

OPTIMIZATION OF A PULTRUSION PROCESS USING FINITE DIFFERENCE AND PARTICLE SWARM ALGORITHMS

L. S. Santos^{1*}, R. L. Pagano², V. M. A. Calado³ and E. C. Biscaia Jr.⁴

¹Departamento de Engenharia Química e de Petróleo, Universidade Federal Fluminense,
Rua Passo da Pátria, 156, CEP: 24210240, Niterói - RJ, Brazil.
E-mail: lizandrosousa@id.uff.br

²DEQ, Universidade Federal de Sergipe, Avenida Marechal Rondon, São Cristovão - SE, Brazil.

³Escola de Química, CT/UFRJ, Bloco E, Rua Horácio Macedo 2030,
Cidade Universitária, CEP: 21941-909, Rio de Janeiro - RJ, Brazil.

⁴PEQ/COPPE, Universidade Federal do Rio de Janeiro, Av. Horácio Macedo 2030, Centro de Tecnologia, Bloco G,
Sala G-115, Cidade Universitária, Ilha do Fundão, C. P. 68502, CEP: 21941914, Rio de Janeiro - RJ, Brazil.

(Submitted: December 20, 2013 ; Revised: June 30, 2014 ; Accepted: July 13, 2014)

Abstract - Pultrusion is one of several manufacturing processes for reinforced polymer composites. In this process fibers are continuously pulled through a resin bath and, after impregnation, the fiber-resin assembly is cured in a heated forming die. In order to obtain a polymeric composite with good properties (high and uniform degree of cure) and a process with a minimum of wasted energy, an optimization procedure is necessary to calculate the optimal temperature profile. The present work suggests a new strategy to minimize the energy rate taking into account the final quality of the product. For this purpose the particle swarm optimization (PSO) algorithm and the computer code DASSL were used to solve the differential algebraic equation that represents the mathematical model. The results of the optimization procedure were compared with results reported in the literature and showed that this strategy may be a good alternative to find the best operational point and to test other heat policies in order to improve the material quality and minimize the energy cost. In addition, the robustness and fast convergence of the algorithm encourage industrial implementation for the inference of the degree of cure and optimization.

Keywords: Polymer-matrix composites; Degree of cure; Computational modeling; Particle swarm optimization.

INTRODUCTION

Polymer composites are materials made of a cured thermosetting or thermoplastic resin and reinforcement elements (Santos *et al.*, 2009). These materials have several good properties, such as corrosion resistance, low density and high compressive and flexural strength, depending on the resin-fiber blending. Composite materials can be manufactured by different processes, such as pultrusion, resin transfer molding and hand lay-up. Among these technologies, pultrusion stands out because of its

ability to join high production rate with high quality of produced parts. In this process, reinforced fibers are saturated in a resin tank and continuously pulled through a heated die, where the resin gradually cures and solidifies, resulting in a composite part with the same cross-sectional profile as the die cavity (Srinivasagupta and Kardos, 2004). The die may be heated by electrical heaters, strip heaters, hot oil or by steam, although electrical heaters are the most common. Outside the die, the composite part is pulled by a continuous pulling system and then a cut-off saw cuts the part into a desired length (Liu and

*To whom correspondence should be addressed

Hillier, 1999). Figure 1 shows a simplified diagram of a pultrusion process.

Cure is characterized by a polymeric exothermic reaction. A non-uniform or low degree of cure implies a product of unreliable quality. Owing to the heat released during the reaction, the reaction system can be overheated and resin degradation can occur (Han *et al.*, 1986). However, the resin needs to be heated at a minimum value of temperature at which the cure reaction may be started. By considering these constraints, an optimization policy must be developed in order to find the minimum energy rate necessary to produce a composite with high degree of cure at the die exit.

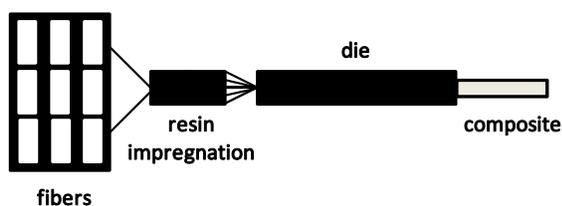


Figure 1: Pultrusion process.

Liu and Hillier (1999) conducted numerical simulations to obtain the temperature and curing profiles for different temperature settings and pull speeds. The predicted temperature profiles were compared with those obtained experimentally. In Liu *et al.* (2000) the governing equations for heat transfer and the resin cure reaction during pultrusion were presented. A finite-element procedure was developed to solve the governing equations. The accuracy and numerical behavior of the procedure were investigated by a number of numerical simulations. Following these studies, Liu (2001) investigated numerically the effect of various process parameters on the temperature and curing profiles in the composite I beams. The results were used to determine preferred process conditions and/or heating configurations for the pultrusion of the composite I beams.

Coelho and Calado (2002) proposed an economical objective-function in order to find the optimal temperature profile. These authors used the simulated annealing optimization strategy and a finite element method to solve the algebraic-differential equations that represent the pultrusion model. The results showed that a considerable profit increase can be obtained if the die temperature is controlled in an optimized way, regarding different heating zones, and if the pull speed is the maximum allowed. According to the author's results, clearly, to attain a high production, the pull speed must be the highest allowed.

Li *et al.* (2002) optimized the pultrusion process based on the uniformity of the degree of cure at the die exit as the objective. An iterative procedure based on the steepest descent method and a finite element method (LUSAS FE package) were used. The effects of the element aspect ratio and different initial heater conditions were studied. The results of the case studies showed that the algorithm developed is numerically stable and provides optimal die temperatures for producing a uniformly cured pultrudate.

Srinivasagupta *et al.* (2003) used a multi-objective optimization procedure that involves control, environmental and quality aspects for pultrusion optimization. The procedure was able to determine the optimal values of the processing parameters, such as heating zone temperatures, as well as the equipment specifications (die and heater dimensions) that satisfy multiple objectives.

In Carlone *et al.* (2006) two different three-dimensional models which take into account thermochemical aspects were compared. Both models considered heat transfer due to heating platens, convective boundary conditions, die-cooler at the die entrance, composite material anisotropy, mass transport effects, and the heat generated by the resin cure reaction. A remarkable accuracy was achieved using the proposed finite element method; which required less computational time with respect to the finite difference model.

Carlone *et al.* (2007) used a hybrid method based on genetic algorithms and the simplex method for minimizing the same function proposed by Li *et al.* (2002) (The objective function considered was based on the variance of the degree of cure, evaluated in the exit cross-section, with respect to a target value). The optimization technique was applied taking into account only the heater temperatures as optimization variables; all the other parameters, such as pull speed or cooling channel temperature were assumed as constants. The optimization procedure was applied to obtain a part characterized by a good quality, i.e., a uniform distribution of the degree of cure. A uniform distribution of the degree of cure, with a satisfactory mean value at the exit of the forming die, could be achieved.

In Santos *et al.* (2012) the pultrusion of an "I" beam composite was simulated by the Finite Volume Method. It was possible to verify that the energy requirements can be reduced by changing the heating configuration of the pultrusion die. Hence, an alternative configuration with internal heaters inside the die body was simulated. The sum of all heating power was considered as the objective function in order to find the minimal energy consumption. Internal

heaters were capable of increasing the thermal efficiency of the process by improving the thermal distribution on the die body. Moreover, this heating arrangement also allowed insulating the die surface.

Stroher *et al.* (2013) applied spatial discretization in bar cross section using the Galerkin finite element method while time integration used a second-order implicit backward difference formula, also known as the Gear method. The computational domain was discretized using an unstructured mesh with triangular elements and an adaptive refinement. Iterative algorithms were used to solve the algebraic equation system. Results showed that, as the temperature and degree of cure along the die extension increased, the volumetric heat capacity and the thermal conductivity also increased. The influence of the pulling speed and the die temperature on the thermal property variation were also analyzed.

More recently, Silva *et al.* (2014) investigated new relative positions for the heaters in order to optimize the heat distribution process and energy consumption. Finite Elements Analysis was applied to identify the best relative position of the heaters in the die. The analysis was firstly developed based on eight cylindrical heaters located in four different location plans. In a second phase, in order to refine the results, a new approach was adopted using sixteen heaters with the same total power. The authors concluded that the correct positioning of the heaters could reduce around 10% the energy consumption, decreasing the production costs.

Our previous research (Santos *et al.*, 2012), nicely demonstrated that the CFD-PSO algorithm can simulate and optimize pultrusion of composite parts with very irregular shapes. Moreover, different heating strategies can be designed. However, one of the drawbacks is that the computational performance can be excruciatingly slow for difficult problems. Roughly speaking, for design purposes, the strategy presented in Santos *et al.* (2012) enjoys good properties. On the other side, if the goal was to provide a fast optimization, i.e., for real time optimization, CFD-PSO would not be recommended. Hence, in this paper we are interested in constructing a more simple and robust algorithm in order to provide a fast simulation-optimization. Our method is based on a mathematical model which joins the code DASSL (Petzold, 1982) and a particle swarm optimization (PSO) method (Kennedy and Eberhart, 1995). The goal is to minimize the energy rate with a constraint of a minimum degree of cure to be achieved at the die exit. A cylindrical composite part, also studied by Santiago *et al.* (2003) and Hunter *et al.* (1986), was used as case-study.

This paper is organized as follows. First, a mathematical formulation of the pultrusion process is presented. Secondly the optimization procedure is explained. In the sequel, the numerical simulation and optimization are realized and discussed. Finally, conclusions of the obtained results are made.

MATHEMATICAL MODEL

In this section, the theoretical background of pultrusion is described (Coelho and Calado, 2002; Santos *et al.*, 2012). Heat transfer in the pultrusion process, with a cylindrical die, can be modeled using the mass and energy balance equations described by Equations (1) and (2), respectively.

$$\frac{d\alpha}{dt} = \frac{r'_a}{C_{a_0}} \cdot \frac{d\alpha}{dz'} = \frac{r_a}{u_z \cdot C_{a_0}} \quad (1)$$

$$\rho_c \cdot c_{p_c} \cdot u_z \cdot \frac{\partial T}{\partial z'} = \frac{1}{r'} \cdot \frac{\partial}{\partial r'} \left[r' \cdot \left(k_c \cdot \frac{\partial T}{\partial r'} \right) \right] - c_{a_0} \cdot r'_a \cdot (1 - \phi_{fi}) \cdot (\Delta H_{re}) \quad (2)$$

where z' and r' are die and radius length (m), respectively, t is time (s), T is temperature (K), u_z is the pulling speed ($m \cdot s^{-1}$), c_{p_c} is the composite specific heat ($J \cdot kg^{-1} \cdot K^{-1}$), c_{a_0} is the resin concentration ($kg \cdot m^{-3}$), α is the degree of cure, ϕ_{fi} is the fiber volume fraction, ΔH_{re} is the energy released by cure reaction ($J \cdot kg^{-1}$), k_c is the composite thermal conductivity ($J \cdot m^{-1} \cdot kg^{-1} \cdot K^{-1}$) and ρ_c is the composite specific mass ($kg \cdot m^{-3}$).

Equation (1) describes the mass balance, which quantifies the reaction inside the pultrusion die. Equation (2) describes the energy balance. As seen, the energy has a convective term in the z direction and a diffusive term in the radial direction; according to the literature (Coelho and Calado, 2002, Santos *et al.*, 2012), the conductive heat transfer along the z -direction can be neglected.

The physical properties of the composite are calculated by the following equations:

$$\rho_c = \phi_{r_e} \rho_{r_e} + \phi_f \rho_f \quad (3)$$

$$\rho_c c_{p_c} = \rho_{r_e} \phi_{r_e} c_{p_{r_e}} + \rho_f \phi_f c_{p_f} \quad (4)$$

$$\frac{1}{k_c} = \phi_{r_e} \frac{1}{k_{r_e}} + \phi_f \frac{1}{k_f} \quad (5)$$

where the subscripts r_e , f and c represent resin, fiber and composite, respectively.

The cure rate of epoxy resin is written as (Santiago *et al.*, 2003):

$$r'_a = c_{a_0} \cdot \frac{\partial \alpha}{\partial t} = -k(T) \cdot (1 - \alpha)^n \quad (6)$$

where $k(T)$ is the reaction order and $k(T)$ is the Arrhenius equation, written as:

$$k(T) = A \cdot e^{\frac{-E_a}{RT}} \quad (7)$$

In this equation, A is a kinetic pre-exponential factor, E_a is the activation energy and R is the universal gas constant. The boundary conditions may be written as:

$$\alpha(r, z' = 0) = 0 \quad (8)$$

which is the degree of cure at die entrance and

$$T(r, z' = 0) = T_0 \quad (9)$$

$$\left. \frac{\partial T(r', z')}{\partial r'} \right|_{r'=0} = 0 \quad (10)$$

$$T(R_c, z') = T(z') \quad (11)$$

where Equations (9), (10) and (11) are the temperatures at the die entrance, die center and die surface, respectively, and R_c is the composite radius.

In this work, mass and energy equations are normalized:

$$\frac{d\alpha}{dz} = D_a \cdot r_a \quad (12)$$

$$\frac{\partial \theta}{\partial z} = \frac{\delta}{r} \cdot \frac{\partial}{\partial r} \left(r \cdot \left(\frac{\partial \theta}{\partial r} \right) \right) - D_a \cdot \beta \cdot (1 - \phi_f) \cdot r_a \cdot c_{a_0} \quad (13)$$

where $\theta = \frac{T}{T_0}$ is the normalized temperature, T_0 is

the feed resin temperature, $z = \frac{z'}{L}$ and $r = \frac{r'}{R_c}$ are

the normalized die length and normalized part radius, respectively, D_a is the Damköhler number, δ , σ

and β are dimensionless constants, defined by following equations:

$$D_a = \frac{L}{u_z} A \cdot e^{\frac{-E_a}{RT_0}}, \quad \delta = \frac{k_c \cdot L}{\rho_c c_{p_c} u_z R_c^2}, \quad (14)$$

$$\sigma = \frac{-E_a}{RT_0}, \quad \beta = \frac{c_{a_0} (\Delta H_{r_e})}{\rho_c c_{p_c} T_0}$$

where L is the die length.

The normalized cure reaction of epoxy resin (Santiago *et al.*, 2003) is written as:

$$r'_a = e^{-\sigma \left(\frac{1}{\theta} - 1 \right)} \cdot (1 - \alpha)^n \quad (15)$$

where n is the order of reaction.

The fiber-resin feed temperature θ_o is regarded as the ambient temperature: $\theta(r, z = 0) = 1$ and the feed degree of cure is zero, $\alpha(r, z = 0) = 0$. Normalized boundary conditions used to simulate this process are:

I) Symmetry condition at $r = 0$:

$$\left. \frac{d\theta(r, z)}{dr} \right|_{r=0} = 0 \quad (16)$$

II) Temperature in the die-composite interface:

$$\theta(r = 1, z) = \theta(z) \quad (17)$$

Taking into account that a second-order derivative can be written using a second-order central finite difference formulation, the partial differential equations were discretized in the radial direction as:

$$\frac{\partial \theta}{\partial r} = \frac{\theta_{i+1} - \theta_{i-1}}{2\Delta r} \quad (18)$$

$$\frac{\partial^2 \theta}{\partial r^2} = \frac{\theta_{i+1} - 2\theta_i + \theta_{i-1}}{\Delta r^2} \quad (19)$$

resulting in a system of algebraic equations:

$$\frac{\partial \theta_i}{\partial z} = \frac{\delta}{r_i} \left(\frac{\theta_{i+1} - \theta_{i-1}}{2\Delta r} \right) + \left(\frac{\theta_{i+1} + 2\theta_i - \theta_{i-1}}{\Delta r^2} \right) + D_a \beta (1 - \phi_f) r_a c_{a_0} \quad (20)$$

$$\frac{d\alpha_i}{dz} = D_a r_a \quad (21)$$

where $i = 1, n_p - 1$, and n_p is the number of nodal points in the radial direction. Then, at $r = 0$, $\theta_o = \theta_1$. Figure 2 shows the grid domain of the finite difference technique. These equations were integrated in the z direction by a version of the integration routine DASSL (Petzold, 1982). The simulation of pultrusion with this procedure is a part of the optimization study that is explained in the next topic.

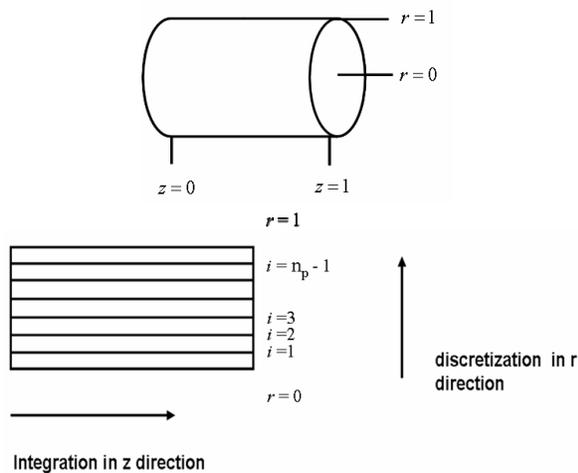


Figure 2: Discretization scheme.

OPTIMIZATION PROCEDURE

In order to optimize the pultrusion process, an objective function similar to that proposed by Coelho and Calado (2002), which represents the energy rate, was used:

$$F(T) = Q = \int_{z'=0}^{z'=L} k_c S \cdot \left. \frac{dT}{dr'} \right|_{r'=R_c} dz' \quad (22)$$

where $S = (2\pi R_c) \cdot L$ is the superficial area of the cylindrical composite surface and Q is the energy rate needed to cure the system. The objective function can be re-formulated in a dimensionless form as:

$$f(\theta) = \eta \cdot \int_{z=0}^{z=L} \left. \frac{d\theta}{dr} \right|_{r=1} dz \quad (23)$$

in which $\eta = 2 \cdot L \cdot \pi \cdot k_c \cdot T_o$.

Considering the objective function written in Equation (23), the purpose of this work was to reduce the energy rate, or minimize $f(\theta)$. In this paper the quality aspect is considered as a constraint. Thus, the constrained non-linear optimization problem is written as:

$$\min_{\theta, g \in \mathbb{R}^n} f(\theta)$$

$$\text{subject to: } g(\alpha) \leq 0$$

$$\theta_L \leq \theta \leq \theta_U \quad (24)$$

where θ_L and θ_U are the lower and upper limits of dimensionless temperature, respectively and:

$$g(\alpha) = \left[\frac{\sum_{i=1}^{n_p} (\alpha_i - \alpha_d)^2}{n_p} \right]_{z=1} - \varepsilon \quad (25)$$

In Equation (25), ε (tolerance factor) measures the maximum allowed sum of square deviation within the degree of cure at a nodal point i to be achieved at the die exit and the desired value of degree of cure, α_d . The choice of this value was made based on the algorithm experiments to evaluate the sensitivity of $g(\alpha)$ by manipulating this parameter. Therefore, constraint $g(\alpha)$ represents the least square deviation function, which evaluates the uniformity of cure for all nodal points at die exit. In order to incorporate the constraints in optimization formulation, a square function was added:

$$f(\theta) = \left(\eta \int_{z=0}^{z=1} \left. \frac{d\theta}{dr} \right|_{r=1} dz \right) + \psi [\max(0, g(\alpha))]^2 \quad (26)$$

where ψ is a positive penalty parameter. In Equation (26), when the constraint is violated, there is a positive contribution to the penalty term (if $g(\alpha) \geq 0$).

The optimization method applied to minimize the objective function is the particle swarm optimization, PSO, developed by Kennedy and Eberhart (1995). In this method, a set of points, denominated particles, is

configured to find the optimal value of the objective function.

PSO estimates values of temperature between the lower and upper limits of temperature of four heating zones (θ_1, θ_2) of the composite surface, as suggested by other authors (Srinivasagupta and Kardos, 2004, Coelho and Calado, 2001) who studied the same type of problem. The assumption of four heating zones implies that the pultrusion die has two heating plates coupled on the die surface. In order to achieve the temperature profile, according to estimated values of θ_1, θ_2 , these values are interpolated by a linear function, resulting in an expression that represents the boundary condition, $\theta(r=1, z) = \theta(z)$, necessary to solve the DAE system.

The PSO model may be written as:

$$v_i^{k_{int}+1} = w \cdot v_i^{k_{int}} + c_1 \cdot r_1 \cdot (p_i^{k_{int}} - x_i^{k_{int}}) + c_2 \cdot r_2 \cdot (p_{global}^{k_{int}} - x_i^{k_{int}}) \quad (27)$$

$$x_i^{k_{int}+1} = x_i^{k_{int}} + v_i^{k_{int}+1} \quad (28)$$

where k_{int} is the PSO iteration, d is the search direction, v is the particle velocity, x is the position in search space, c_1 and c_2 are positive constants, called cognitive and social parameters, respectively, r_1 and r_2 are random numbers that vary at each iteration between the interval $[0 \ 1]$, p_i is the best point of an individual particle i and p_{global} is the best value found for all particles. The choice of parameters c_1, c_2 and inertia weight w depends on the problem formulation. According to our experience, 30 particles were sufficient for optimization. In addition, a maximum of 20 iterations (this value is the maximal allowable iteration for PSO to find the best solution) was used. The choice of PSO parameters was made according to Schwaab *et al.* (2008). Therefore, no further study was realized to investigate the influence of these parameters. The algorithm was written in FORTRAN 90 code as sketched in Figure 3(a). Figure 3(b) illustrates the PSO flowsheet.

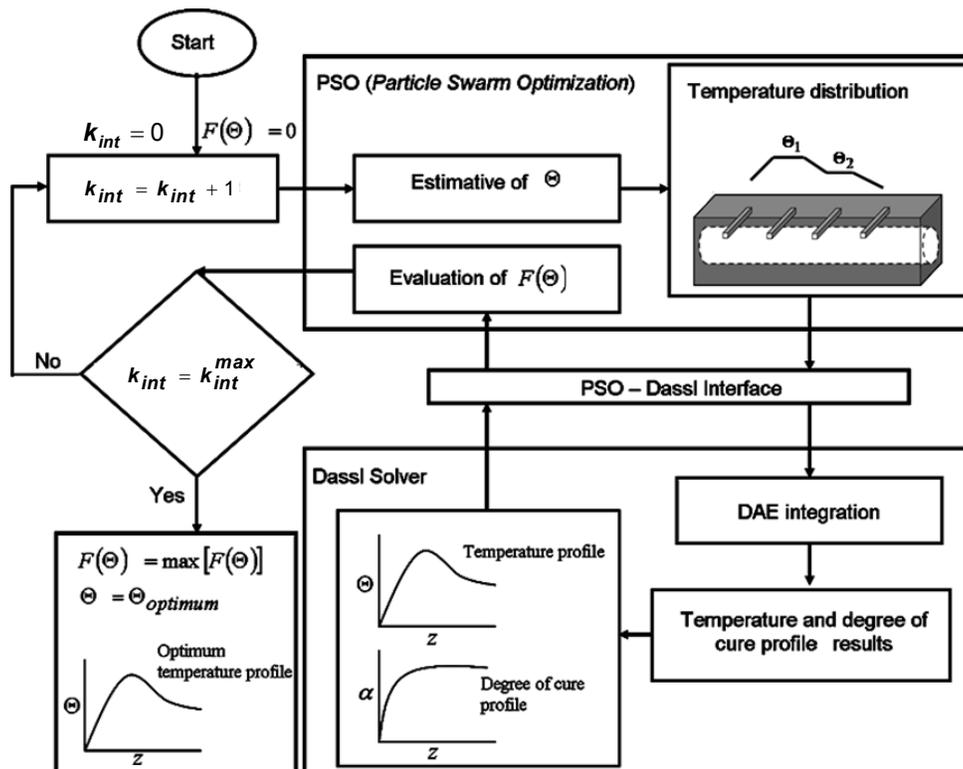


Figure 3(a): Optimization procedure.

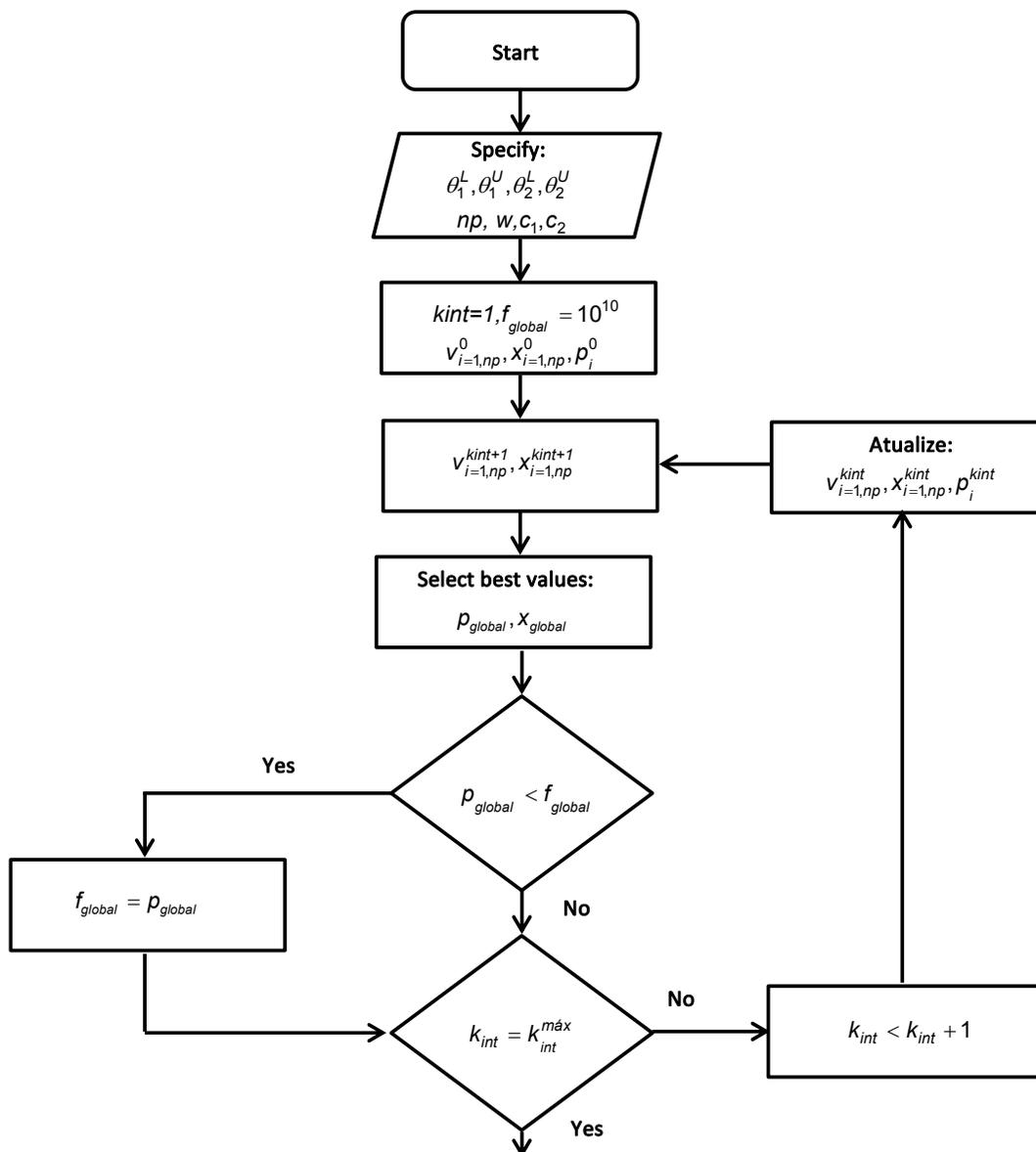


Figure 3(b): Flowsheet of the PSO algorithm.

RESULTS: SIMULATION

Before the optimization study, the proposed model was validated by comparing our results with experimental results from Hunter *et al.* (1986), according to the processing conditions in Tables 1 and 2.

Figure 4 shows the temperature profiles of the composite center for $np = 40 - 10$. As observed, the temperature profile at the composite center presents a very similar behavior when compared with the results obtained by Hunter *et al.* (2006) at the same location. Santiago *et al.* (2003) also carried out the same comparative study. As seen, the error is more

pronounced between the interval $0.4 \leq z \leq 0.7$, where the temperature reaches the maximum value, approximately at $z = 0.55$. The reason lies in the fact that experimental errors occurred during the study, as highlighted also by Santiago *et al.* (2003).

Table 1: Kinetic parameters (Santiago *et al.*, 2003).

Symbol	Unit	Description	Value
A	s^{-1}	pre-exponential factor	2.918×10^6
E_a	$kJ \cdot mol^{-1}$	activation energy	25.57×10^3
n	-	order of reaction	2

Table 2: Parameters of resin, fiber and composite and physical properties of the pultrusion model (Santiago *et al.*, 2003).

Symbol	Unit	Description	Value
φ	-	volume fraction	0.72
ρ_f	$kg \cdot m^{-3}$	fiber specific mass	2580
ρ_{re}	$kg \cdot m^{-3}$	resin specific mass	1234
$C_{p_{re}}$	$J \cdot kg^{-1} \cdot K^{-1}$	resin specific heat	1833
C_{p_f}	$J \cdot kg^{-1} \cdot K^{-1}$	fiber specific heat	920
k_{re}	$J \cdot m^{-1} \cdot kg^{-1} \cdot K^{-1}$	resin thermal conductivity	1693
k_{fi}	$J \cdot m^{-1} \cdot kg^{-1} \cdot K^{-1}$	fiber thermal conductivity	1967
T_o	K	feed resin-fiber temperature	300
Ca_o	$kg \cdot m^{-3}$	feed resin concentration	1100
R_c	m	composite radius	6.352×10^{-3}
L	m	die length	1.254
u_z	$m \cdot s^{-1}$	speed in pull direction	3.0×10^{-3}
ΔH_{re}	$J \cdot kg^{-1}$	released energy	-398.44×10^3

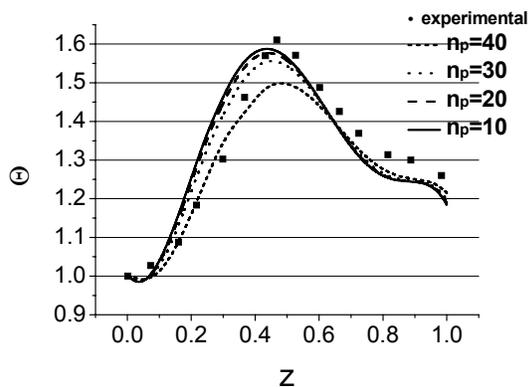


Figure 4: Comparison between theoretical and experimental results (Hunter *et al.*, 1986) for the temperature profile at the composite center ($r = 0$).

In this present study, the following equation is considered as an error estimation of the temperature profile:

$$e_{\theta} = \frac{\sqrt{\sum_{i=1}^{N_e} (\theta_{\text{exp}}^i - \theta_t^i)^2}}{N_e} \quad (29)$$

in which N_e is the number of experimental points and θ_t is the simulated temperature. A statistical

analysis was carried out for error evaluation. In this light, the same procedure was realized for $n_p = 10, 20, 30$ and 40 points, resulting in errors equal to $0.31, 0.25, 0.23$ and 0.23 , respectively. Clearly, the algorithm performs better for finer grids, in our case, $n_p = 40$. Then, $n_p = 40$ was established for the following numerical experiments.

The degree of cure profile is sketched at four regions of the composite part in Figure 5. As observed, the degree of cure achieves the maximum value at the die exit. According to the results, the composite surface achieves a degree of cure of 1.0 at approximately $z = 0.2$, while the composite center achieves this condition at $z = 0.6$. Obviously, after this point, there is no need to heat the material because the resin is completely cured. It is easy to figure out that a reduction of temperature would not affect the material quality (degree of cure).

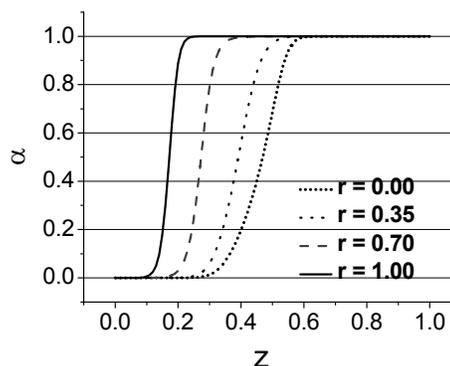


Figure 5: Profile of the degree of cure at four axial composite sections.

The case study of Santiago *et al.* (2003) was chosen as the reference for a comparative study between optimized and experimental results realized by Hunter *et al.* (1986). According to the proposed objective function (Equation (22)), the energy rate equals to 138 W. In the next topic, the proposed optimization procedure will be applied in order to minimize this value and keep the material quality.

RESULTS: OPTIMIZATION STUDY

The optimization parameters for this problem are shown in Table 3. Note that the degree of cure set point is 0.95 . Based on results from the literature (Liu, 2001; Calado *et al.*, 2001), it was assumed that a significant amount of cure reaction happens in the post-die region. Table 4 displays the temperature values of four heating zones, separated by two heaters (θ_1, θ_2), located at $z = [0.25 \ 0.5]$ and $z = [0.75 \ 1.0]$,

also the energy rate at each iteration of the optimization algorithm. The minimum objective function is equivalent to 94.13 W, 31% smaller than the total energy rate spent in the reference operational conditions of Santiago *et al.* (2003). In addition, the computation performance of the PSO-DASSL routine is very fast, around 2 min implemented on a 2.3 GHz – 4 GB PC.

Table 3: Optimization parameters.

Symbol	Unit	Description	Value
θ_L	-	lower limit of dimensionless temperature	1.1
θ_U	-	upper limit of dimensionless temperature	1.7
ψ	-	penalty parameter	100
α_d	-	degree of cure set point	0.95
ε	-	tolerance factor	0.01

Table 4: Temperature and objective function.

Iteration	θ_1	θ_2	Energy rate
1	1.410	1.168	114.564
2	1.475	1.100	98.855
3	1.475	1.100	98.855
4	1.430	1.100	94.388
5	1.430	1.100	94.388

In Figure 6 the predicted optimal temperature profile is depicted. It is interesting to note that the temperature at the composite surface increases at the die entrance and reaches its maximum value equal to 430 K (dimensionless value of 1.43). After this point, the temperature is reduced to the value of 330 K (dimensionless value of 1.1). Clearly, the proposed strategy is to heat the material during the first stage of the cure reaction, while reusing the released heat during the second stage θ_2 . This configuration can be explained by the fact that a considerable amount of energy is released by the cure reaction and, therefore, the material does not need to be excessively heated after this stage. This peak can be identified in Figure 6, where the temperature profiles are inverted around $z = 0.7$.

The optimal degree of cure profile is depicted in Figure 7. It is clear that the cure profile presents a smoother curve, if compared with the cure profile of the reference case (Figure 5). Here, the rate of degree of cure in relation to the die length is considerably smaller than the same results obtained in Santiago *et al.*, (2003); however, the degree of cure constraint (Equation (25)) is kept.

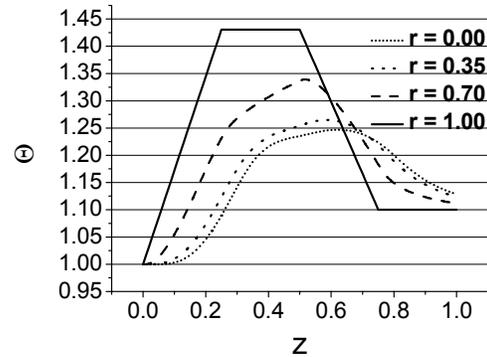


Figure 6: Optimum temperature profile at four axial composite sections.

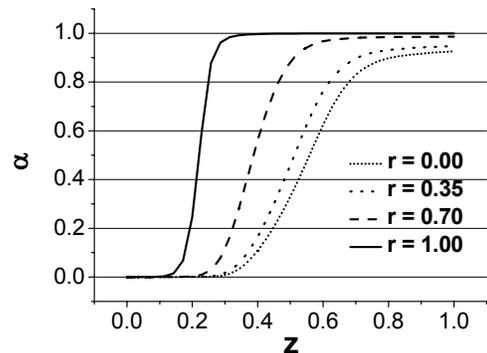


Figure 7: Optimum degree of cure profile.

In order to evaluate the influence of α_d , a comparative study was carried out with different values of α_d : 0.7, 0.8, 0.95 and 1.0. The results are displayed in Table 5. It can be seen that the energy rate presents higher values for higher values of α_d , as expected. It is easy to check that, in the last case, for $\alpha_d = 1$, the energy rate necessary to cure the material is higher. On the other hand, for small values of α_d , less amount of energy is needed.

Table 5: Influence of α_d .

Iteration	Energy rate in W for $\alpha_d = 0.7$	Energy rate in W for $\alpha_d = 0.8$	Energy rate in W for $\alpha_d = 0.95$	Energy rate in W for $\alpha_d = 1.00$
1	88.166	87.193	114.564	97.145
2	83.175	87.193	98.855	97.145
3	82.010	87.193	98.855	97.145
4	82.010	87.193	94.388	97.145
5	82.008	87.193	94.388	97.123

Additional studies were realized in order to compare the influence of some optimization parameters.

Table 6 shows the influence of the penalty parameter, ψ , on the objective function, regarding

$\alpha_d = 0.95$. Obviously, when the value of ψ is zero, there is no constraint on the optimization problem. Then, the computed energy rate is very small. Contrary to this, as ψ increases, the penalty term becomes larger for any value of $g(\alpha)$, which violates the equality constraints. Hence, the computed energy rate becomes higher.

Table 7 shows the influence of the tolerance ε on optimization results. The choice of ε is a measure of the importance of the constraint written in Equation (25). In other words, if one desires a high degree of cure uniformity, a small ε must be required.

Table 6: Influence of the penalty parameter ψ .

ψ	Energy rate in W
0	27.59238
0.001	27.94128
0.01	31.08143
0.1	57.86185
1	57.86185
10	94.38852
100	94.38954

Table 7: Influence of the parameter ε .

ε	Energy rate in W
0.001	94.47151
0.01	94.38852
0.1	94.38852
1	94.38852

CONCLUSIONS

This paper describes an algorithm to optimize the pultrusion process. The model was solved by the DASSL routine with very few points and the results show a good prediction of temperature profile and degree of cure. A PSO-DASSL routine was written for pultrusion optimization and several optimization runs were done in order to evaluate the effect of constraints on optimal operation strategy.

The present optimization algorithm can reduce the temperature at the second heating zone, taking into account the released heat of cure reaction. The obtained results encourage us to apply the optimization code to improve industrial performance of pultrusion. The main aspect is the relative facility of calculating the optimal temperature profile and consequently the minimum energy rate. It is important to emphasize that the optimal temperature profile

depends of the control strategy implemented to warm the pultrusion die. In other words, the assumption of four heating zones is a restriction that can be changed to obtain different optimal control strategies. Moreover, the high computational performance of our strategy allows possible industrial implementations for industrial monitoring and optimization.

ACKNOWLEDGEMENTS

We thank the Brazilian Agencies CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico) and FAPERJ (Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro) for providing scholarships and for supporting this work. The authors would like to dedicate this paper to Professor Alberto Luiz Coimbra, on the 50th anniversary of COPPE (1963–2013), the Graduate School of Engineering of the Federal University of Rio de Janeiro.

REFERENCES

- Carlone, P., Palazzo, G. S., Pasquito, R., Pultrusion manufacturing process development by computational modelling and methods. *Mathematical and Computer Modelling*, 44, 701-709 (2006)
- Carlone, P., Palazzo, G. S., Pasquito, R., Pultrusion manufacturing process development: Cure optimization by hybrid computational methods. *Computers and Mathematics with Applications*, 53, 1464-1471 (2007).
- Coelho, R. M. L., Calado, V. M. A., An optimization procedure for the pultrusion process based on a finite element formulation. *Polymer Composites*, 23, 329-341 (2002).
- Han, C. H., Lee, D. S., Chin, H. B., Development of a mathematical model for the pultrusion process. *Polymer Engineering and Science* 26, 393-404 (1986).
- Hunter, G. A., In: 43rd Annual Conf. Composites Institute. The Society of Plastics Industry (1986).
- Kennedy, J., Eberhart, R., Particle Swarm Optimization. In: *Proc. IEEE International Conference on Neural Networks* (1995).
- Li, J., Joshi, C. S., Lam, Y. C., Curing optimization for pultruded composite sections. *Composite Science and Technology*, 62, 457-467 (2002).
- Liu, X. L., Numerical modeling on pultrusion of composite I beam. *Composites Part A: Applied Science and Manufacturing*, 32, 663-681 (2001).

- Liu, X. L. and Hillier, W., Heat transfer and cure analysis for the pultrusion of a fiberglass-vinyl ester I beam. *Composite Structures*, 47, 581-588 (1999).
- Liu, X. L., Grouch, I. G., Lam, I. C., Simulation of heat transfer and cure in pultrusion with a general-purpose finite element package. *Composite Science and Technology*, 60, 857-864 (2000).
- Petzold, L. A., Description of DASSL: A Differential/Algebraic Systems Solver. *IMACS Trans. on Scientific Computation* (1982).
- Sala, G. and Cutolo, D., The pultrusion of powder-impregnated thermoplastic composites. *Composites Part A: Applied Science and Manufacturing*, 28, 637-646 (1997).
- Santiago, D., Lombera, G., Urquiza, S., Moschiar, S. M., Modelado numérico del proceso de pultrusion en materiales compuestos. *Materials Research*, 6, 583-589 (2003). (In Spanish).
- Santos, L. S., Pagano, R. L., Calado, V. M. A., Biscaia Jr., E. C., Optimum heating configuration of pultrusion process. *Computer Aided Chemical Engineering*, 27, 705-710 (2009).
- Santos, L. S., Pagano, R. L., Calado, V. M. A., Biscaia Jr., E. C., CFD-optimization algorithm to optimize the energy transport in pultruded polymer composites. *Brazilian Journal of Chemical Engineering*, 29, 559-566 (2012).
- Schwaab, M., Biscaia Jr., E. C., Monteiro, J. L., Pinto, J. C., Nonlinear parameter estimation through particle swarm optimization. *Chemical Engineering Science*, 63, 1542-1552 (2008).
- Silva, F. J. G., Ferreira, F., Ribeiro, M. C. S., Castro, A. C. M., Castro, M. R. A., Dinis, M. L., Fiúza, A., Optimising the energy consumption on pultrusion process. *Composites Part B.*, 57, 13-20 (2014).
- Srinivasagupta, D., Kardos, J. L., Ecologically and economically conscious design of the injected pultrusion process via multi-objective optimization. *Modeling and Simulation in Materials Science and Engineering*, 12, S205 (2004).
- Srinivasagupta, D., Kardos, J. L., Joseph, B., Rigorous dynamic model-based economic design of the injected pultrusion process with controllability considerations. *Journal of Composite Materials*, 37, 1851-1880 (2003).
- Stroher, G. R., Zapparoli, E. L., de Andrade, C. R., Parabolic modeling of the pultrusion process with thermal property variation. *International Communications in Heat and Mass Transfer*, 42, 32-37 (2013).