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# NATURAL CONVECTION HEAT TRANSFER IN PARTIALLY OPEN ENCLOSURES CONTAINING AN INTERNAL LOCAL HEAT SOURCE

V. C. Mariani<sup>1\*</sup> and L. S. Coelho<sup>2</sup>

<sup>1</sup>Pontifical Catholic University of Paraná, Graduate Program in Mechanical Engineering, PUCPR/CCET/PPGEM, Imaculada Conceição 1155, CEP: 80215-901, Curitiba PR, Brazil E-mail: viviana.mariani@pucpr.br

<sup>2</sup>Pontifical Catholic University of Paraná, Graduate Program in Industrial Systems and Engineering, Automation and Systems Laboratory, PUCPR/CCET/PPGEPS, Imaculada Conceição 1155, CEP: 80215-901, Curitiba - PR, Brazil. E-mail: leandro.coelho@pucpr.br

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**Abstract** - A numerical study was conducted to investigate steady heat transfer and flow phenomena of natural convection of air in enclosures, with three aspect ratios (H/W = 1, 2, and 4), within which there is a local heat source on the bottom wall at three different positions,  $W_h$ . This heat source occupies 1% of the total volume of the enclosure. The vertical walls in the enclosures are insulated and there is an opening on the right wall. The natural convection is influenced by the difference in temperature between the left and right walls, represented by a Rayleigh number (Ra<sub>e</sub>), and by local heat source, represented by a Rayleigh number (Ra<sub>i</sub>). Numerical simulations were performed for several values of the Rayleigh number ranging between  $10^3$  and  $10^6$ , while the intensity of the two effects – the difference in temperature on the vertical walls and the local heat source – was evaluated based on the Ra<sub>i</sub>/Ra<sub>e</sub> ratio in the range between 0 and 2500. The analysis proceeds by observing variations in the streamlines and isotherms with respect to the different Ra<sub>e</sub>, R ratios, aspect ratios, of the radius and positions of the local heat source. The average Nusselt numbers on the hot and cold walls are influenced by different values of the parameters R, Ra<sub>e</sub>, W<sub>h</sub>, and H/W. Results show the presence of different flow patterns in the enclosures studied. Thus, the flow and heat transfer can be controlled by external heating, and local heat source.

Keywords: Natural convection; Average Nusselt number; Rectangular enclosures; Numerical study; heat source.

# **INTRODUCTION**

In recent years, numerical modeling of the convective heat transfer problem has been an area of great interest due to its broad applications in engineering. Compared to the experimental method, numerical analysis provides a more direct way to enhance/reduce heat transfer effectively so as to improve the performance or to optimize the structure of a thermal device.

Natural convection in enclosures has been studied both experimentally and numerically, due to the considerable interest in its many engineering applications, such as building insulation, solar energy collection, cooling of heat-generating components in the electrical and nuclear industries, and flows in rooms due to thermal energy sources (Yang, 1987).

Numerical studies of natural convection heat transfer and flow in closed enclosures without a local heat source are reported in the literature; we can cite the work of Davis (1983), Hortmann et al. (1990), Le Quéré (1991), Mohamad (1998), Corcione (2003), and Ben-Nakhi and Chamkha (2006).

<sup>\*</sup>To whom correspondence should be addressed

Other authors have studied the natural convection caused by a heat-generating conducting body located inside an enclosure: Chu and Churchill, 1976; Khalilollahi and Sammakia, 1986; Keyhani et al., 1988; Farouk, 1988; Ho and Chang, 1994; Ha et al., 1999; Deng and Tang, 2002; Oztop et al., 2004; Bazylak et al., 2006.

Numerous studies on natural convection caused only by external heating in partially open enclosures have been conducted by Chan and Tien, 1985; Angirasa et al., 1995; Polat and Bilgen, 2002; Bilgen and Oztop, 2005; Lauriat and Desrayaud, 2006. However, few results have been reported for natural convection caused simultaneously by both external heating in partially open enclosures and an internal local heat sources although problems of this type are frequently important and their study is necessary for understanding the performance of complex natural convection flow and heat transfer.

Indirectly related to the present study, Xia and Zhou (1992) studied a square and partially open enclosure with an internal heat source. These authors change the position on the bottom wall or left vertical wall for only three R ratios. They found that the opening was advantageous to the flow and heat transfer in the cavity. In this case, the characteristics of flow and heat transfer changed with heat source location, external and internal Rayleigh number, and opening size. Reinehr et al. (2002) examined natural convection using the aspect ratio H/W = 2, with an internal heat source whose position was varied only on the bottom wall. In that work, no heat transfer results were reported and a limited number of Rae and R ratios were also studied.

The present work is a numerical study of natural convection due to the temperature difference between left and right walls and an internal local heat source in three partially open enclosures, for which few results have been reported in the literature. The enclosures have an opening in the cooled right vertical wall, while the left vertical wall is heated and the upper and lower walls are adiabatic. Natural convection is induced by the difference in temperature between the vertical walls, and it is represented by the Rayleigh number (Ra<sub>e</sub>) and by an internal local heat source represented by the Rayleigh number (Ra<sub>i</sub>), occupying approximately 1% of the enclosure volume.

The study is conducted numerically under the assumption of steady laminar flow for three different values of both the height-to-width aspect ratio of the enclosure of 1, 2, and 4 and the Rayleigh number based on enclosure height in the range between  $10^3$  and  $10^6$ . The Ra<sub>i</sub>/Ra<sub>e</sub> ratio in the range between 0 and

2500 and the internal local heat source position at  $W_h = 0.25$ , 0.5, and 0.75 on the bottom wall are evaluated. In this context, the influence on flow patterns, temperature distributions and heat transfer rates is analyzed and discussed.

#### MATHEMATICAL FORMULATION

To model the flow under study, we use the conservation equations for mass, momentum, and energy for the two-dimensional, steady, and laminar flow. For the moderate temperature difference considered in this work, all the physical properties of the fluid,  $\mu$ , k, and  $c_p$ , are considered constant except density, in the buoyancy term, which obeys the Boussinesq approximation. In the energy conservation equation, we neglect the effects of compressibility and viscous dissipation. Thus, the dimensionless equations that govern the flow are

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right)$$
(2)

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + Ra_e Pr\theta \quad (3)$$

$$U\frac{\partial\theta}{\partial X} + V\frac{\partial\theta}{\partial Y} = \left(\frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2}\right) + R \tag{4}$$

Definitions of the dimensionless parameters are listed in the Nomenclature section. The fluid in the interior environment is atmospheric air with the Prandtl number, Pr=0.71 (air is the working fluid). The Rayleigh number ( $Ra_e$ ) is represented by the difference in temperature between the vertical walls,  $10^3 \le Ra_e \le 10^6$ . The intensity of heat produced by the source is represented by the Rayleigh number ( $Ra_i$ ), which is based on the volumetric heat generation rate. The influence of the intensity of the two Rayleigh numbers is evaluated by means of the equation,

$$R = \frac{Ra_i}{Ra_e} \tag{5}$$

where  $0 \le R \le 2500$ .

The stream function is defined by the following equations

$$\mathbf{u} = \partial \mathbf{\psi} / \partial \mathbf{y} \tag{6}$$

$$\mathbf{v} = -\partial \mathbf{\psi}/\partial \mathbf{x} \tag{7}$$

where the numerical results illustrating the fluid's behavior are presented in terms of the dimensionless stream function,

$$\psi_{\rm dim} = \frac{\psi}{\alpha} \tag{8}$$

Other physical quantities of interest in the present study are the average Nusselt numbers for the hot and cold walls; these variables are defined respectively as

$$\overline{Nu_h} = -\int_0^H \frac{\partial \theta}{\partial X} \Big|_{X=0} dY$$
 (9)

$$\overline{Nu_{c}} = -\int_{H/2}^{H} \frac{\partial \theta}{\partial X} \Big|_{X=W} dY$$
 (10)

The air is studied in three rectangular enclosures with width W and height H heated on the left-hand vertical wall and cooled on the upper half of the right-hand vertical wall. These two walls have a prescribed temperature, while the horizontal walls

are adiabatic. Schematic representations of the configurations studied are presented in Fig. 1.

The position  $W_h = 0.25$ , 0.5, and 0.75, illustrated in Fig. 1 is the distance from the left lateral wall to the center of the heat source. The internal heat generation source is located on the adiabatic bottom wall at different places and occupies 1% of the total volume of the enclosure. The lower half of the right wall is open and is in contact with the air outside the enclosure. The boundary conditions used are

$$U = V = 0$$
 and  $\theta = 1$  at  $X = 0$  and  $0 \le Y \le H$  (11a)

$$U = V = 0$$
 and  $\theta = 0$  at  $X = W$  and  $H/2 \le Y \le H$  (11b)

$$U = V = 0$$
 and  $\partial \theta / \partial Y = 0$  at  $0 \le X \le W$  and  $Y = 0$  (11c)

$$U = V = 0$$
 and  $\partial \theta / \partial Y = 0$  at  $0 \le X \le W$  and  $Y = H$  (11d)

$$K \partial \theta_s / \partial n = \partial \theta_f / \partial n$$
 at the source-fluid interfaces (11e)

where K is the ratio of the thermal conductivities between the heated source and the fluid,  $\theta_s$  is the dimensionless temperature in the heated source, and  $\theta_f$  is the dimensionless temperature in the fluid. The velocity and temperature profiles for the opening are assumed to be,

$$\partial\theta/\partial X=0$$
 if  $U>0$  or else  $\theta=0;~V=0$  at  $X=W$  and  $0\leq Y\leq H/2$ .

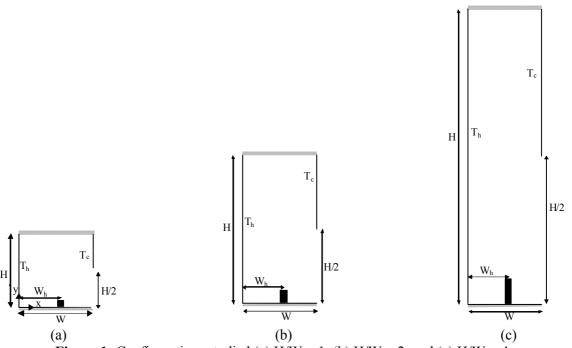


Figure 1: Configurations studied (a) H/W = 1, (b) H/W = 2, and (c) H/W = 4.

### **SOLUTION PROCEDURE**

The differential equations, represented by equations (1) to (4), together with respective boundary conditions, equations (11) and (12), are solved using the finite volume method (MVF) described in Patankar (1980). In this method the solution domain is divided into small finite control volumes. The differential equations are integrated into each of those control volumes. From this integration there were algebraic equations which, when solved simultaneously or separately, supplied pressure and velocity components. A power law scheme is adopted for the convection-diffusion formulation. For the pressure-velocity coupling the SIMPLEC algorithm (semi implicit method for pressure linked equations consistency) is used (Patankar, 1980).

The discretized equations are solved iteratively, using the line-by-line method known as the Thomas algorithm or TDMA (tridiagonal matrix algorithm). An underrelaxation parameter of 0.5 was used in order to obtain a stable convergence for the solution of momentum and energy equations, while there was no need for such a parameter in the solution of the pressure equation.

Validation of the computer code for this work was verified for the natural convection problem in a closed enclosure without a local heat source. The results presented here are for two Rayleigh numbers,  $Ra_e = 10^5$  and  $Ra_e = 10^6$ . Hortman et al. (1990) found the average Nusselt numbers 4.616 and 4.525 for  $Ra_e = 10^5$  and grids 42  $\times$  42 and 82  $\times$  82, respectively, while in our study 4.604 and 4.535, respectively, were found. For the  $Ra_e = 10^6$  they found the average Nusselt numbers 9.422 and 8.977 for the same grids, while in our study 9.487 and 8.975, respectively, were found.

Grid-independence tests were conducted for all the configurations studied in this work. Three different grid sizes ( $22 \times 22$ ,  $42 \times 42$  and  $82 \times 82$ ) were used and, for example, for H/W = 1, Ra<sub>e</sub> = 2500, and R = 1000 the average Nusselt numbers for the hot wall obtained were -3.87, -4.01, and -4.06 and for the cold wall 4.45, 4.40, and 4.38, respectively, were found for the grids. Because of the small differences for the  $42 \times 42$  and  $82 \times 82$  grids, a  $42 \times 42$  uniform grid was chosen for all the simulations presented in this work. Staggered storage of the variables was used. The numerical solution is considered to be converged when the maximum absolute value of the mass conservation was smaller than  $10^{-10}$ .

#### NUMERICAL RESULTS AND DISCUSSION

In order to compare the numerical code specifically developed for the present study, some solutions obtained in the square cavity, H/W=1, when the internal heat source was centrally located at the bottom horizontal wall (at position  $W_h=0.5$ ) were compared with results of Xia and Zhou (1992) and Reinehr et al. (2002), showing good agreement. These results are presented in Table 1, where different flow patterns occur with the change in  $Ra_e$  or R. A comparative analysis between the values in Table 1 shows relative deviation (RD),

$$RD = (100.|\Delta\theta_{\text{max}}|/\theta_{\text{max}})\%$$
 (13)

where  $\theta_{max}$  is proposed in Xia and Zhou (1992) and Reinehr et al. (2002) and  $\Delta\theta_{max}$  is the difference between the results of this work and those of the two studies mentioned. The results of Xia and Zhou (1992) are situated in the interval of  $3.11\% \le \theta_{max} \le 8.3\%$ , while for Reinehr et al. (2002) the interval is  $0.2\% \le \theta_{max} \le 7\%$ , showing good agreement between the results.

Table 1: Maximum dimensionless temperature ( $\theta_{max}$ ) for H/W = 1 and W<sub>h</sub> = 0.5.

	Xia and Zhou (1992)	Reinehr et al. (1992)	Present work
$R = 400, Ra_e = 10^5$	1.80	1.82	1.95
$R = 1000, Ra_e = 10^5$	3.90	4.07	4.19
$R = 2500, Ra_e = 10^3$	15.40	15.69	16.08
$R = 2500, Ra_e = 10^4$	10.90	11.50	11.47
$R = 2500, Ra_e = 10^5$	8.30	8.46	8.56
$R = 2500 Ra = 10^6$	6.10	_	6.29

According to Table 2 the maximum dimensionless temperature increases considerably with the increasing R ratio, keeping Ra<sub>e</sub> constant, independently of the position, heat sources and

aspect ratio H/W. The maximum dimensionless temperature decreases with the increasing Ra<sub>e</sub> when the R ratio is kept constant. A comparison of the maximum dimensionless temperatures at the same

values of R and Ra<sub>e</sub>, with the heat source at different positions inside the enclosures indicates that the differences are not significant. When the height of the enclosures increases, the dimensionless temperature also increases, as shown in Table 2.

In Table 3 the dependence of  $\overline{Nu_h}$  on R,  $\underline{Ra_e}$ , H/W, and  $W_h$  is shown. In the range studied,  $\overline{Nu_h}$  undergoes a process from positive to negative with the increasing R for all enclosures studied. When the effect of external heating ( $Ra_e$ ) on heat transfer in the partially open enclosures is larger than that of the local heat source,  $\overline{Nu_h}$  is positive and decreases with increasing R at a given  $Ra_e$ , and the hot wall plays a role in heating the fluid in the enclosures, but when  $Ra_e$  is smaller,  $\overline{Nu_h}$  is negative

and  $|\overline{Nu_h}|$  increases with R and the hot wall plays a role in cooling the fluid in the enclosures. This behavior was also noted by Xia and Zhou (1992). The  $\overline{Nu_h}$  changes with the different positions of local heat source, generally increasing with  $W_h$ . The  $|\overline{Nu_h}|$  value increases with H/W. It can be observed that the value of  $Nu_h$  generally doubles with the increasing of enclosures height from H/W = 1 to H/W = 2 and H/W = 2 to H/W = 4. In Table 4 the dependence of  $\overline{Nu_c}$  on R,  $Ra_e$ , H/W, and  $W_h$  is shown. In the range studied,  $\overline{Nu_c}$  increases with R and  $Ra_e$ .  $\overline{Nu_c}$  is generally larger when the local heat source is closer to the cold wall. When the enclosure height increases,  $\overline{Nu_c}$  increases significantly.

Table 2: Maximum dimensionless temperature for H/W = 1, 2, and 4, respectively.

		Ra <sub>e</sub>			
$\mathbf{W_h}$	R	10 <sup>3</sup>	104	10 <sup>5</sup>	$10^{6}$
	400	3.29; 4.39; 5.20	2.66; 3.49; 3.89	2.06; 2.53; 2.84	1.49; 1.83; 2.10
0.25	1000	6.95; 9.39; 10.97	5.56; 7.15; 7.94	4.19; 4.78; 5.55	3.05; 3.49; 4.01
	2500	15.34; 20.77; 23.54	11.97; 14.58; 16.61	8.54; 9.64; 11.36	6.32; 7.22; 8.25
	400	3.50; 4.92; 6.02	2.62; 3.60; 3.89	1.95; 2.47; 2.73	1.36; 1.71; 2.00
0.5	1000	7.64; 10.52; 12.02	5.62; 7.14; 7.74	4.19; 4.77; 5.40	2.97; 3.45; 3.91
	2500	16.08; 21.20; 38.31	11.47; 14.41; 15.99	8.56; 9.68; 11.18	6.29; 7.22; 8.14
	400	3.01; 4.39; 5.37	2.46; 3.72; 3.79	1.86; 2.40; 2.72	1.33; 1.58; 1.95
0.75	1000	7.08; 10.10; 11.19	5.58; 7.16; 7.64	4.13; 4.73; 5.37	2.93; 3.39; 3.88
	2500	15.29; 20.28; 23.24	11.50; 14.33; 15.89	8.48; 9.60; 11.13	6.33; 7.17; 8.11

Table 3: Average Nusselt number on the hot wall for H/W = 1, 2, and 4, respectively.

		$Ra_e$			
$\mathbf{W_h}$	R	10 <sup>3</sup>	104	10 <sup>5</sup>	10 <sup>6</sup>
	400	-2.14; -4.59; -8.74	-0.94; -1.77; -3.49	1.95; 4.88; 7.33	8.61; 14.81; 23.33
0.25	1000	-6.87;-14.42;-26.74	-5.25;-10.50;-21.02	-1.92; -3.51; -9.40	4.83; 5.87; 8.61
	2500	-18.03;-35.19;-68.5	-15.59;-29.9;-62.48	-11.53;-24.15;-51.9	-4.98;-12.44;-28.48
	400	-1.26; -3.41; -6.39	-0.50; -0.97; -2.28	2.30; 5.35; 8.37	9.13; 14.90; 24.55
0.5	1000	-4.50;-11.09;-20.99	-3.38; -7.85; -18.1	-0.78; -3.46;-9.47	5.69; 8.85; 11.82
	2500	-11.28;-20.27;-55.0	-9.20;-23.17;-55.43	-8.03;-19.97;-44.25	-1.50; -7.07;-19.77
	400	-0.29; -0.85; -2.03	-0.03; -0.29; -0.59	2.47; 5.19; 6.54	9.46; 15.25; 25.11
0.75	1000	-1.82; -6.34;-13.56	-2.06; -5.99; -15.04	-0.27; -2.24; -6.25	5.21; 9.82; 14.38
	2500	-5.89;-16.79;-43.45	-8.67;-18.12;-48.79	-5.49;-15.95;-39.21	-1.15; -4.80; -15.42

Table 4: Average Nusselt number on the cold wall for H/W = 1, 2, and 4, respectively.

		Ra <sub>e</sub>			
$\mathbf{W_h}$	R	$10^{3}$	104	10 <sup>5</sup>	$10^{6}$
	400	1.27; 2.23; 3.26	2.32; 4.27; 7.09	3.70; 6.52; 11.47	6.83; 12.02; 20.68
0.25	1000	2.22; 3.22; 3.62	3.45; 5.32; 8.11	4.42; 7.39; 12.80	7.28; 12.82; 21.54
	2500	5.13; 6.07; 4.77	6.52; 7.20; 10.39	6.40; 9.95; 17.28	8.64; 14.60; 25.25
	400	1.50; 2.38; 3.39	2.61; 4.58; 7.31	3.79; 6.69; 11.56	6.90; 12.40; 20.70
0.5	1000	3.48; 4.42; 3.95	4.76; 5.91; 8.48	4.80; 8.59; 14.55	7.33; 12.53; 21.65
	2500	9.87; 13.37; 6.21	11.29; 8.32; 11.04	7.53; 11.62; 17.21	8.49; 14.33; 25.69
	400	1.39; 2.06; 3.24	2.55; 4.52; 7.28	3.86; 6.86; 12.39	6.94; 12.20; 20.69
0.75	1000	2.88; 3.33; 3.77	5.30; 6.16; 8.57	5.05; 8.95; 14.12	7.61; 12.54; 21.31
	2500	7.17; 12.51; 5.43	10.97; 9.28; 11.34	8.80; 12.01; 17.99	9.26; 14.42; 25.69

Streamlines and isotherms for R=2500 are shown in Figs. 2 to 7 for all enclosures and  $W_h=0.25,\,0.5,\,$  and 0.75. An analysis of these figures indicates that when the heating source is located at the center,  $W_h=0.5,\,$  or close to the opening of the enclosure,  $W_h=0.75,\,$  various differences occur in the flow pattern according to the change in  $Ra_e$  or R. When the local heat source is located at the position  $W_h=0.25$  close to the hot wall, its influence on the flow patterns and on the heat transfer is negligible.

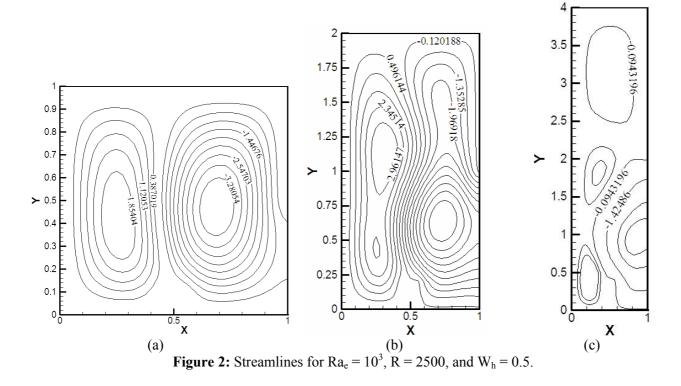
Verifying the position  $W_h = 0.5$  in Figs. 2 and 3, it can be seen that for  $Ra_e = 10^3$  two eddies appear inside the enclosures with H/W = 1 and 2 (see Fig. 2a and 2b) and three eddies appear inside the enclosure with H/W = 4 (see Fig. 2c). They circulate counterclockwise at the left of the enclosure and clockwise at the bottom right, where the intensity of both eddies is important for H/W = 1, i.e., flow and heat transfer are controlled by the internal local heat source and the difference in temperature on the vertical walls has a negligible influence. The left eddy close to the hot wall for H/W = 2 and 4 is less intense than the main flow because the local heat source becomes less important than the difference in temperature between the vertical walls.

In Figs. 4 and 5, for the position  $W_h = 0.75$  it can be seen that at  $Ra_e = 10^4$  two main eddies emerge in the enclosure, one circulating counterclockwise close to the hot wall and the other circulating clockwise

close to the cold wall. In Figs. 4a and 5a the flow and heat transfer are controlled by the internal local heat source because the intensity of the counterclockwise eddy is much greater than that of the clockwise eddy, while in Figs. 4b, 4c, 5b, and 5c, the flow and heat transfer are controlled by the difference in temperature on the vertical walls.

In Figs. 6 and 7, it can be observed that at  $W_h = 0.25$  for  $Ra_e = 10^5$  the enclosures have only a very weak counterclockwise eddy at the backwind side of the local heat source and the larger eddy is a clockwise flow moving upwards along the hot wall and downwards along the cold wall, through the inflow and outflow openings. In these enclosures the flow and heat transfer are controlled by the difference in temperature of the vertical walls.

In Fig. 8 the fluid dynamic behavior of the air in the square enclosure (H/W=1) is shown. Note that the increase in temperature differences between the vertical walls ( $Ra_e$  number) affects the fluid dynamic behavior, increasing the intensity of the flow in the enclosure. This behavior was observed for the various values of R employed, but is illustrated here only for R=2500 and for the local heat source at the central position on the bottom wall. The main eddy increases with the  $Ra_e$  number, although it reaches the full height of the enclosure. In Fig. 9 some of the isotherms obtained for differents  $Ra_e$  are shown. Note that the thermal behavior changes as a function of the increasing  $Ra_e$  number.



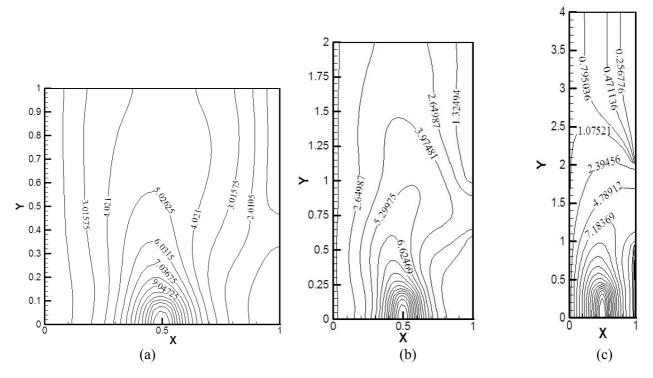


Figure 3: Isotherms for  $Ra_e = 10^3$ , R = 2500, and  $W_h = 0.5$ .

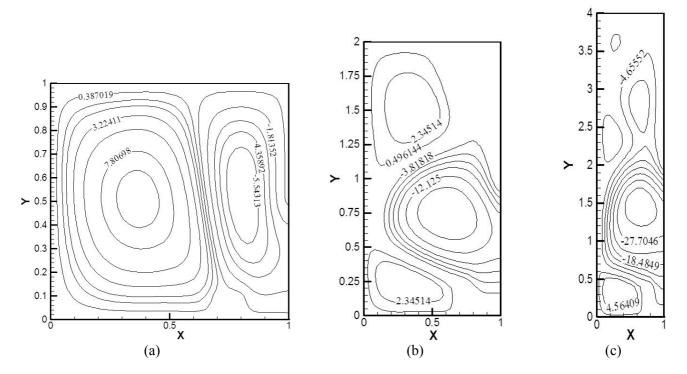


Figure 4: Streamlines for  $Ra_e = 10^4$ , R = 2500, and  $W_h = 0.75$ .

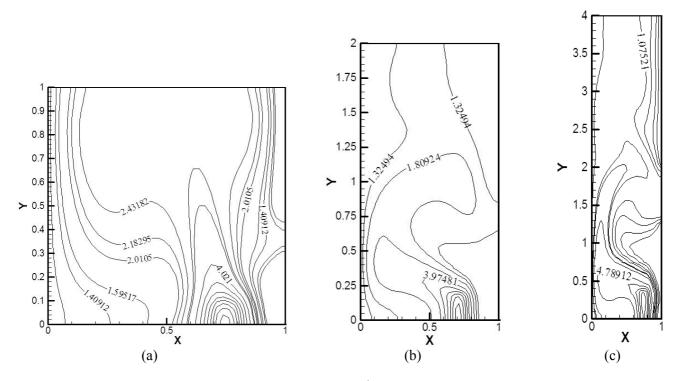
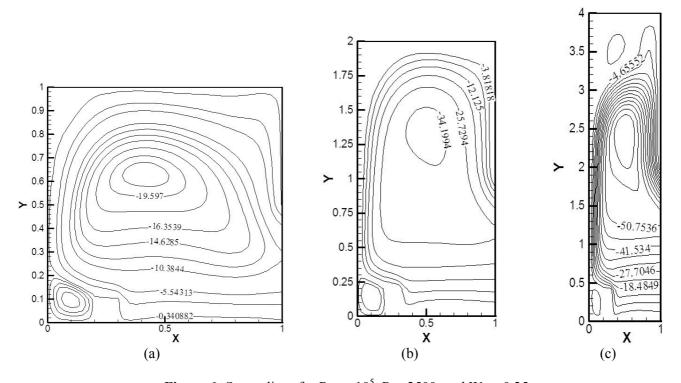
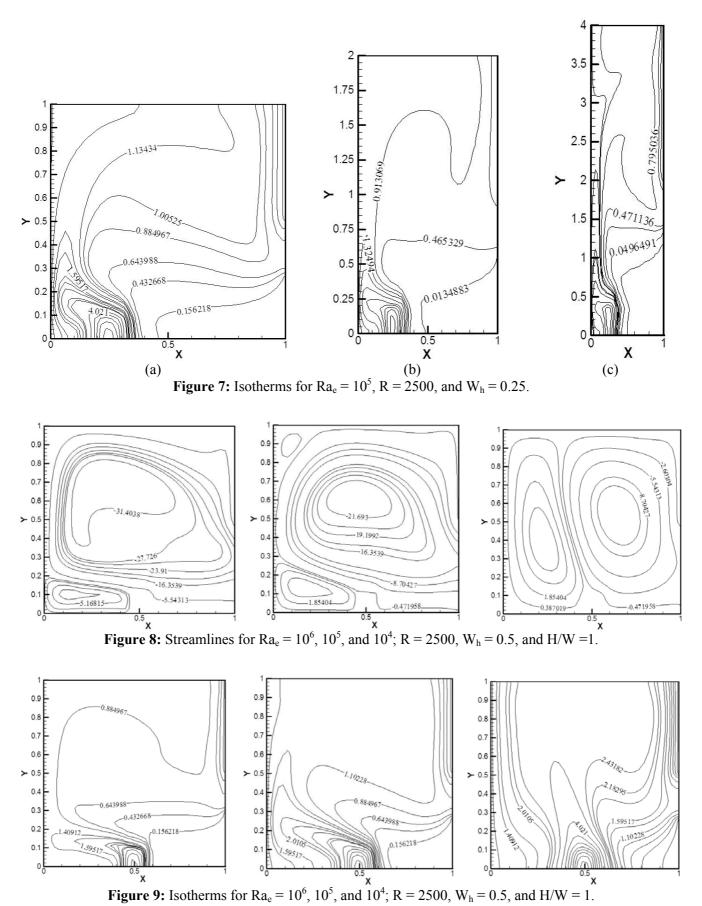


Figure 5: Isotherms for  $Ra_e = 10^4$ , R = 2500, and  $W_h = 0.75$ .

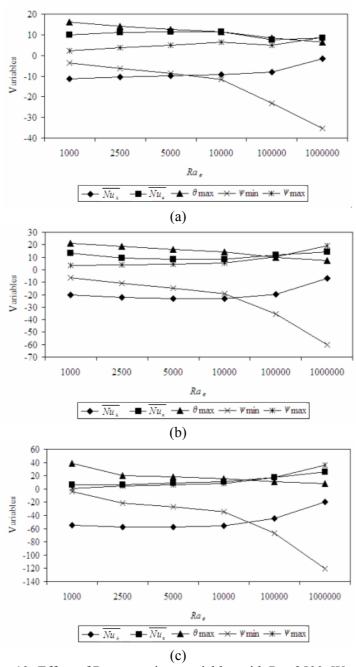


**Figure 6:** Streamlines for  $Ra_e = 10^5$ , R = 2500, and  $W_h = 0.25$ .



The  $\overline{\text{Nu}}$  on the hot and cold walls are shown in Figs. 10a, 10b, and 10c, respectively, for H/W = 1, 2, and 4 at W<sub>h</sub> = 0.5 and R = 2500, as a function of Ra<sub>e</sub>. As shown in the isotherms in Fig. 3 and by the dimensionless temperature in Table 2, the fluid temperature decreases with increasing Ra<sub>e</sub> for the same R in all enclosures studied in this work. The temperature gradient on the hot wall increases with Ra<sub>e</sub>, as shown in Table 3. In this table for approximately R > 400, the  $\overline{\text{Nu}_h}$  has negative values, meaning that the fluid temperature next to the

hot wall is higher than the hot wall temperature and the heat flow changes direction from the enclosure to the hot wall, while  $Nu_c$  increases with  $Ra_e$ , meaning that the heat flow from the enclosure to the cold wall increases with this parameter. For example,  $\overline{Nu_c}$  and  $\overline{Nu_h}$  for  $Ra_e=10^5$  are larger than those for  $Ra_e=10^4$  for the same R and all enclosures, due to the increasing convective heat transfer with increasing  $Ra_e$ . The dimensionless stream functions  $\psi_{max}$  and  $|\psi_{min}|$  increase with  $Ra_e$ .



**Figure 10:** Effect of Ra<sub>e</sub> on various variables with R = 2500,  $W_h = 0.5$ , and H/W = (a) 1, (b) 2, and (c) 4, respectively.

In Fig. 11 the variation in  $\overline{Nu_h}$ ,  $\overline{Nu_c}$ ,  $\theta_{max}$ , dimensionless  $\psi_{max}$ , and  $\psi_{min}$  with the R ratio is shown. In the range studied, the curves are similar, independent of the H/W value; nevertheless the values obtained are different. The  $\overline{Nu_c}$  increases with R, meaning that the heat flow from the enclosure to the cold wall increases with this parameter, and on the hot wall,  $\overline{Nu_h}$  undergoes a process from positive to negative with increasing R; this effect was shown in the results in Table 3. The dimensionless stream functions  $\psi_{max}$  and  $|\psi_{min}|$  increase with R. The other variables increase with R for fixed Ra<sub>e</sub>. If the effect of external heating on heat transfer in the partially open enclosure is larger than

that of the local heat source, then Nu<sub>h</sub> is positive and decreases with the increasing R at a given Ra<sub>e</sub>.

Fig. 12 illustrates the variation in  $Nu_h$ ,  $Nu_c$ ,  $\theta_{max}$ , dimensionless  $\psi_{max}$ , and  $\psi_{min}$  with the location of local heat source for  $Ra_e=10^4$  and R=1000. For this configuration, when the local heat source moves on the adiabatic wall (bottom wall) small changes in the average Nusselt number and in the other variables occur. When the heat source is located near the opening of the enclosures, the  $\overline{Nu_h}$ ,  $\overline{Nu_c}$  (generally) and  $\psi_{max}$  values become larger in all enclosures studied. It can be seen in Fig. 12 that the values of all variables are modified smoothly with the change in aspect ratio of all enclosures.

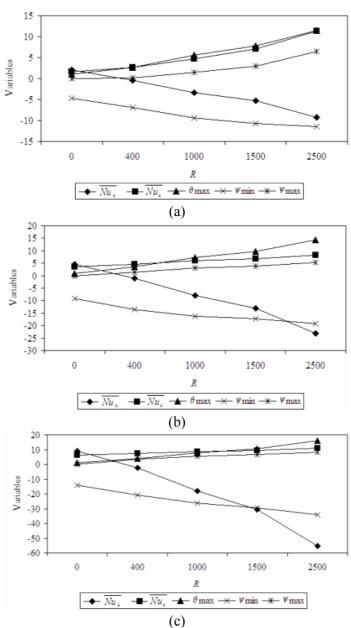
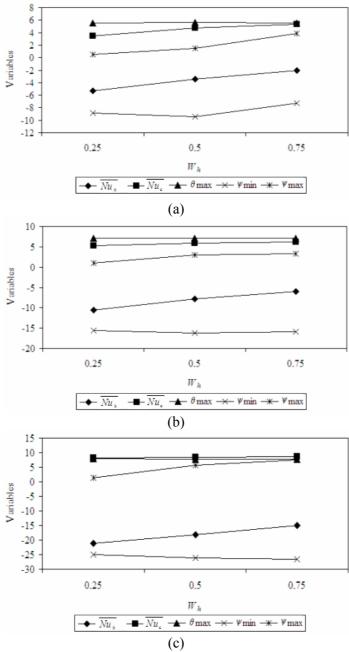


Figure 11: Effect of R on various variables with  $Ra_e = 10^4$ ,  $W_h = 0.5$ , and H/W = (a) 1, (b) 2, and (c) 4, respectively.



**Figure 12:** Effect of location of heat source on various variables with R = 1000,  $Ra_e = 10^4$ , and H/W = (a) 1, (b) 2, and (c) 4, respectively.

## **CONCLUSIONS**

This work was concerned with the numerical modeling of natural convection in three enclosures, within which there is a local heat source, occupying 1% of the total volume of the enclosures, located on the bottom wall at three different positions. The governing parameters were the Rayleigh number and the R ratio characterizing the heat transfer regime in natural convection. In view of the results, the findings may be summarized as follows:

- i) with the increase in Rayleigh number (Ra<sub>e</sub>), i. e., in the difference in temperature between the vertical walls, the maximum dimensionless temperature in the internal enclosures decreases, maintaining the R ratio constant, while  $\overline{Nu_h}$  and  $\overline{Nu_c}$  increase.
- ii) with the increase in the R ratio, the maximum dimensionless temperature in the enclosure increases, maintaining the Rayleigh number constant for all positions of the local heat source analyzed, while  $\overline{Nu_h}$  decreases and  $\overline{Nu_c}$  increases.

iii) the position of the local heat source,  $W_h$ , influences the fluid dynamics of the air as well as the heat transfer rate in the enclosures.

iv) the aspect ratio H/W influences meaningfully the results obtained (for example, for maximum dimensionless temperature, small enclosures have lower temperatures, however large enclosures have higher temperatures for the same configurations) for  $\overline{Nu_h}$ , when the value is positive then it increases and when  $\overline{Nu_h}$  is negative it decreases with increasing height of the enclosure, while  $\overline{Nu_c}$  increases with increasing height of the enclosure.

v) The opening is advantageous for the flow and heat transfer in the enclosures, and its characteristics are complicated and change with location of internal heat source and external and internal Rayleigh numbers.

#### **NOMENCLATURE**

$c_p$	heat capacity	(J/kg K)
g	acceleration of gravity	$(m/s^2)$
H	enclosure height	(m)
k	thermal conductivity	(W/m K)
K	ratio of the thermal	(-)
IX.	conductivities between the	()
	heated source and the fluid	
	average Nusselt number	(-)
Nu	average Nussent number	
p	pressure	(Pa)
P	dimensionless pressure	$(=(p+\rho gy)H^3$
		$/\rho\alpha^2$ )
$P_r$	Prandtl number	$(= v/\alpha)$
q	rate of local heat generation	$(W/m^2)$
1	by the heated protrusion	,
R	Rayleigh number ratio	(-)
$Ra_e$	external Rayleigh number	$(=(g\beta(T_h -$
		$T_c$ ) $H^3/v\alpha$ ))
Rai	internal Rayleigh number	$(=(g\beta qH^5)$
1	<i>j</i> 8	(= (gpq11 / ναk))
т	toman anatoma	**
T	temperature	(K)
u	velocity in x direction	(m/s)
V	velocity in y direction	(m/s)
U	dimensionless velocity in x	$(= uH/\alpha)$
	direction	
V	dimensionless velocity in y	$(= vH/\alpha)$
	direction	
W	enclosure width	(m)
$W_h$	heat generation source	(m)
	location	

37 37	Cartesian coordinates	(m)
x, y		(m)
X, Y	dimensionless Cartesian	(= x/H, = y/H)
	coordinates	
α	thermal diffusivity	$(=k/\rho c_p)$
	•	$(m^2/s)$
β	volumetric coefficient of	$(K^{-1})$
Р		(11)
	thermal expansion	
θ	dimensionless temperature	$(=(T-T_c)/(T_h-T_c))$
$\theta_{\rm s}$	dimensionless temperature	(-)
5	in the heated source	,
$\theta_{ m f}$	dimensionless temperature	(-)
O <sub>I</sub>	in the fluid	( )
		(Pa s)
μ	dynamic viscosity	, ,
ν	kinematic viscosity	$(m^2/s)$
ρ	fluid density	$(Kg/m^3)$
Ψ	stream function	$(m^2/s)$
т		()

# Subscripts

c	cold	(-)
dim	dimensionless	(-)
e	external	(-)
h	hot	(-)
i	internal	(-)
max	maximum value	(-)
min	minimum value	(-)
W	wall	(-)

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