USING THE WEIBULL FUNCTION FOR PROGNOSIS OF YIELD BY DIAMETER CLASS IN *Eucalyptus urophylla* STANDS

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ABSTRACT: This study aimed to fit the Weibull probability density function to stand data so as to represent the distribution of number of trees by diameter class, and also to fit models representing stand attributes so as to project growth and yield by diameter class and also yield of multiple wood products at the relevant rotation age (7 years). Data were collected from *Eucalyptus urophylla* stands owned by Anglo Americana plc (CODEMIM), a company situated in the municipality of Niquelândia, Goiás state. Initially a 3P-Weibull probability function was fitted by the percentile method for the aggregate stand. Then models were fitted and selected of stand attributes using residuals plot, residual standard error (s_{yx}) and coefficient of determination (R^2) as selection criteria. The hypsometric model of Trorey was then used to derive the average height of possible diameter classes, and the taper model of degree 5 and volumetric model of Takata were used to check compatibility of generated volumes. Prognosticated results were consistent with local productivity when compared to previous inventories complied by the same company of stands within the same age group.

Key words: Modeling, stand attributes, implicit models.

UTILIZAÇÃO DA FUNÇÃO WEIBULL PARA PROGNOSE DA PRODUÇÃO POR CLASSE DIAMÉTRICA PARA POVOAMENTOS DE *Eucalyptus urophylla*

RESUMO: *Nesta pesquisa, objetivou-se ajustar a função de densidade de probabilidade Weibull aos dados do povoamento, visando a representar a distribuição do número de árvores por classes de diâmetro bem como ajustar modelos que representem seus atributos para projetar o crescimento e a produção por classe diamétrica e o rendimento dos múltiplos produtos da madeira na idade da rotação adotada pela empresa (sete anos). Os dados provieram de povoamentos de Eucalyptus urophylla da empresa Anglo Americana plc (CODEMIM), localizada no município de Niquelândia no estado de Goiás. Inicialmente foi ajustada a função de probabilidade Weibull três parâmetros pelo método dos percentis, para o povoamento como um todo. Posteriormente, foram ajustados e selecionados modelos dos atributos do povoamento, utilizando-se como critérios de seleção o Gráfico de resíduos, Erro Padrão Residual* (s_{yx}) *e o Coeficiente de Determinação* (R^2)*. Em seguida, foi utilizado o modelo hipsométrico de Trorey para obtenção da altura média das possíveis classes diamétricas e utilizado o modelo de afilamento de 5° grau e o modelo volumétrico de Takata, visando à compatibilidade dos volumes por eles gerados. Os resultados prognosticados foram coerentes com a produção do local, quando comparados com inventários já realizados na empresa em povoamento com essa faixa de idade.*

Palavras-chave: Modelagem, atributos do povoamento, modelos implícitos.

1 INTRODUCTION

The structure of a forest consists of the distribution of trees and their respective size in relation to a unit area, being a result of species growth patterns, environmental conditions and forest management practices. In studying forest structure, diameter distribution is a basic, widely disseminated and applied tool that constitutes the simplest and most effective way to describe the characteristics of a given stand (BARTOSZECK 2000).

Diameter distribution is an efficient indicator of forest growth and the most powerful way to describe the properties of a stand. The diameter variable is derived from direct measurement of trees and is well correlated to other important variables, including volume, production quality and exploration costs (BAILEY & DELL 1973).

According to Scolforo (1998), knowledge of diameter distribution in planted forests is a critical requirement to ensure that prediction or prognosis of yield is implemented. In analyzing diameter structure,

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there is an interest in describing diameter frequency distribution by using probability density functions. A probability density function defines the probability associated to each value of the variable in question, or else it can describe the relative and/or absolute frequency distribution of the various tree sizes (CAMPOS & LEITE 2006).

Diameter distribution models can estimate the number of trees per hectare per diameter class at present and future ages. Then the use of a volume, taper or volumetric ratio equation allows estimating yield per diameter class, it being an important tool in situations where multiple wood products are concerned.

The Weibull distribution function was first applied to forest environment by Bailey & Dell (1973). Since then, several works on growth modeling have used the Weibull probability distribution, in Brazil and abroad, due to its efficacy (EISFELD 2004). Péllico Netto (1993) mentioned that its superiority in the large majority of cases is attributed to great flexibility, it being suitable for fitting skewed diameter distributions because its parameters have a correlation with stand variables.

Eucalyptus is the most extensively planted genus in the world, being used in various market segments, and that justifies sustained efforts in pursuing more detailed growth and yield projections with which to prescribe suitable management regimes seeking final product quality and to allow the forest industry to outline a supply plan always seeking to maximize profits.

The objectives of this study include:

- to fit the Weibull probability density function (p.d.f.) using the percentile method for species *Eucalyptus urophylla*;

- to estimate growth and yield by diameter class using the cumulative function and models representing attributes for the age interval 5 to 7 years;

- to check compatibility of estimated volume by the volumetric model and the taper model;

- to estimate the yield of multiple wood products for the relevant stand at the scheduled age of technical/ silvicultural rotation.

2 MATERIAL AND METHODS

2.1 Study site

The study site is owned by Grupo Anglo American plc (CODEMIN), a company situated in the municipality of Niquelândia, Goiás state, at an average altitude of 535 m. The local soil, according to Embrapa classification (1999), is predominantly nonhydromorphic red-yellow latosol with a latosolic B horizon, a low alumina/silica molar ratio, below 1.9, and color shifting from red to yellow with in-between shades. They usually are very deep or deep, with sequential A, B and C horizons and diffuse, gradual transitions between subhorizons, notably well drained. They are mostly alic, which means the percentage of aluminum saturation is above 50%, potentially reaching around 95%. They present medium texture and flat to rugged relief.

The local climate is Aw, according to Köppen classification, which is typical of tropical wet conditions, with two very defined seasons—dry winters, from May to August, and wet summers, from October to March—, and an average annual temperature of 28°C.

2.2 Stand database

Information from local forest inventory for ages three and five years, along with silvicultural data (initial density, survival) were the chief constituents to make the projection of stand attributes necessary to make the prognosis per diameter class.

21 permanent rectangular plots were used, each with 480 m². The experimental procedure used in two years of measurements was simple random sampling.

Information was collected from each sampling unit concerning: diameter at breast height (DBH), total height (Ht) and height of dominant trees (Hdom), according to Assmann description (1961).

2.3 Site classification

The forest sites were classified using curves developed by Miguel et al. (2008).

2.4 Fit of Weibull density function

Three fit methods have been described in literature by various authors for obtaining Weibull distribution coefficients: maximum likelihood, moments and percentiles. The Weibull distribution consists of three parameters, ‘a’, ‘b’ and ‘c’. The ‘a’ parameter (location parameter) represents the lower bound of the distribution. The Weibull distribution is known to be a very flexible function, assuming different forms according to diameter distribution requirements. The ‘c’ parameter represents different inclinations in the distribution (shape parameter) while the ‘b’ parameter represents the scale (scale parameter).

The location parameter in all three fit methods is an independent term that must be known prior to obtaining the scale and shape parameters.

The 3-P Weibull distribution used is represented as follows:

$$f(x) = \left(\frac{c}{b}\right) * \left(\frac{x-a}{b}\right)^{c-1} * \exp\left(-\left(\frac{x-a}{b}\right)\right)^{c}$$

where:
a = location parameter;
b = scale parameter;
c = shape parameter;
x = variable of interest (diameter class midpoint).

The method chosen to fit the Weibull function was the percentile method, considering the 30 and 90 percentile diameters, according to several authors who worked with this type of modeling for forest stands, including Scolforo (1998).

With the 'a' parameter defined, which represents the smallest measured diameter in the stand, the other parameters are obtained as follows:

$$c = \frac{\left[-Ln(1-p_1)/-Ln(1-p_2)\right]}{Ln\left[(X_{p1}-a)/(X_{p2}-a)\right]}$$

$$b = \frac{X_{p1}-a}{\left(-Ln(1-p_1)\right)^{1/c}}$$

where:
X_{p1} = percentile diameter 1;
X_{p2} = percentile diameter 2;
p_1 = percentile 1;
p_2 = percentile 2;
Ln = natural logarithm.

With the Weibull distribution coefficients derived, it is possible to describe the theoretical frequency and compare it to the observed frequency in the different diameter classes. A Kolmogorov (1933) nonparametric goodness-of-fit test was used to compare observed frequency with cumulative frequency. There being divergence between these two distributions, a comparison is made between the test D-value and a tabulated value, allowing to check if the distribution provides goodness-of-fit.

Let $F_0(X)$ be a theoretical cumulative distribution and $S_n(X)$ an observed distribution in a sample of 'n' observations. The maximum difference (D) between $F_0(X)$ and $S_n(X)$ is given as follows:

$$D = \max\left|F_0(X) - S_n(X)\right|$$

2.5 Selecting models of stand attributes

Estimating stand attributes such as basal area, minimum diameter, maximum diameter, average diameter, X_{P1} and X_{P2} percentile diameter, survival, along with the Weibull distribution, is a necessary step to make a projection of the number of individuals for the reference age (seven years). According to Schneider et al. (1988) and Souza (1999), this is the average rotation age for Eucalyptus in Brazil and is also the age of choice here. A number of models were developed for each attribute and the equations providing the best fit and accuracy statistics were then selected using the coefficient of determination (R^2.adj) and standard error of estimate for the variable of interest (s_{yx}) and in percentage (s_{yx}%) to express each attribute. These models were also tested and applied by Abreu et al. (2002), Scolforo & Machado (1996) and Thiersch (2002), and are presented in Scolforo (2006). The models to predict minimum diameter, maximum diameter, average diameter and 30 and 90 percentile diameters were derived using the Stepwise regression analysis procedure.

2.6 Growth and yield projection

Based on estimations of the above attributes using hypsometric, volumetric and taper models, a projection of growth and yield can be made in different diameter classes, following the next steps:

a) With the estimated parameters for the Weibull distribution function, the probability of trees occurring in each diameter class was determined. The product of this probability and the estimated number of surviving trees in the reference year allowed estimating the number of trees in each diameter class.

b) Having the number of trees in each diameter class, with the selected hypsometric equation, it was possible to obtain the total height of trees according to each diameter class midpoint value.

c) Diameter and height values of each projected diameter class were used, in the volume equation fit to generate the compatibility of the taper function, according to specifications in Table 1.

Table 1 – Specification of multiple wood products for *Eucalyptus urophylla*.

***Tabela 1** – Especificação dos múltiplos produtos da madeira para o Eucalyptus urophylla.*

Small-end diameter (cm)	Length (m)	Market
25	4.0	Sawn wood
18	2.8	Pallets
7	2.6	Pulp
< 7	-	Energy

3 RESULTS AND DISCUSSION

3.1 Forest Inventory

Characteristics and statistics of the stand at age 5 years are illustrated in Table 2, based on analysis of the 21 reference permanent plots.

Table 2 – Characteristics and statistics of the *Eucalyptus urophylla* stand.

***Tabela 2** – Características e estatísticas da povoamento de Eucalyptus urophylla.*

Characteristics/statistics	
Inventory Area (ha)	243
S.U. size (m²)	480
Initial Density	1666
No. of Possible S.U. (N)	5062
Admissible Error (%)	10
No. of Allocated S.U.	21
Average DBH (cm)	11.30
Average Height (m)	19.86
Dmin (cm)	5.00
Dmax (cm)	18.62
Average Individuals / ha	1550
Average mortality	116
Average volume per S.U. (m³)	6.3161
Standard Deviation of Volume per S.U. (m³)	0.5681
Standard Error of Mean per S.U.(m³)	0.1230
Standard Error of Mean per S.U. (%)	1.95
Coefficient of Variation (%)	9.00
Volume per Hectare (m³)	131.5855
Sampling Error (m³)	0.2573
Sampling Error (%)	5.53
Lower Confidence Interval per ha (m³)	125.0025
Upper Confidence Interval per ha (m³)	138.7275

S.U.: sampling unit; DBH: diameter at breast height (1.3m); Dmin: minimal diameter, Dmax: maximum diameter.

Results of estimated total volume with bark were found compatible with other inventories of the company in previous years, or in existing literature, according to works of Scolforo (1998) in northern Minas Gerais state, a region with similar soil types where the average annual growth (IMA) of *Eucalyptus urophylla* stands ranges from 25 to 50 m³. In this study, the IMA value was around 27 m³/ha.

3.2 Fit of Weibull distribution

The observed and estimated frequencies by the Weibull function (estimated a, b and c of 5.03, 4.71 and 7.61 respectively) are presented in Figure 1, for age 5 years.

The fit of the Weibull function was suitable to estimate the number of trees per diameter class by the Kolmogorov-Smirnov test, having in mind that the calculated D-value (0.031) was less than the tabulated D-value (0.035) for α = 0.05, an indication of goodness-of-fit of the model to the data.

3.3 Prognosis of growth and yield

The prognosis of growth and yield was based on the 3-P Weibull function as defined by the percentile method, and selected equations to express attributes constituting the basis of yield modeling by diameter class are illustrated in Table 3.

To estimate survival, the model proposed by Lenhart (1971) was selected. The basal area model presented satisfactory statistics, with unbiased estimation of the variable about the regression line, as with the models predicting minimum diameter, maximum diameter, average diameter, and 30 and 90 percentile diameters. All selected equations presented satisfactory statistics, with no over- or underestimation of the variables, which means they can be used without harm or implications for the estimations.

To estimate total height, the model proposed by Trorey (1932) was selected due to its better statistics. To estimate

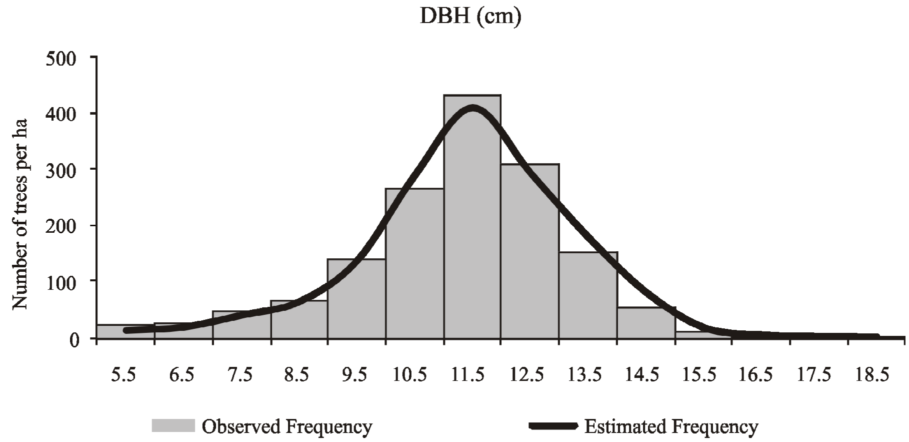

Figure 1 – Observed and estimated diameter distribution by the 3-P Weibull Function.

***Figura 1** – Distribuição diamétrica observada e estimada pela Função Weibull 3 parâmetros.*

Table 3 – Equations expressing attributes of the *Eucalyptus urophylla* stand in northern Goiás state.

***Tabela 3** – Equações que expressam os atributos do povoamento de Eucalyptus urophylla para região norte do estado de Goiás.*

Attributes	Selected equations	R² adj.	s_{yx}	$s_{yx(\%)}$
Site	$Hdom = 23.6772 * \left(1 - \exp\left(-30.1997 * Id\right)^{0.0140213}\right)$	0.85	1.18 m	6.31
Survival	$N_2 = N_1 * \exp\left(-0.0602076 * (Id_2 - Id_1) + 0.0844695 * \left(\frac{LnId_2}{LnId_1}\right)\right)$	0.96	20.0 tree	1.37
Basal Area	$G = 631.022 - 9.1375 * Hdon + 0.0770259 * N - 93.9344 * Ln(N)$	0.94	0.86 m²	5.73
Dmin	$D\min = 33.7934 - 6.14222 * Id + 0.3961 * Hdon + 9902.8 * \left(\frac{1}{N}\right)$	0.89	0.52 m	9.09
Dmax	$D\max = 7.7165 + 0.0598148 * Hdom$	0.92	1.22 m	8.22
Dmed	$Dmed = 9.25251 - 0.0040878 * N_2 + 0.147689 * G_2 + 0.27483 * Hdon_2$	0.96	0.20 m	1.84
Dp30	$Dp30 = 2.86731 - (0.00016034 * N_2) - (0.075231 * D\max) + (0.8156 * Dmed)$	0.86	0.13 cm	1.27
Dp90	$Dp90 = -5.3748 + (0.00015079 * N_2) - (0.31875 * D\max) + (1.0182 * Dmed)$	0.92	0.20 cm	1.55
Hypsom. Mod.	$Ht = 2.9604022 + (1.96985 * DAP) + (-0.045685 * (DAP^2))$	0.758	1.20 m	6.40
Takata Mod.	$Vt = \frac{DAP^2 * Ht}{29466.2422041 + (10.975966 * DAP)}$	0.988	0.0044m³	8.86
Taper Model	$d_i = DAP * \left[118505 - 3.34565 * \left(\frac{h_i}{Ht}\right) + 13.0994 * \left(\frac{h_i}{Ht}\right)^2 - 28.9369 * \left(\frac{h_i}{Ht}\right)^3 + 28.7884 * \left(\frac{h_i}{Ht}\right)^4 - 10.7912 \left(\frac{h_i}{Ht}\right)^5\right]$	0.9875	0.008 m³	8.85

R² adj: adjusted coefficient of determination; s_{yx}: standard error of mean adjusted for the variable of interest; Id_1 and Id_2: age, $Hdom_2$: dominant height at age Id_2; $Hdom_1$: dominant height at age Id_1; N_1 and N_2: number of surviving trees at respective age; *G*: basal area; $Dmin_n$: minimum diameter; $Dmax_x$: maximum diameter; *Dmed*: average diameter; D_{p30}: 30 percentile diameter; D_{p90}: 90 percentile diameter and *Ln*: natural logarithm; *Vt*: total volume; *Ht*: total height; *DBH*: diameter at breast height; *h*: height at different diameters; *di*: diameter at different heights.

volume, the model proposed by Takata (1959) was selected for its good statistics and unbiased dispersion of residuals. And lastly, the selected taper model was the polynomial model of degree 5 as proposed by Schöepfer (1966).

3.4 Prognosis of number of individuals per diameter class

Using equations relating to stand attributes and site classification it was possible to project attributes for age 7 and, so, with the Weibull probability distribution by the percentile method the diameter structure was estimated for age three (3), then for age five (5) and then the structure for age seven (7) was projected. Figure 2 illustrates the evolution of tree frequency by diameter class over the years.

As can be observed in Figure 2, the distribution curve of individuals frequency by diameter class becomes flatter as age increases, and with it the mean shifts to the right, representing a reduction in density yet with larger diameters and thus higher volumetric yield.

To improve volumetric estimators for the projected age (age seven), the diameter distribution of the stand, as described by the Weibull function, was fitted according to site yield level. In each site (three), location (a), scale (b), and shape (c) coefficients of the Weibull p.d.f. were derived by the percentile method for the projected age (7), whose values are illustrated in Table 4.

With the a, b and c parameters estimated, the occurrence probability of the number of individuals per diameter class was derived, which, multiplied by the number of trees (N), together with the class range in the reference age, allows obtaining the estimated frequency per site in each class based on the minimum diameter, as illustrated in Table 5.

With the number of trees per class obtained by the Weibull probability function, together with the diameter class midpoint value, and the adjusted hypsometric relationship it is possible to obtain the average height of the diameter class. The volume can be thus obtained by diameter class in different sites using a volume model, or using a taper function.

To check compatibility between the volume function (Takata model) and the taper function (polynomial of degree 5), as projected for year seven, Tables 6 (Site I), 7 (Site II), and 8 (Site III) provide values derived from each equation per hectare per diameter class as a

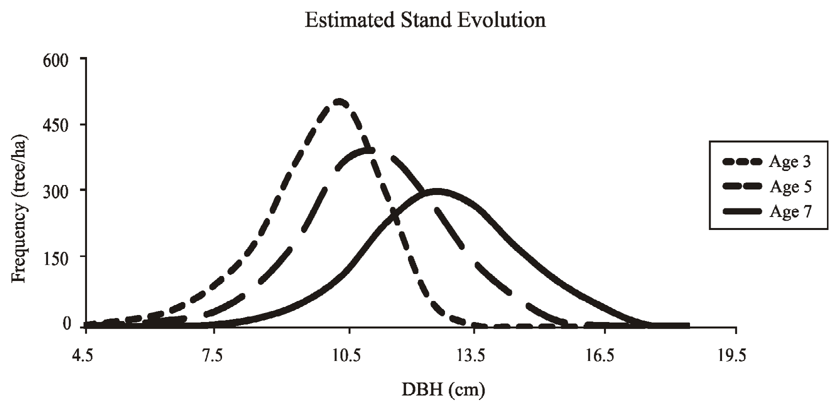

Figure 2 – Diameter evolution in the *Eucalyptus urophylla* stand, as predicted by the Weibull function, from year three to year five and then to year seven.

***Figura 2** – Evolução diamétrica do povoamento de Eucalyptus urophylla, predito pela Função Weibull, do ano três, para o ano cinco e posteriormente para o ano sete.*

Table 4 – a, b and c coefficients of the Weibull function by site at the projected age.

***Tabela 4** – Coeficientes a, b e c da Função Weibull por sítio na idade projetada.*

Parameters	Site I	Site II	Site III
a	7.39	6.34	5.52
b	8.68	5.91	6.91
c	5.86	3.21	4.00

Table 5 – Estimation of number of trees per hectare at age 7 per DBH class and per site.

***Tabela 5** – Estimativa do número de árvores por hectare aos 7 anos por classes de DAP e por sítio.*

DBH class midpoint value (cm)	Number of trees per hectare		
	Site I	Site II	Site III
6.5	-	-	2
7.5	6	10	19
8.5	20	75	64
9.5	45	130	140
10.5	67	210	233
11.5	80	230	300
12.5	200	273	294
13.5	250	180	210
14.5	360	110	101
15.5	310	101	31
16.5	160	89	5
17.5	35	49	0
18.5	3	22	0
19.5	2	1	0
20.5	1	-	0
21.5	-	-	0
Total	1540	1479	1400

function of DBH (diameter at breast height) and Ht (total height), multiplied by the likely number of trees in each diameter class as projected by the Weibull function.

It can be noted from Tables 6, 7 and 8 that the volumes projected for year seven by the volumetric model of Takata and by the taper model of degree 5 for different sites are very close, despite invariably larger volumes being provided by the taper model in different diameter classes in the three sites, around 3% more, not affecting model compatibility. According to the Student's t-test, in site I the calculated $t_{0.05}$ value was 0.064 and the tabulated value was 2.160, while in site II the calculated $t_{0.05}$ value was 0.111 and the tabulated value was 2.178, and in site III the calculated $t_{0.05}$ value was 0.051 and the tabulated value was 2.228. It is thus clear that no significant difference exists in deriving volume, regardless of the model used.

3.5 Yield estimations of multiple products

Criteria for definition of multiple products are illustrated in Table 1. Results obtained with prognosis by diameter class and with the taper functions fitted for each site separately allowed estimating possible total volume with bark to be yielded in each site for each product at age seven, as is illustrated in Figure 3.

Site I had a total volume with bark yield of 223.8996 m³/ha, at age seven, 150.1314 m³ of which can be destined for pulp production and 73.7682 m³ for energy production.

Products such as sawn wood and pallets could not be obtained due to the small-end diameter being less than the minimum requirement, reached only in later ages.

Site II had a total volume with bark yield of 165.8957 m³/ha, 111.6337m³ of which can be destined for pulp, and 54.2320m³ for energy production. And lastly, site III had a total volume with bark yield of 128.1017 m³/ha, 88.3717m³ of which can be used in pulp production and 32.7299m³ can be used in energy production.

In the different sites of the stand, volume prognoses for year seven are consistent with other inventories conducted by the company referred to earlier on in stands of the same age. It is clear and evident that if the stand is managed correctly, even at later ages, products demanding larger small-end diameters can be obtained. In

Table 6 – Volume projected for year 7 per DBH class, using the Takata model and the polynomial of degree 5, in Site I.

Tabela 6 *– Volume projetado para o ano 7 por classes de DAP, pelo modelo volumétrico de Takata e pelo polinômio do 5° grau, para o Sítio I.*

DBH (cm)	Total H (m)	N/ha	Volumetric model	Taper model
6.5	13.83	0	0.0000	0.0000
7.5	14.65	7	0.1732	0.2078
8.5	15.10	21	0.6819	0.7370
9.5	15.73	46	2.0533	2.1613
10.5	16.05	67	4.0078	4.1286
11.5	17.98	80	6.4283	6.6244
12.5	18.95	200	20.0040	20.6221
13.5	20.08	250	30.8936	31.8602
14.5	21.05	360	53.7807	55.4830
15.5	22.15	310	55.6639	57.4478
16.5	22.89	158	33.2112	34.2883
17.5	23.44	34	8.2300	8.4994
18.5	23.77	3	0.8225	0.849902
19.5	24.00	2	0.6150	0.6356
20.5	24.14	1	0.3417	0.3533
21.5	24.19		0.0000	0.0000
Total		1539	216.9071	223.8996

N/ha: number of trees per ha

Table 7 – Volume projected for year 7 per DBH class, using the Takata volumetric model and the polynomial of degree 5, in Site II.

Tabela 7 *– Volume projetado para o ano sete por classes de DAP, pelo modelo volumétrico de Takata e pelo polinômio do 5° grau, para o Sítio II.*

DBH (cm)	Total H (m)	N/ha	Volumetric model	Taper model
6.5	13.83	0	0.0000	0.0000
7.5	14.65	10	0.2887	0.2969
8.5	15.10	75	2.5573	2.6323
9.5	15.73	130	5.9316	6.1081
10.5	16.05	210	12.5618	12.9401
11.5	17.98	230	18.4813	19.0453
12.5	18.95	273	27.3055	28.1492
13.5	20.08	180	22.2434	22.9392
14.5	21.05	110	16.4330	16.9533
15.5	22.15	101	18.1357	18.7168
16.5	22.89	89	18.7076	19.3143
17.5	23.44	49	11.8608	12.2491
18.5	23.77	22	6.0316	6.2326
19.5	24.00	1	0.3075	0.3178
20.5	24.14	0	0.0000	0.0000
21.5	24.19	0	0.0000	0.0000
Total		1480	160.8457	165.8957

N/ha: number of trees per ha

Table 8 – Volume projected for year 7 per diameter class, using the Takata volumetric model and the polynomial of degree 5, in Site III.

Tabela 8 *– Volume projetado para o ano 7 por classe diamétrica, pelo modelo volumétrico de Takata, e pelo modelo de afilamento do Polinômio do 5° Grau, para o Sítio III.*

DBH (cm)	Total H (m)	N/ha	Volumetric model	Taper model
6.5	13.83	2	0.0396	0.0407
7.5	14.65	19	0.5485	0.5642
8.5	15.10	64	2.1822	2.2463
9.5	15.73	140	6.3879	6.5779
10.5	16.05	233	13.9376	14.3576
11.5	17.98	300	24.1060	24.8417
12.5	18.95	294	29.4059	30.3145
13.5	20.08	210	25.9506	26.7624
14.5	21.05	101	15.0885	15.5662
15.5	22.15	31	5.5664	5.7448
16.5	22.89	5	1.0510	1.0851
17.5	23.44	0	0.0000	0.0000
18.5	23.77	0	0.0000	0.0000
19.5	24.00	0	0.0000	0.0000
20.5	24.14	0	0.0000	0.0000
21.5	24.19	0	0.0000	0.0000
Total		1399	124.2642	128.1017

N/ha: number of trees per ha

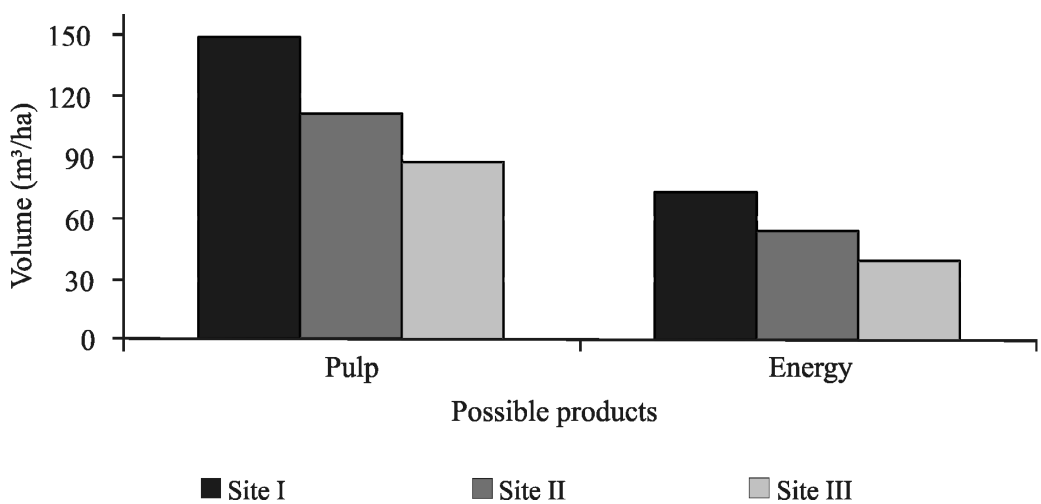

Figure 3 – Volumetric yield of possible commercial products in a *Eucalyptus urophylla* stand at age seven.

Figura 3 *– Rendimento volumétrico para os possíveis produtos comerciais em um povoamento de Eucalyptus urophylla para uma idade de sete anos.*

order to obtain larger-sized pieces, thinning becomes a highly recommended practice.

4 CONCLUSIONS

The fit of the Weibull probability density function by the percentile method at the point of maximum difference between observed and theoretical curves provided satisfactory goodness-of-fit when submitted to the nonparametric Kolmogorov-Smirnov test for ($\alpha = 0.05$).

Growth and production estimations for the *Eucalyptus urophylla* stand in northern Goiás are consistent with other inventories conducted in similar areas, and with previous inventories conducted by the company subject of this study.

The $t_{0.05}$ values showed no significant difference in obtaining the total volume variable regardless of the model being used, taper or volumetric.

In none of the three sites was it possible to obtain timber for sawn wood or pallets, due to larger small-end diameter requirements that the age in question was unable to meet, or perhaps because of local yield characteristics.

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