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NEWTONIAN HEATING, THERMAL-DIFFUSION AND DIFFUSION-THERMO EFFECTS IN AN AXISYMMETRIC FLOW OF A JEFFERY FLUID OVER A STRETCHING SURFACE

M. Awais[1*], T. Hayat[2,3], M. Nawaz[4] and A. Alsaedi[3]

[1]Department of Mathematics, COMSATS Institute of Information Technology, Attock, Pakistan.
[2]Deparment of Mathematics, Quaid-i-Azam University, Islamabad, Pakistan.
[3]Department of Mathematics, Faculty of Science, King Abdulaziz University, P. O. Box 80257, Jeddah 21589, Saudi Arabia.
[4]Department of Applied Mathematics and Statistics, Institute of Space Technology, Islamabad, Pakistan.
E-mail: awais_mm@yahoo.com

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Abstract - In this communication we have investigated the phenomenon of Newtonian heating under the application of a uniform magnetic field when thermal-diffusion “Soret” and diffusion-thermo “Dufour” effects appear in the energy and concentration equations in a flow of a Jeffery fluid. The flow is induced by the stretching of a disk in the radial direction. The solutions of the nonlinear equations governing the velocity, temperature and concentration profiles are solved analytically “using HAM” and graphical results for the resulting parameters are displayed and discussed. Numerical values of local Nusselt and Sherwood numbers for different values of physical parameters are computed and shown. It is shown that the magnetic field retards the flow, whereas Newtonian heating acts as a boosting agent which enhances the flow. It is also noted that the combined Soret and Dufour effects on the temperature and concentration profiles are opposite.
Keywords: Soret and Dufour effects; Heat and mass transfer; Newtonian heating; Radial stretching.

INTRODUCTION

The dynamics of non-Newtonian fluids are of great importance in science and technology. Such flows have wide application in paint manufacturing, salt solutions, polymers, food items, cosmetics, etc. It is generally recognized that the governing equations analyzing such flow characteristics are of higher order and cannot be examined by a single constitutive equation. In view of this, various non-Newtonian models have been proposed in the past. A vast amount of literature is available on non-Newtonian fluid models. However, the rate-type fluids (a subclass of non-Newtonian fluids) are not properly addressed by other fluid models. This is because of the various complications that arise in terms of derivatives and constitutive equations of the rate-type fluid models. Thus, in recent years researchers Fetecau *et al.* (2009), Jamil and Fetecau (2010), Fetecau *et al.* (2010), Wang and Tan (2011) and Hayat and Awais (2011) have started analyzing the flow dynamics of rate-type fluids under different flow aspects including suction/injection at the boundaries, magnetohydrodynamics (MHD), heat and mass transport processes, thermal-diffusion and diffusion-thermo effects, thermal radiations etc.

Heat and mass transport processes are important in many physical situations. Such phenomena have wide applications in engineering and industries. Mass transport phenomena are basically the net

*To whom correspondence should be addressed

movement of mass from one location to another in the system. This process is used in different scientific disciplines for various systems and mechanisms that involve molecular and convective transport of atoms and molecules. The evaporation of water, the diffusion of chemical contamination in rivers and oceans from natural or artificial sources, separation of chemicals in distillation procedures, chemical species transfer between two phases through an interface or diffusion through a phase, in distillation etc. are some common examples of mass transport phenomena. Similarly, heat transfer is concerned with the exchange of thermal energy from one physical system to another. Merkin (1994) presented the most recent heating phenomenon, namely Newtonian heating (NH), in which the heat transfer rate from the bounding surface with a finite heat capacity is proportional to the local surface temperature and which is usually termed conjugate convective flow. Various authors have analyzed the simultaneous effects of heat and mass transport phenomena for the flows of non-Newtonian fluids under these conditions. For instance, hydromagnetic flow over a stretching sheet in the presence of heat and mass transfer was presented by Liu (2005). Cortell (2007) discussed mass transfer with chemically reactive species for two classes of viscoelastic fluid over a porous stretching sheet. Misra *et al.* (2008) presented the flow and heat transfer of a MHD viscoelastic fluid in a channel with stretching walls, with applications to haemodynamics. Hayat *et al.* (2012) presented mixed convection three-dimensional flow of an upper-convected Maxwell (UCM) fluid under magnetic field, thermal-diffusion and diffusion-thermo effects.

In this paper the flow of non-Newtonian fluid is analyzed by incorporating thermal-diffusion, diffusion-thermo, magnetic field and Newtonian heating effects over a radially stretching surface. Equations are modeled for the Jeffery fluid model. The Jeffrey model "Kothandapani and Srinivas (2008), Hayat *et al.* (2013), Nadeem *et al.* (2014) and Hayat *et al.* (2013)" is a relatively simpler linear model using time derivatives instead of convected derivatives as, for example, the Maxwell model or an Oldroyd-B model does. This fluid model represents a rheology different from the Newtonian fluid since this fluid model can easily predict relaxation and retardation time characteristics. The derived nonlinear differential equations are solved analytically by employing the homotopy analysis method (HAM). We have preferred HAM in view of its following advantages over the other numerical and analytical approaches. First, it gives the solution for each point within the domain of interest, unlike the numerical solutions, which are available for a particular run only for a set of discrete points in the domain. Secondly, compared to a numerical solution, a nicely produced approximate solution, requiring a minimal effort and having a reasonable amount of accuracy, is always handy for an engineer, scientist or an applied mathematician, who can obtain a solution quickly, thereby gaining valuable insight into the essentials of the problem. Thirdly, even with most of the scientific packages, some initial guess is required for the solution, as the algorithms, in general, are not globally convergent. In such situations approximate solutions can provide an excellent starting guess that can be readily refined to the exact numerical solution in a few iterations. Finally, aesthetically an approximate solution, if it is analytical, is more pleasing than a numerical solution. Due to all of the above mentioned advantages HAM has successfully been applied by various researchers in the field. For instance, Liao (2004) discussed and utilized the homotopy analysis method for the construction of the solutions of the nonlinear problems. Abbasbandy *et al.* (2011) investigated the mathematical properties of h-curve in the framework of the homotopy analysis method (HAM). Analytical approximate solutions for steady flow over a rotating disk in a porous medium with heat transfer have been analyzed by the homotopy analysis method by Rashidi *et al.* (2012). Awais *et al.* (2014) presented the time-dependent three-dimensional flow of an upper-convected Maxwell (UCM) fluid over a bidirectionally stretching surface. Unsteady flow induced by squeezing parallel disks has been studied by Qayyum *et al.* (2012).

In the present work, various graphical and numerical results are presented in detail to analyze the physical insight of the problem. Plots of residual errors are constructed to ensure the validity of the results. Tables are constructed to discuss the effects of local Nusselt and local Sherwood numbers in details. Comparison with published data is shown numerically and good agreement found between the present and previous results.

MATHEMATICAL FORMULATION

Let us consider the flow of a Jeffery fluid near the region of a stagnation point. The flow is induced due to the stretching of a surface in the radial direction located at $z=0$, whereas the fluid occupies the region $z \geq 0$. A constant magnetic field B_0 is applied perpendicular to the sheet. The constitutive relationships satisfying the Jeffery fluid model are:

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S}, \tag{1}$$

$$\mathbf{S} = \frac{\mu}{1+\lambda_1}\left(A_1 + \lambda_2 \frac{dA_1}{dt} \right), \tag{2}$$

in which p, $\mathbf{I}$, μ denote the pressure, the identity tensor and the dynamic viscosity, respectively, λ_1 the ratio of relaxation and retardation times and λ_2 the retardation time. Moreover, the quantity A_1 is defined as:

$$A_1 = \nabla\mathbf{V} + (\nabla\mathbf{V})^T, \tag{3}$$

$$\frac{d}{dt}(A_1) = \frac{\partial}{\partial t}(A_1) + (\mathbf{V}\cdot\nabla)A_1. \tag{4}$$

The laws of conservation of mass, momentum, energy and concentration under the boundary layer approach for the axisymmetric case yield:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{5}$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = \frac{\nu}{1+\lambda_1}\left[\frac{\partial^2 u}{\partial z^2} + \lambda_2\left(u\frac{\partial^3 u}{\partial r\partial z^2} + w\frac{\partial^3 u}{\partial z^3} + \frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial r\partial z} + \frac{\partial w}{\partial z}\frac{\partial^2 u}{\partial z^2}\right)\right] + \frac{\sigma B_0^2}{\rho}(u_e - u) + u_e\frac{du_e}{dr}, \tag{6}$$

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial z^2}\right) + \frac{Dk_T}{c_p c_s}\left(\frac{\partial^2 C}{\partial z^2}\right), \tag{7}$$

$$u\frac{\partial C}{\partial r} + w\frac{\partial C}{\partial z} = D\left(\frac{\partial^2 C}{\partial z^2}\right) + \frac{Dk_T}{T_m}\left(\frac{\partial^2 T}{\partial z^2}\right), \tag{8}$$

and the boundary conditions are given by:

$$\begin{aligned} &u = cr,\ w = 0, \frac{\partial T}{\partial z} = -h_s T,\ \frac{\partial C}{\partial z} = -h_c C \text{ at } z = 0, \\ &u = ar, \quad T \to T_\infty,\ C \to C_\infty \quad \text{as } z \to \infty, \end{aligned} \tag{9}$$

where u and w are the components of velocity in r and z directions, respectively, T and C are the temperature and concentration fields, c_p, k, ρ, ν, h_s, T_∞ are the specific heat of the fluid, the thermal conductivity, the density, kinematic viscosity, heat transfer parameter and the ambient temperature, respectively.

Utilizing the following similarity transforms:

$$\begin{aligned} &\eta = \sqrt{\frac{c}{\nu}}z,\ u = crf'(\eta),\ w = -2\sqrt{c\nu}f(\eta), \\ &\theta(\eta) = \frac{T - T_\infty}{T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_\infty}, \end{aligned} \tag{10}$$

the continuity equation is satisfied identically and Equations (6)-(8) reduce to the following equations:

$$\begin{aligned} &f''' + \beta\left(f''^2 - ff'''' - 2ff''''\right) \\ &\quad + (1+\lambda_1)\left[M^2(A - f') + A^2 + 2ff'' - f'^2\right] = 0, \end{aligned} \tag{11}$$

$$\theta'' + 2\Pr f\theta' + Du\Pr\phi'' = 0, \tag{12}$$

$$\phi'' + 2\Pr f\phi' + ScSr\phi'' = 0 \tag{13}$$

$$\begin{aligned} &f'(\eta) = 1, f(\eta) = 0,\ \theta'(\eta) = -\gamma(1 + \theta(\eta)), \\ &\phi'(\eta) = -\beta_1(1 + \phi(\eta)) \text{ at } \quad \eta = 0, \\ &f'(\eta) = A,\ \theta(\eta) = 0, \quad \phi(\eta) = 0, \text{ as } \quad \eta \to \infty, \end{aligned} \tag{14}$$

In above equations, β represent the Deborah number, M the Hartman number, A the ratio of free stream velocity to the stretching velocity, Pr the Prandtl number, Du the Dufour number, Sr the Soret number, γ the conjugate parameter for Newtonian heating and β_1 is the conjugate parameter for concentration. These are defined as:

$$\begin{aligned} &\beta = \lambda_2 c,\ M^2 = \frac{\sigma B_0^2}{\rho c},\ A = \frac{a}{c}, \\ &\Pr = \frac{\nu}{\alpha},\ \gamma = h_s\sqrt{\frac{\nu}{c}}, \beta_1 = h_c\sqrt{\frac{\nu}{c}}. \end{aligned} \tag{15}$$

The local Nusselt Nu_r and Sherwood numbers Sh_r are

$$Nu_r = \frac{rq_w}{k(T - T_\infty)}, \tag{16}$$

$$Sh_r = \frac{rj_w}{D(TC - C_\infty)}, \tag{17}$$

where the fluxes are:

$$q_w = -k(\frac{\partial T}{\partial z}) \; |_{z=0}, \tag{18}$$

$$j_w = -D(\frac{\partial C}{\partial z}) \; |_{z=0}, \tag{19}$$

In dimensionless form these quantities are:

$$(R_{e_r})^{-1/2} Nu_r = \gamma\left(1+\frac{1}{\theta(0)}\right), \tag{20}$$

$$(R_{e_r})^{-1/2} Sh_r = \beta_1\left(1+\frac{1}{\phi(0)}\right), \tag{21}$$

where $R_{er} = (ar^2 / \nu)$ denotes the local Reynolds number.

RESULTS

Equations (11)-(14) combined with the boundary conditions (15) were solved analytically by employing the HAM. Convergence of the results is shown in Table 1. It is noted that the series solutions converge in the whole region of η $(0<\eta<\infty)$ for $\hbar_f = -0.5 = \hbar_\theta$.

Table 1: **Convergence of the homotopy solutions for different order of approximation when $A=0.1=\gamma=\beta_1$, $M = 1.0$, $\beta = 0.2 = \lambda_1 = Du = Sr$ and $\Pr = 1.0$.**

Order of approximation	$-f''=(0)$	$-\theta'(0)$	$-\phi(0)$
1	1.32300	0.11234	0.11265
5	1.47271	0.11428	0.11511
10	1.47593	0.11454	0.11551
15	1.47596	0.11455	0.11554
25	1.47596	0.11455	0.11554
35	1.47596	0.11455	0.11554

Figures (1-3) are presented to show the graphs of residual errors in the velocity, temperature and concentration fields. From these figures it is noted that the error in our computation is negligible. Table 1 presents the convergence of the derived series solutions. It is noted that the convergent solutions are obtained only at the 15th order of approximation. Figures (4-11) analyze the results obtained for the velocity, temperature and concentration fields. Figure 4 shows that an increase in A causes an increase in velocity and the boundary layer thickness $(0 \le A < 1)$. It is also noted that the boundary layer thickness vanishes when $A = 1.0$. Furthermore when the free stream velocity is greater than the velocity of the stretching sheet, i.e., $A > 1$, the velocity increases and the boundary layer thickness decreases upon increasing A. Physically, the larger values of A accompany the higher free stream velocity, which results in an increase in the fluid motion. The effect of the Deborah number β on f' is presented in Figure 5. From this figure. it is noted that the fluid's velocity and the associated boundary layer thickness increase with an increase in β. Since the Deborah number β is dependent upon λ_2 (retardation time), physically a larger retardation time of any material makes it less viscous, resulting in an increase in its motion. Figure 6 elucidates the effects of λ_1 on f'. It is seen that an increase in λ_1, a viscoelastic parameter, retards the flow. Figure 7 presents the effects of the magnetic field M on the velocity profile. It is noted that the magnetic field retards the flow. From the physical point of view, when a magnetic field is applied to any fluid the apparent viscosity of the fluid increases to the point of becoming a viscoelastic solid. It is of great interest that the yield stress of the fluid can be controlled very accurately through variation of the magnetic field intensity. The result is that the ability of the fluid to transmit force can be controlled with the help of an electromagnet, which gives rise to many possible control-based applications, including MHD power generation, electromagnetic casting of metals, MHD ion propulsion etc. Figure 8 delineates the effects of the conjugate parameter γ on the temperature $\theta(\eta)$. The temperature increases rapidly with an increase in γ near the surface and a significant deviation in the temperature profile is observed for large values of γ. Moreover, the thermal boundary layer thickness is also an increasing function of γ. The simultaneous Soret (Sr) and Dufour (Du) effects are portrayed in Figure 9. It is observed that the temperature field increases with the Soret and Dufour numbers. Thus, we can conclude that the conjugate parameter for Newtonian heating γ, combined with the Soret and Dufour numbers, act as boosting agent to increase the temperature rapidly. Further, the effects of the concentration parameter β_1 are plotted in Figure 10. It is observed that the concentration field increases with an increase in β_1. The combined effects of Soret (Sr) and Dufour (Du) on the concentration field are shown in Figure 11. This graph shows that concentration field decreases rapidly near the surface with an increase in Sr and Du. Thus, we can conclude that Su and Du have opposite effects on the temperature and concentration fields.

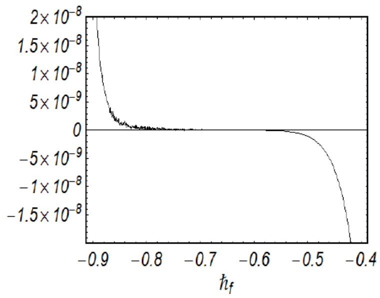

Figure 1: Residual error in f.

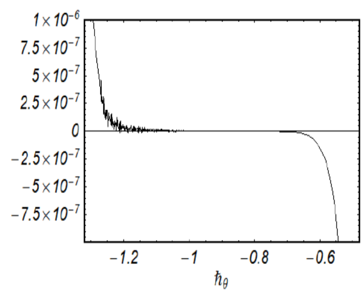

Figure 2: Residual error in θ.

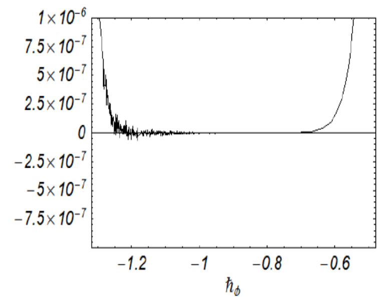

Figure 3: Residual error in ϕ.

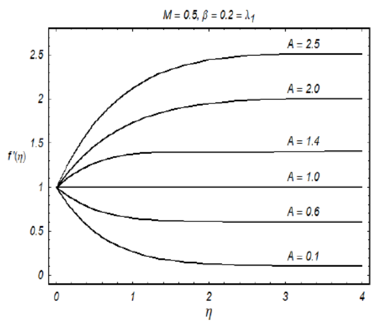

Figure 4: Influence of A on f'.

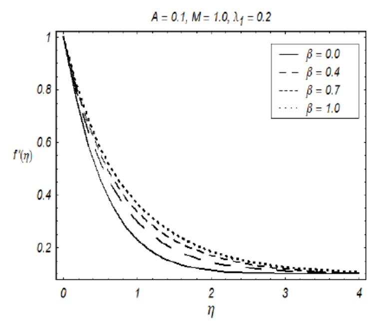

Figure 5: Influence of β on f'.

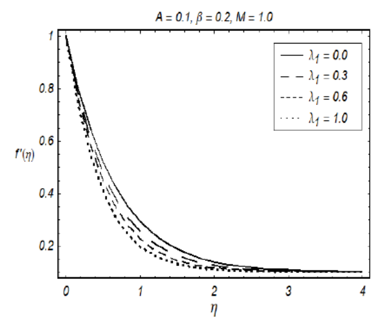

Figure 6: Influence of λ_1 on f'.

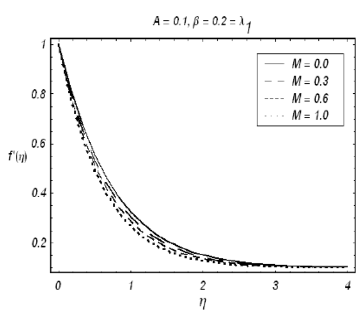

Figure 7: Influence of M on f'.

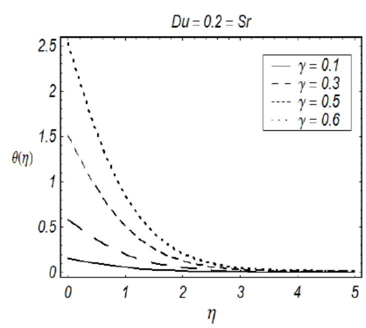

Figure 8: Influence of γ on θ.

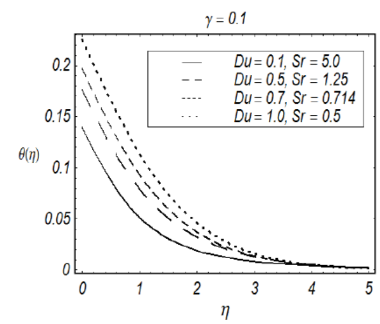

Figure 9: Influence of Du and Sr on θ.

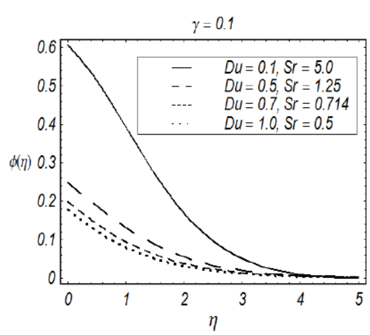

Figure 10: Influence of Du and Sr on ϕ.

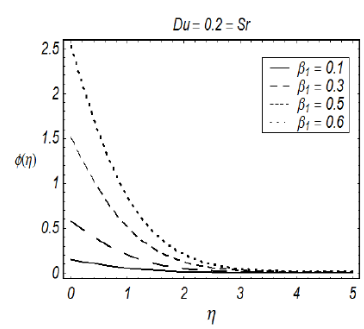

Figure 11: Influence of β_1 on ϕ.

Table 2 depicts a comparison of the present results with published work in the limiting sense "Attia (2007)". It is noted that the present results are in a good agreement with the published results "Attia (2007)". Numerical values of the local Nusselt and local Sherwood numbers for various values of embedding parameters are shown in Table 3.

Table 2: Comparison of the values of $f''(0)$ in the limiting case when $\beta = 0.0 = \lambda_1$.

	Attia (2007) results		Present results	
A	***M* = 0.0**	***M* = 1.0**	***M* = 0.0**	***M* = 1.0**
0.1	−1.1246	−1.4334	−1.124601	−1.433473
0.5	−0.7534	−0.9002	−0.753297	−0.899974
1.0	0.0	0.0	0.0	0.0
1.1	0.1821	0.2070	0.181935	0.206817
1.5	1.0009	1.1157	0.993184	1.094581

Table 3: Local Nusselt number for various values of embedding parameters.

M	A	β	γ	Sr	Df	$(R_{e_r})^{-1/2} Nu_r$	$(R_{e_r})^{-1/2} Sh_r$
1.0	0.0	0.2	0.1	0.2	0.1	-0.75283	-0.53155
	0.15					-0.76698	-0.55472
	0.3					-0.77653	-0.57085
	0.1	0.0				-0.75805	-0.53950
		0.2				-0.76296	-0.54807
		0.4				-0.76674	-0.55456
0.0		0.2				-0.77091	-0.56185
0.7						-0.76522	-0.55193
1.5						-0.75986	-0.54231
1.0			0.1			-0.76326	-0.54819
			0.2			-0.54590	-0.53141
			0.3			-0.32853	-0.50935
			0.1	0.1	0.2	-0.74676	-0.55461
				0.2	0.1	-0.75283	-0.53155
				0.3	0.0667	-0.76872	0.54188
				0.2	0.1	-0.75283	-0.53155
				0.1	0.2	-0.74676	-0.55461
				0.0667	0.3	-0.73007	-0.55674

FINAL OUTCOMES

In this communication, an axisymmetric stagnation point flow of a Jeffery fluid is considered when thermal-diffusion, diffusion-thermo, magnetic field and Newtonian heating effects are present. Graphs of residual errors are shown to validate the results. The final outcomes of the investigation are:

- Plots of residual errors show the validity of the analytical solution.
- It is noted that the magnetic field opposes the flow.
- Newtonian heating enhances the temperature of the fluid.
- Effects of thermal-diffusion and diffusion-thermo on the temperature and concentration profiles are in the opposite direction.
- The velocity of the fluid can be increased by increasing its retarding time.

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