decisions of the problem.

MR:
Max NPV=GAS+OIL-DR-CON (1)
s.t.
constraints (2) and (3)
(9) to (17)
(23) and (24)

$$DR = \sum_i (100 \times M_i + 10 \times \sum_j \sum_t x_{i,j,t}) \times 10000 \qquad (25)$$

$$CON = \sum_i \sum_j \sum_t COST_{i,j} \times x_{i,j,t} \qquad (26)$$

$$F_{s,j,t} \ge 0 \qquad \forall s,j,t \qquad (27)$$

Note that Constraints 18 and 19 were eliminated because the lower bounds for the flow rates (FOMIN and FGMIN) are set to zero. On the other hand, non negativity constraints for the flow rates are imposed in the model in 27.

Model MO uses variable $X_{i,j}$ to connect platform i to well j and $x_{i,j,t}$ to connect platform i to well j at time t. Furthermore, variables $Y_{i,t}$ denote the time t at which platform i is installed. All these decisions can be represented by $x_{i,j,t}$. Therefore, Equations 20 to 22 are unnecessary and Equations 5 and 6 are transformed into Equations 25 and 26, respectively.

RESULTS

In this section we present in detail a case study as the one introduced by Tsarbopoulou (2000) that provides a comparison between MO and the proposed model MR. For this case, 16 platforms and 30 wells are considered for a horizon of 10 years that is divided into equal time periods of 1 year each. In this problem, a rectangular oilfield of 10,000 ft by 15,000 ft is considered (Figure 2). Upper production limits of oil and gas in each well are 1,250,000 ft^3 and 875,000 ft^3, respectively.

Interest and inflation rates were set by Tsarbopoulou (2000) to 15% and 3%, respectively. Data regarding productivity indexes (PI), initial amounts of substances (oil and gas), the coordinates in the field, and depth (WD) from each well are given in Table 1.

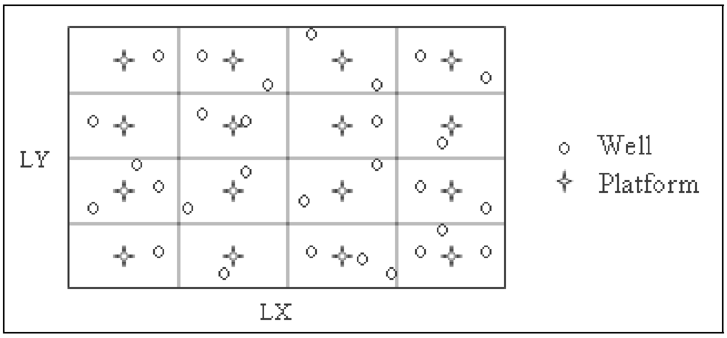


Figure 2: Configuration of field

Table 1: Data for each well

j	WX (ft)	WY (ft)	WD (ft)	PI $\left(\frac{\text{barrel}}{\text{yr.bar}}\right)$	INVAL (10^5 barrel/year) Oil	INVAL (10^5 barrel/year) Gas
1	5336	1183	6.27	1840	8.5	5.95
2	6136	4283	5.26	2000	11.0	7.70
3	6338	6640	5.34	1760	12.0	8.40
4	12911	1082	5.61	1920	9.5	6.65
5	4528	8700	5.92	1980	10.0	7.00
6	10862	8990	5.16	1680	10.5	7.35
7	9683	4679	5.42	1620	8.0	5.60
8	2716	2677	5.11	1629	9.0	6.30
9	8808	4510	5.82	1740	10.0	7.00
10	6007	5702	5.66	1940	11.5	8.05
11	2999	6058	5.00	1840	8.5	5.95
12	13090	2313	6.22	2000	11.0	7.70
13	13855	5889	6.25	1760	12.0	8.40
14	7713	6440	4.90	1920	9.5	6.65
15	4369	2773	5.59	1980	10.0	7.00
16	10260	8099	5.26	1680	10.5	7.35
17	11416	4973	6.03	1620	8.0	5.60
18	6648	3866	5.17	1629	9.0	6.30
19	9834	3451	5.57	1740	10.0	7.00
20	8006	3679	5.73	1940	11.5	8.05
21	12096	2913	4.88	1840	8.5	5.95
22	7000	7869	4.58	2000	11.0	7.70
23	3477	1774	5.78	1760	12.0	8.40
24	9153	3104	6.08	1920	9.5	6.65
25	617	1034	4.76	1980	10.0	7.00
26	1071	3328	5.06	1680	10.5	7.35
27	4095	1249	5.06	1620	8.0	5.60
28	7440	9979	5.98	1629	9.0	6.30
29	7155	9232	6.29	1740	10.0	7.00
30	1095	7980	6.36	1940	11.5	8.05

The problem is solved to illustrate the performance of the models and of the solution strategy. The MILP problems were modeled using GAMS (Brooke et al., 1998) and solved in full space using the LP-based branch and bound method implemented in the CPLEX solver (ILOG, 1999).

The reformulated model (MR) presented better computational performance with respect to the original model (MO) proposed by Tsarbopoulou (2000), as shown in Table 2 that presents the CPU times obtained for a problem with 16 platforms as a function of the number of wells (NW). Interestingly, the integrality gap is the same for both models and increases with problem size.

Table 3 presents the sizes of MO and MR, such as the number of single equations (SE), the number of continuous variables (SV) and the number of discrete variables (DV) for several numbers of wells (NW) and 16 platforms. Note from Table 3 that there is a linear increase in the number of equations as well as in the binary and continuous variables with the increment of the number of wells.

It is important to note that the two models in principle might not have the same integrality gap because they are based on different formulations (Williams, 1999). In that respect, modeling is largely an art that has a large impact in mixed-integer programming (Biegler et al., 1997) and computational experiments are necessary to test and compare formulations. Solution performance depends on several factors such as model size (constraints, binary variables and continuous variables) and model formulation. In this particular case, the former played the most significant role in reducing computational effort.

Figure 3 illustrates the computational time for MO and MR for different numbers of wells and 16 platforms and shows that the latter is smaller than the one for MO under any configuration. However, the computational effort presents a non-linear behavior with the number of wells.

Table 2: Computational performance of the models

NW	CPU time (s)		Gap
	MO	MR	(%)
05	0.9	0.7	0.10
10	2.6	1.8	0.10
15	6.4	4.0	0.10
20	23.8	17.7	0.13
25	269.5	179.0	0.17
30	8473.953	7605.562	0.18
35	*	*	-

* No integer solution obtained after 18,000 CPU s.

Table 3: Dimensions of MO and MR

NW	MO			MR		
	SE	SV	DV	SE	SV	DV
05	1486	295	1056	490	295	816
10	2911	545	1936	935	545	1616
15	4336	795	2816	1380	795	2416
20	5761	1045	3696	1825	1045	3216
25	7186	1295	4576	2270	1295	4016
30	8611	1545	5456	2715	1545	4816

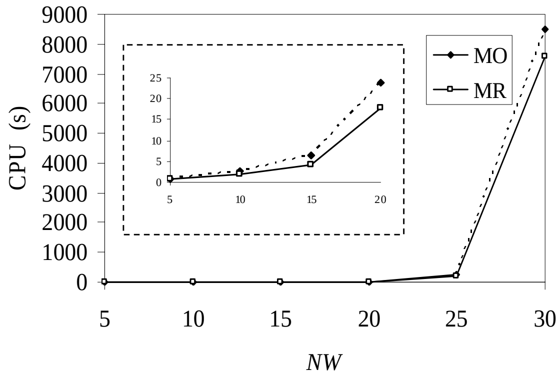


Figure 3: CPU times for MO and MR

Sensitivity Analysis

A sensitivity analysis of the main model parameters was performed. Figures 4 to 6 show the relative change of the objective function (NPV) with respect to the base case the parameters in the -20% to +20% range.

Figure 4 shows the results for the main parameters that impact the objective function that are INTEREST, INFLATION, PCO and PCG. Note that the first two parameters directly affect depreciation. The ones that most significantly impact NPV are PCO and INTEREST.

Figure 5 presents the influence of FOMAX, FGMAX and INVAL. It can be seen that parameter INVAL has a small impact on the objective function, whereas there is no sensitivity of NPV on the two others because constraints (16) to (19) are not active at any time period.

The main purpose of Figure 6 is to verify the influence of the parameters that take part in the pressure constraint (Equation 10). These parameters are α and β denote the intercept and the slope of Equation 10 (values 100 and 8.0×10^{-6} in the base case), respectively. Hence, it can be observed from Figure 6 that the initial pressure has a stronger influence than the decrease rate.

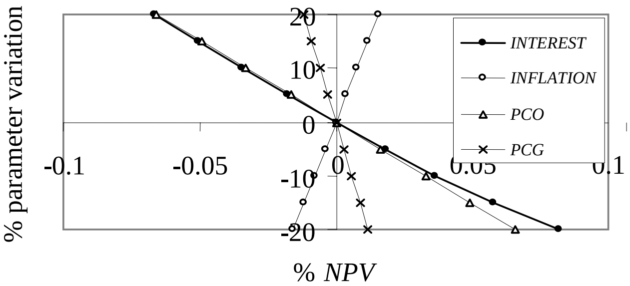


Figure 4: Sensitivity of the objective function parameters

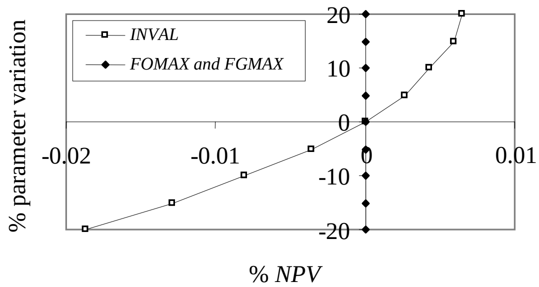


Figure 5: Sensitivity of FOMAX, FGMAX and INVAL

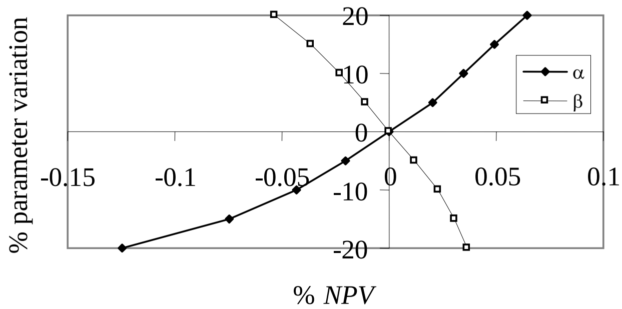


Figure 6: Sensitivity of the pressure constraints

BILEVEL DECOMPOSITION APPROACH

From the results of the previous section it becomes clear that neither model can efficiently solve problems of larger sizes if MILP solvers were to tackle them in full-scale. Therefore, a decomposition approach is applied to model MR.

Iyer and Grossmann (1998) proposed a two-level decomposition approach for the planning of process networks. Van der Heever and Grossmann (2000) then applied this technique to an oilfield infrastructure-planning model. In this section, a similar approach is applied to the reformulated model MR. The resulting model is denoted as MD that is decomposed into two subproblems: the master subproblem that solves a model that assigns platforms to wells (problem AP) and the timing subproblem (problem TP). The latter relies on the assignments that are obtained in the master subproblem and decides on when to install the platforms. The decomposition algorithm as applied to model MR can be seen in Figure 7. In Figure 7, design cuts correspond to Constraints 35 to 37 that are described in item 5.1. The proposed technique is similar to the one proposed by Van den Heever and Grossmann (2000), which however have considered non convex nonlinearities in the sub-problem and therefore could not guarantee global solutions.

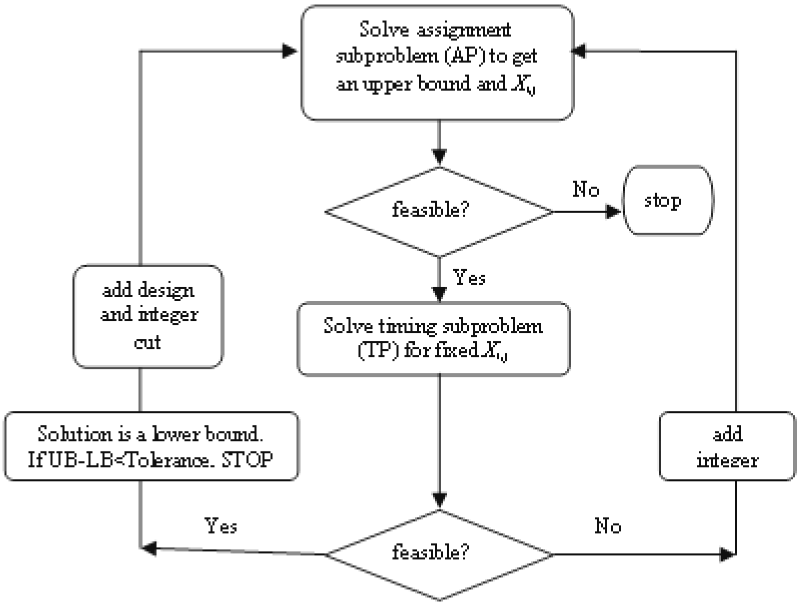


Figure 7: Bilevel decomposition algorithm.

Model MD

The model is solved iteratively such that the two MILP sub-problems AP and TP are optimized in each iteration r.

The assignment problem (AP) is defined as follows:

max NPV=GAS+OIL-DR-CON (1)

s.t.

constraints (2) and (3)
(5) and (6)
(9) to (14) and 27

$$A_j = \sum_i X_{i,j} \qquad \forall j \tag{28}$$

$$F_{o,j,t} \le FOMAX \times A_j \qquad \forall j,t \tag{29}$$

$$F_{g,j,t} \le FGMAX \times A_j \qquad \forall j,t \tag{30}$$

$$\sum_i X_{i,j} \le 1 \qquad \forall j \tag{31}$$

$$X_{ij} \le M_i \qquad \forall i,j \tag{32}$$

In (28), variable A_j is assigned to one when a well is opened. Note that the availability of the well is no longer associated to time and that the assignment variable $X_{i,j}$ originally defined by Tsarbopoulou (2000) is introduced.

In (29) and (30), the flow rates of both oil and gas should never be above specific limits FOMAX and FGMAX, respectively.

In (31) it is clear that both wells and platforms are connected only once within the horizon. Furthermore, Equation 32 states that one well is connected to a platform only if this was installed.

The solution of AP provides values for $X_{i,j}$. If this variable is fixed, denoted by $\overline{X_{i,j}}$, a feasible solution for TP is a feasible solution for MR and generates a lower bound for this problem.

The timing problem (TP^R), at iteration R, is defined as follows:

max NPV=GAS+OIL-DR-CON (1)

s.t.

constraints (2) and (3)
(9) to (17)
(23) to (27)

$$x_{i,j,t} \le \overline{X_{i,j}} \qquad \forall i,j,t \tag{33}$$

$$a_{j,t} \le \overline{A_j} \qquad \forall j,t \tag{34}$$

Similarly to Iyer and Grossmann (1998), Constraints 33 and 34 select a subset of assignments for the planning problem.

The following are the constraints (design cuts) used in the AP model in the algorithm to avoid subsets and supersets that would result in suboptimal solutions:

$$\sum_{(n1,n2)\in Z_1^r} X_{n1,n2} + X_{i,j}^r \le \left|Z_1^r\right| \quad \forall (i,j)\in Z_0^r, r=1...R \tag{35}$$

$$\sum_{(n1,n2)\in Z_0^r} X_{n1,n2} + X_{i,j}^r \ge 1 \quad \forall (i,j)\in Z_1^r, r=1...R \tag{36}$$

$$\sum_{(i,j)\in M_r} \overline{X_{i,j}^r} - \sum_{(i,j)\in N_r} \overline{X_{i,j}^r} \le \left|M_r\right| - 1 \quad r=1...R \tag{37}$$

where

$$M_r = \left\{(i,j) / \overline{X}_{i,j}^r = 1 \text{ for configuration in iteration } r\right\}$$

$$N_r = \left\{(i,j) / \overline{X}_{i,j}^r = 0 \text{ for configuration in iteration } r\right\}$$

$$Z_1^r = \left\{(i,j) / \overline{X_{i,j}^r} = 1\right\}$$

$$Z_0^r = \left\{(i,j) / \overline{X_{i,j}^r} = 0\right\}$$

Similarly to Iyer and Grossmann (1998), Equation 35 states that if in any solution all the $\overline{X_{i,j}}$ variables in any set Z_1^r are 1, then all remaining variables must be zero in order to prevent a superset of Z_1^r from entering the solution of AP. Equation 36 shows cuts for precluding subsets of Z_1^r. Equation 37 has the effect of establishing the basis for deriving integer cuts on supersets and subsets of the configurations predicted by the assignment problem. This property of supersets and subsets is the basis for deriving the integer cuts. Note that the design cuts (35) to (37) accumulate along the iterations.

Bounding Properties of DP

A very important property of the decomposition strategy is that AP represents a rigorous upper bounding problem to TP. This can be verified by comparing the feasible region of the two problems. Note that all constraints of AP are also present in TP, with exception of (5), (6), and (28) to (32). Firstly, it is important to establish the relationship between the logical (binary) variables of AP – X_{ij} and A_j – and those of TP, given by $x_{ij,t}$ and $a_{j,t}$. The variables that define the connection of platform i to well j are related in (20). The variables that define the availability of wells are related in (38).

$$X_{i,j} = \sum_t x_{i,j,t} \quad \forall i,j \tag{20}$$

$$A_j = \sum_t \left(a_{j,t} - a_{j,t-1}\right) \quad \forall j \tag{38}$$

By definition, A_j is one when the well is made available during any time period within the time horizon. This is verified by examining variables $a_{j,t}$ at every two consecutive time periods. If the well is made available at time t, then $a_{j,t}$ - $a_{j,t-1}$ = 1; otherwise, the difference is zero (the well is not available or it was made available at a previous period, in which case $a_{j,t} = a_{j,t-1} = 1$).

Regarding the constraints of AP that do not belong to TP, Equations (5) and (6) are obtained by substituting (20) into (25) and (26), respectively. Constraint (28) can be obtained as follows. Consider equation (15):

$$a_{j,t} = a_{j,t-1} + \sum_i x_{i,j,t} \quad \forall j,t \tag{15}$$

Rearranging (15) and summing over t:

$$\sum_t \left(a_{j,t} - a_{j,t-1}\right) = \sum_t \sum_i x_{i,j,t} \quad \forall j \tag{39}$$

Using (38) and (20) yields Equation (28):

$$A_j = \sum_i X_{i,j} \quad \forall j \tag{28}$$

Constraints (29) and (30) represent relaxations of constraints (16) and (17), respectively; this can be shown by the relationship between A_j and $a_{j,t}$ given in (34). Moreover, constraints (31) and (32) are obtained by replacing (20) into (23) and (24), respectively.

Results

From Table 2 it is clear that models MO and MR are unable to solve problems with more than 35 wells, despite a relatively small integrality gap verified for the smaller instances. Nevertheless, when MR is subject to the decomposition strategy proposed in the previous section (denoted as MD), the computational gain is remarkable. The CPU times obtained for a problem with 16 platforms as a function of NW are compared to those from MR in Figure 8.

Figure 8 also illustrates the computational time for MD for different numbers of wells and platforms, ranging from 16 and 25.

Table 4 presents the corresponding sizes of problem MD, for several values of the number of wells. SV and DV are maintained at each iteration, whereas there is an average increase of 20% in the number of equations from iteration 1 to 2, due to the cut generation step.

It can be seen from Table 4 that the reduction in the number of discrete variables (DV) in MD is not significant with respect to MR. However the introduction of Constraints (33) and (34) greatly reduces the search space and therefore the computational effort.

The optimal values obtained with MO, MR and MD are the same for all cases and only 3 subproblems are required for MD for all instances.

The well-platform assignments obtained for MD are given in Table 5. Note that, besides the objective function value, the decision variables are expressed by platform (wells). For the sake of illustration, ten iterations of the algorithm are shown in Table 5. It is important to note however that the algorithm converges in a single iteration.

Note that Constraints 35 to 37 do not allow the repetition of assignments neither the generation of sub and supersets. In this sense, there is no significant change in the allocation obtained in AP in consecutive iterations. For instance from the first to the second iteration, the only modification is the allocation of well 9 to platform 11 in place of the assignment of well 9 to platform 10.

Note also from Table 5 that all platforms are installed and connected to the wells in the first time period, since no investment constraints are imposed in the model. Therefore assumption A15 does not represent any simplification to the model. Results in Table 5 show that there is a more significant decrease in NPV in the first four iterations. Finally, 29 allocations are made for all iterations.

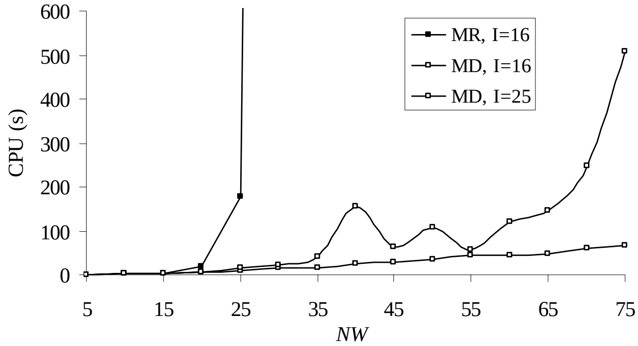


Figure 8: Computational performance for MR and MD

Table 4: Size of problem MD

NW	Sub problem	1st iteration				
		SE	SV	DV	CPU (s)	NPV
5	AP	445	245	101	1.5	5.0969×10^{7}
	TP	1265	245	866		
10	AP	845	445	186	2.0	8.6692×10^{7}
	TP	2485	445	1716		
15	AP	1245	645	271	2.8	1.0378×10^{8}
	TP	3705	645	2566		
20	AP	1645	845	356	7.5	1.0989×10^{8}
	TP	4925	845	3416		
25	AP	2045	1045	441	9.1	1.1438×10^{8}
	TP	6093	1045	4266		
30	AP	2445	1245	526	15.5	1.1840×10^{8}
	TP	7313	1245	5116		
35	AP	2845	1445	611	15.3	1.2312×10^{8}
	TP	8585	1445	5966		

Table 5: Assignments for 10 iterations of MD

Iterations (r)	NPV (10^8)	allocation $X_{i,j}$ j (i| $X_{i,j}$=1)
01	1.1840	2(3,6,7,11,14,23); 3(12,13,15,19); 4(4,10); 9(1,8,22,24,29,30); 10(2,5,9,20); 11(25,28); 13(18,27); 15(17,21); 16(16)
02	1.1835	2(3,6,7,11,14,23); 3(12,13,15,19); 4(4,10); 9(1,8,22,24,29,30); 10(2,5,20); 11(9,25,28); 13(18,27); 15(17,21); 16(16)
03	1.1828	1,26; 2(3,6,7,11,14,23); 3(12,13,15,19); 4(4,10); 9(1,8,22,24,29,30); 10(2,5,9,20); 11(25,28); 13(18,27); 15(17,21)
04	1.1823	1,26; 2(3,6,7,11,14,23); 3(12,13,15,19); 4(4,10); 9(1,8,22,24,29,30); 10(2,5,20); 11(9,25,28); 13(18,27); 15(17,21)
05	1.1813	2(3,6,7,11,14,23); 3(12,13,15,19); 4(4,10); 9(1,22,24,29,30); 10(2,5,9,20); 11(25,28); 13(8,18,27); 15(17,21); 16(16)
06	1.1810	2(3,6,7,11,14,23); 3(12,13,15,19); 8(4,10); 9(1,8,22,24,29,30); 10(2,5,9,20); 11(25,28); 13(18,27); 15(17,21); 16(16)
07	1.1808	2(3,6,7,11,14,23); 3(12,13,15,19); 4(4,10); 9(1,22,24,29,30); 10(2,5,20); 11(9,25,28); 13(8,18,27); 15(17,21); 16(16)
08	1.1806	2(3,6,7,11,14,23); 3(12,13,15,19); 8(4,10); 9(1,8,22,24,29,30); 10(2,5,20); 11(9,25,28); 13(18,27); 15(17,21); 16(16)
09	1.1806	1,26; 2(3,6,7,11,14,23); 3(12,13,15,19); 4(4,10); 9(1,8,22,24,29,30); 10(2,5,20); 11(25,28); 13(18,27); 15(17,21); 16(16)
10	1.1805	2(3,6,7,11,14,23); 3(12,13,15,19); 4(4,10); 9(1,8,22,24,29,30); 10(2,5,9,20); 11(25,28); 13(18,27); 15(17,21); 16(16)

DECOMPOSITION APPROACH USING HEURISTIC

From Figure 9 it becomes clear that most of the computational effort lies on the solution of AP due to the large combinatorial aspect that results from allocating wells to platforms. As solving AP is intrinsically related with the well-platform connection, it becomes interesting to limit some of the possible connections.

According to Grimmett and Starzman (1988), it is common the use of the constraint that enforces a maximum horizontal distance (radius) that a well can be drilled from a fixed surface location. This constraint is important because it limits the number of wells that can be directionally drilled from a platform. Such type of constraint can be included in the mathematical programming formulation of the location-allocation problem, as illustrated in Figure 10.

Model MH

Models MO, MR and MD consider all associations between wells and platforms. However, Equation 7 shows connection costs that are directly related to the horizontal distance between well and platform.

Figure 10 illustrates how the wells and platforms are distributed, as suggested by assumptions A9 and A10. The area of the field is divided in N^2 smaller rectangles of equal size and in each of them the number of potential platforms allocated to each well at every time period.

A constraint that assigns zero values to the $X_{i,j}$ variable when the distance is larger than the smallest distance of the smaller rectangles of the field is added to model MD. So, MD, now denoted MH, has a smaller number of possible associations between wells and platforms, and therefore may be able to solve the problem that includes heuristics with lower computational effort.

In order to evaluate the impact of the heuristics in the model, two limiting values of association between wells and platforms were tested. The radius is determined by the horizontal distance that a well can be drilled from a fixed surface and two connection limits are defined as follows: for MH1 the maximum horizontal distance is given by the maximum between LY/N and LX/N, whereas for MH2, the radius is defined as the one that connects the center to the vertices of the small rectangle (Figure 10).

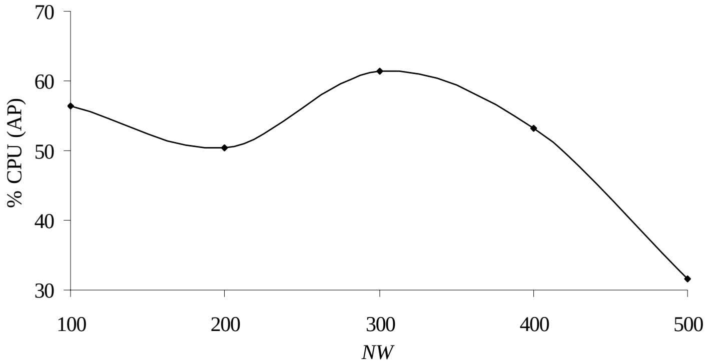


Figure 9: Percentage of computational time to solve AP

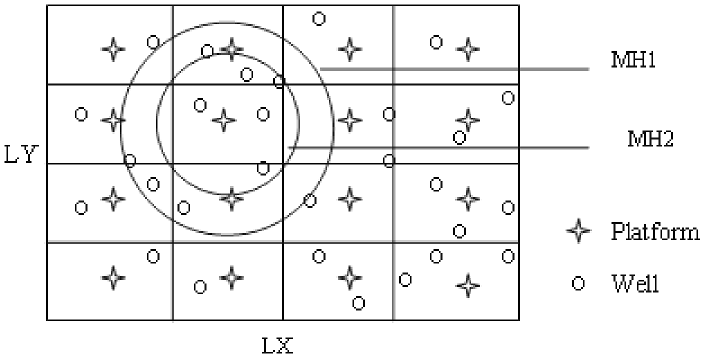


Figure 10: Configuration of field with heuristic

Results

Values of LX and LY represent the dimensions of the field, and originally admit values of 15,000 ft and 10,000 ft, respectively. The value of N as well as the size of the field were increased with the objective of analyzing the effect of problem dimension in the computational effort. Values are shown in Table 6.

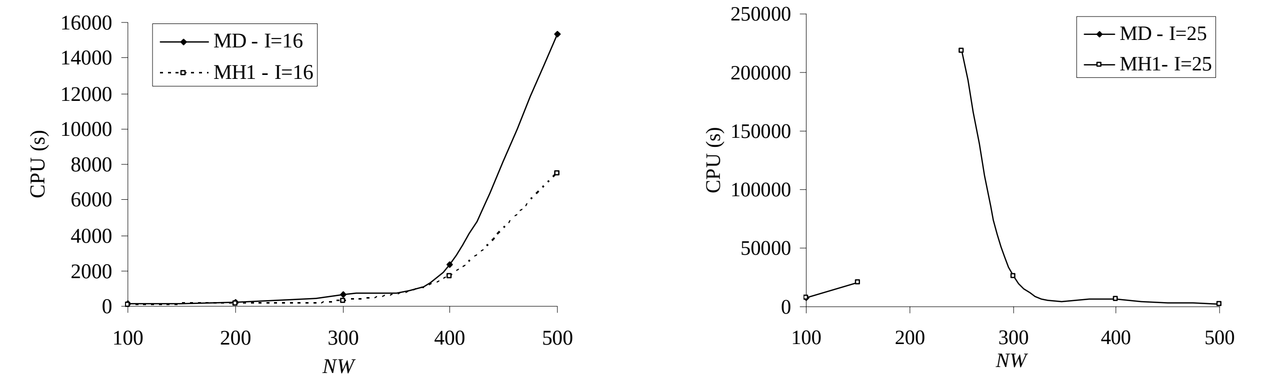
Figures 11a and 11b illustrate results for MD and MH1 for different numbers of wells and platforms.

Attributions well-platform along time for MH1, considering 30 wells and 16 platforms are the same as the ones for model MD as well as the value of the objective function found. However, CPU time was 2.4 seconds that is 35% smaller than that requested by MD and 60% smaller than required from MO. Note also that in Figure 11b only the case with 100 wells could be solved when 25 potential platforms are defined.

Figures 12a and 12b illustrate the impact of the heuristic on the models MILP. The same value for the objective function was found in all models. However, model MH1 requires a larger computational time than model MH2. This result should be expected because essentially the radius determines the search area. In this sense, the heuristics that uses a smaller radius tends to obtain results with smaller computational effort. As the radius used in MH1 and MH2 are similar, their computational time is also similar. The radius utilized in MH1 was chosen in order to cover the smallest rectangle. In other words, to allow connection of the platform location in the center of the rectangle to all potential wells in this rectangle. The radius utilized in MH2 was chosen in order to evaluate the search in relation the MH1 and to verify how the radius could influence the computational time.

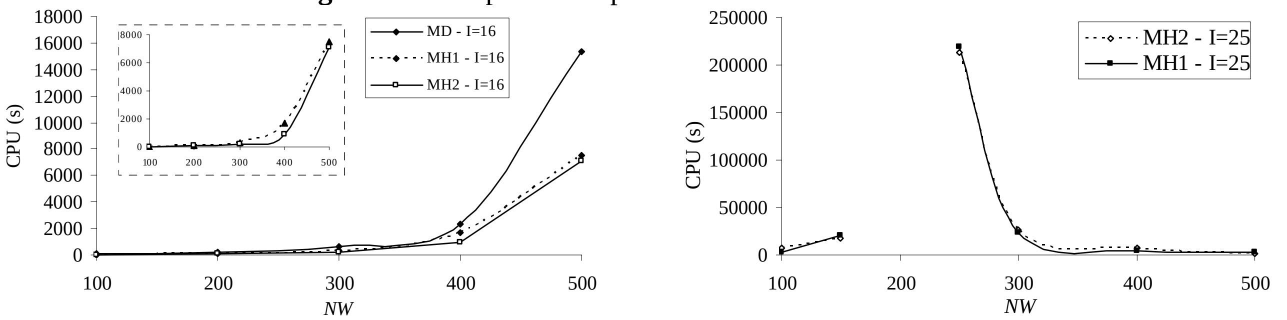
Table 6: Size of the field

N	WP	LX (ft)	LY (ft)
4	16	15,000	10,000
5	25	23,438	15,625
6	36	33,750	22,500
7	49	45,938	30,625
8	64	60,000	40,000



(a) (b)

Figure 11: Computational performance for MD and MH1



(a) (b)

Figure 12: Computational performance for MH1 and MH2

According to Figures 11b and 12b, the model cannot be solved to global optimality for 200 wells and 25 platforms. Several values of the relative optimality criterion (OPTCR) were used in the range of 1 to 10% in model MH2. OPTCR is defined as (best estimate-best integer)/ best estimate, in which "best integer" is the best solution that satisfies all integer requirements found so far and "best estimate" provides a bound for the optimal integer solution. Table 7 and Figure 13 show the results and compare the objective function and computational time for each criterion. Despite different values of OPTCR, the resulting objective function value is the same.

In order to evaluate the impact of the optimality criterion for another instance of the problem, results were optimized initially for the case in which an optimal solution was obtained (100 wells and 25 platforms). Results are represented by Figure 14 (also shown in Table 7). Note that the computational time presents a significant increase when the optimally criterion is smaller than 1%, and the corresponding objective function presented a small increase. The computational time to non null optimality criteria is smaller for all cases, except for 500 wells. For 300 wells and optimality criterion 1%, the decrease in CPU is 95% and represents a reduction in the objective function of only 0.5% in relation to the optimal solution.

Table 7: Computational time for 25 platforms

NW	OPTCR	Objective Function	CPU (s)
100	10	1.4313×10^8	98. 7
	5	1.4313×10^8	111.8
	2.5	1.4556×10^8	121.8
	1	1.4490×10^8	159.6
	0.5	1.4536×10^8	1896.9
	0	1.4610×10^8	3462.1
200	10	1.7422×10^8	217.3
	5	1.7422×10^8	185.2
	2.5	1.7422×10^8	269.9
	1	1.7372×10^8	286.9
300	1	1.9498×10^8	650.3
	0	1.9589×10^8	23577.5
400	1	2.1428×10^8	1045.8
	0	2.1582×10^8	3849.0
500	1	2.3303×10^8	1055.9
	0	2.3430×10^8	2298.2

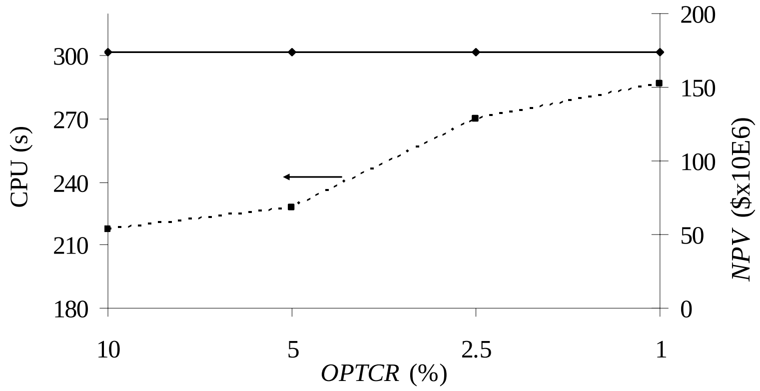


Figure 13: Computational time and objective function for 25 platforms and 200 wells

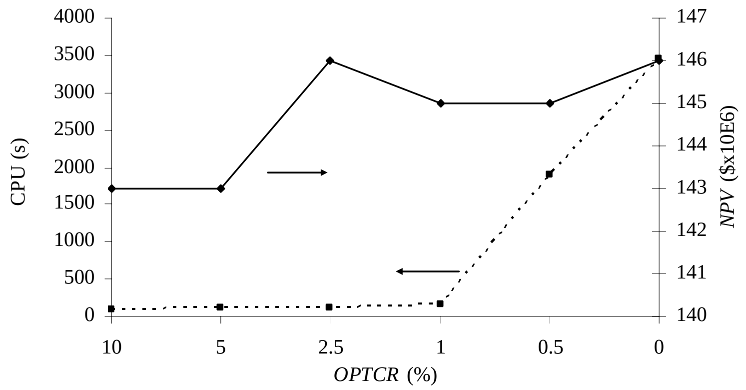


Figure 14: Computational time and objective function for 25 platforms and 100 wells

CONCLUSIONS

This paper addressed the long term planning of the oilfield infrastructure. Firstly, we proposed a reformulated MILP that presents a significant reduction in the number of discrete variables for the same relaxation gap with respect to the model developed by Tsarbopoulou (2000). Moreover, a bilevel decomposition approach that relies on the disaggregation of the assignment and timing decisions in analogy to the one proposed by Iyer and Grossmann (1998) has been presented. Results show that computational performance is greatly improved, whereas global optimality is guaranteed. Problems of 25 platforms and 400 wells are efficiently solved for a 10-year horizon. Finally, heuristics that limit the assignment of platforms to wells were proposed. Results show that gains of up to 86% in CPU time were obtained with the addition of the heuristic rule without compromising the solution quality.

ACKNOWLEDGMENT

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NOMECLATURE

Indices

g	gas
i	platform
j	well
o	oil
s	substance (gas or oil)
t	time period

Parameters

APO_t	annual oil price at time period t
APG_t	annual gas price at time period t
$COST_{i,j}$	cost between wells and platforms
D_t	depreciation at time period t
$H_{i,j}$	horizontal distance between well j to be connected to platform i
FGMAX	upper bound of gas flow
FGMIN	lower bound of gas flow
FOMAX	upper bound of oil flow
FOMIN	lower bound of oil flow
INFLATION	inflation rate
INTEREST	interest rate
$INVAL_{s,j}$	initial value for substance s in

	well j
LX	x-dimension of the oilfield
LY	y-dimension of the oilfield
N	number of grids in which each distance is divided (N^2 rectangles are formed)
NW	overall number of wells
PCG	annual production costs for gas
PCO	annual production costs for oil
PI_j	productivity index for well j
PX_i	x co-ordinate of platform i
PY_i	y co-ordinate of platform i
$Q_{s,t}$	upper production limit for substance s at time period t
WD_j	depth of well j
WX_j	x co-ordinate of well j
WY_j	y co-ordinate of well j

Continuous Variables

A_j	availability of well j
CON	connection cost
$CUM_{s,t}$	cumulative production of substance s up to time period t
DR	overall drilling cost
$F_{s,j,t}$	flow rate of substance s from well j during time period t
$FMAX_{s,j,t}$	maximum flow of substance s from well j at time period t
GAS	revenues of gas
NPV	objective function variable
OIL	revenues of oil
P_t	pressure of all wells at time period t

Binary Variables

$a_{j,t}$	availability of well j at time period t
M_i	existence of platform i
$x_{i,j,t}$	connection of platform i to well j at time period t
$X_{i,j}$	connection of platform i to well j
$Y_{i,t}$	time period t at which platform i is installed

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