Modelagem e otimização da operação de veículos na mineração de cobre subterrânea

Modeling and optimization of vehicle operations in underground copper mining

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Resumo

Na mineração subterrânea, diariamente, uma frota de veículo LHD (load-haul-dump) deve ser alocada à rede de galerias para extrair o minério segundo um plano estratégico de trabalho. Esse plano é definido pela alta gerência e contém o número de cargas a serem extraídas desde os pontos de remoção de uma dada galeria para cada turno de trabalho. Nesse trabalho, formula-se um modelo de programação inteira (PI), para minimizar o makespan de uma galeria de trabalho, e propõe-se um algoritmo ótimo em tempo polinomial, para a sua resolução. Em seguida, obtém-se um conjunto de regras de decisão, a partir do algoritmo proposto, o qual é integrado a um processo de tomada de decisão (PTD). Esse PTD é de simples execução para os operadores de LHDs, determina o makespan ótimo e, se existe a possibilidade de conclusão dentro do turno de trabalho. Finalmente, realiza-se uma análise comparativa entre o PTD proposto e àquele utilizado na mina chilena subterrânea de cobre *El Teniente*. Os resultados mostram que a experiência acumulada dos operadores converge para soluções próximas ao makespan ótimo.

Palavras-chave: Pesquisa operacional na mineração, algoritmo ótimo em tempo polinomial, processo de tomada de decisão, gestão da frota dos veículos da mineração.

Abstract

In underground mining, daily a fleet of LHDs must be allocated to a haulage network of drifts for extracting ore according to a plan-driven strategy. This plan is hierarchically decided by a higher management level and it contains the number of ore bucketfuls to be extracted from each drawpoint within a drift for each working shift. In this paper, an integer programming (IP) model for minimizing the makespan of drift workload is formulated and a polynomial time optimal algorithm for its resolution is proposed. Next, a set of decision rules obtained from the algorithm above is integrated into the decision-making process (DMP). This DMP is simple to execute for LHD operators, determines the optimal makespan, and whether or not it can be carried out in the working shift. Finally, a comparative analysis between the DMP proposed and the DMP used in El Teniente copper underground Chilean mine is studied. The results show that the cumulative operators experience has converged to solutions near to optimal makespan.

Keywords: *Operations research in mining, polynomial time optimal algorithm, decision making process, mining vehicles fleet management.*

1. Introduction

In block caving underground mining, good production-level performance is the key for all the other levels vertically positioned as reduction and transportation. At this latter level, there is a haulage network containing parallel drifts, turning points and drawpoints, where ore must be extracted by LHDs (load-haul-dump) according to the fleet management established.

LHD fleet management addresses three decision problems: dispatching, routing and scheduling. The aim of dispatching is to choose, for one or many vehicles, a new destination (loading or dumping sites). Routing consists of choosing the best route (road segments) from the origin to the destination. Finally, scheduling consists of deciding the speeds and waiting times of vehicles on road segments of a route to avoid conflicts between vehicles (Gamache et al., 2005).

Ideally, the optimal decision must solve the three problems in a unique model. Nevertheless, its development and later implementation assumes that data on vehicle location are available in a certain period, that waiting and stop areas at the drift are known and fixed, and that decisions of dispatching, routing and scheduling are integrated (Bealieu and Gamache, 2006).

In practice, scheduling and dispatching problems are solved through a plan-driven strategy for a working shift elaborated by high-level management. In this plan, the production goal is computed as a number of ore bucketfuls to extract from a set of drawpoints within a drift, given the average operating speeds and bucket capacity of LHDs (Córdova et al., 2008).

At the beginning of each working shift, the supervisor allocates operators for the LHDs and drifts, taking into account the current resources status. Thereafter, the LHD operator decides by himself the routing. In general, the operator does not have any kind of decision support system for the routing and he uses rules of thumb according to his experience (Córdova et. al, 2004).

In this paper, the LHD operation problem to minimize makespan of drift workload is studied. The work objective is to propose a simple and efficient decision making process (DMP) for an LHD operator. The paper is organized as follows: in section two, the integer programming (IP) model is formulated; in section three, the polynomial time optimal algorithm is presented, its integration into a decision making process (DMP) is made and an illlustrative example is shown; in section four, the comparative analysis between the DMP proposed and the DMP used in the *El Teniente* copper underground Chilean mine is studied; and finally, in section five, the conclusions and directions for further research are given.

2. Integer programming model

The integer programming (IP) model is based on haulage network representation through a digraph, according to drift characteristics and plan-driven strategy data. The model objective is to determine the LHD work path, in order to minimize the makespan subject to the operational constraints. The model is defined as follows.

Let $k \in \{L,R\}$ denote the "k" side of drift, where $k=L$ is left side and $k=R$ is right side. Let $STSD=SSDL \cup SSDR$ be the set of drawpoints selected by the plan -driven strategy, where $SSDL \neq \emptyset$ and $SSDR \neq \emptyset$ are the sets of drawpoints located to left side and right side of drift, respectively. Each drawpoint $i \in STSD$ has a number of ore bucketfuls to be extracted B_i, a transfer time between it and the initial point IT_i; and a the transfer time between it and the dumping site / turning point FT_i. On the other hand, LHD vehicle has a transfer time between the initial point and the dumping site / turning point TL, a turning time TT, and a loading time LT and an unloading time UT for an ore bucketful.

In addition, given the topology of the mine, the characteristics of the drifts and the operational constraints, such as, LHD must enter to drawpoint with its shovel in front position, and it must travel to a turning point and return in opposite direction due to narrow angle; two simplifications are introduced to reduce the number of variables from the model:

a) If LHD is at the dumping site/turning point and is ready to extract ore at the *i-th* drawpoint, then it will work in this *i-th* drawpoint until its workload is completed and.

b) LHD can turn to the opposite side of the drift, if and only if the workload of a side drift is completed.

Thus, the variables of the representative digraph define the LHD operation by four types of movements: 1) from the initial point (with empty bucket) to the first drawpoint in the path, 2) from a drawpoint to the dumping site and vice versa (until workload is completed), 3) from turning point to the first drawpoint on the other side of the drift, and 4) from the dumping site back to the initial point. The variables, constraints and objective function of LHD operation problem are presented as follows.

Variables

$$X_{1k} = \begin{cases} 1 \text{ if the LHD vehicle begins working at the } k \text{ side of the drift, } k \in \{L,R\} \\ 0 \text{ in other case} \end{cases}$$

$$X_{2i} = \begin{cases} 1 \text{ if the LHD vehicle begins working the } i\text{-th} \text{ drift coming from the entrance} \\ 0 \text{ in other case} \end{cases}$$

$$X_{3i} = \begin{cases} 1 \text{ if the LHD vehicle begins working the } i\text{-th} \text{ drawpoint from the dumping site / turn point} \\ 0 \text{ in other case} \end{cases}$$

$$X_{4k} = \begin{cases} 1 \text{ if the LHD vehicle turns back from the } k \text{ side of the drift to the other side } k \in \{L,R\} \\ 0 \text{ in other case} \end{cases}$$

$$X_{5k} = \begin{cases} 1 \text{ if the LHD vehicle turns back from the } k \text{ side to the initial point of the drift } k \in \{L,R\} \\ 0 \text{ in other case} \end{cases}$$

Constraints

$$X_{1L} + X_{1R} = 1 \quad (1)$$

$$\sum_{i \in SSDL} X_{2i} = X_{1L} \quad (2)$$

$$\sum_{i \in SSDR} X_{2i} = X_{1R} \quad (3)$$

$$X_{2i} + X_{3i} = 1 \qquad \forall i, i \in STSD \quad (4)$$

$$X_{1L} = X_{4L} \quad (5)$$

$$X_{1R} = X_{4R} \quad (6)$$

$$X_{4L} = X_{5R} \quad (7)$$

$$X_{4R} = X_{5L} \quad (8)$$

$$\sum_{i \in SSDL} X_{3i} + X_{5L} + X_{4L} = \sum_{i \in SSDL} 1 + X_{4R} \quad (9)$$

$$\sum_{i \in SSDR} X_{3i} + X_{5R} + X_{4R} = \sum_{i \in SSDR} 1 + X_{4L} \quad (10)$$

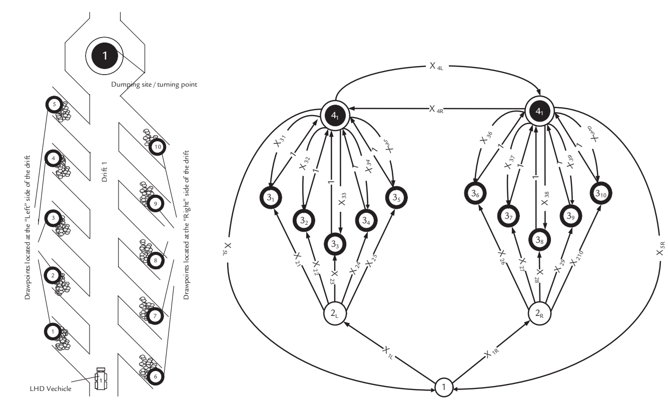
$$X_{5L} + X_{5R} = X_{1L} + X_{1R} \quad (11)$$

Objective function

$$M = \sum_{i \in STSD} (2B_i - 1 + X_{3i})\, FT_i + X_{2i} IT_i + (LT + UT) B_i + \sum_{k \in \{L,R\}} X_{4k} TT + \sum_{k \in \{L,R\}} X_{5k} TL \quad (12)$$

Figure 1 shows an example of a digraph representing a haulage network according to the plan-driven strategy data.

Date	1/10/2007				
Drift	1				
Working shift	2				
LHD vehicle number	3				
	Left side				
Drawpoint	**1**	**2**	**3**	**4**	**5**
Bucketfuls	10	13	13	11	9
	Right side				
Drawpoint	**6**	**7**	**8**	**9**	**10**
Bucketfuls	7	10	13	14	10
Dumping site /turning point	1				



(a) Plan-driven strategy data. (b) Graphical view of drift. (c) Digraph representing a haulage network.

Figure 1
An example of a digraph representing a haulage network according to plan-driven strategy data.

In the digraph, edge one represents the beginning of work at the drift entrance. Edge $2k$ is the *k-th* side of the drift, with $k \in \{L,R\}$. Edge $3i$ is the *i-th* drawpoint, with $i \in STSD$. The edges number four represent the dumping site / turning point of the drift. This representation of dumping site / turning point by two edges allows to model the turn of the vehicle when changing direction. The arc joining the *i-th* drawpoint with the dumping site / turning point is equal to one and it stands for the last trip. This arc is meant to establish a flow balance at the *i-th* drawpoint; that is, given that a last trip to the dumping site must take place, then an arrival to that drift must occur. Constraints (1), (2), (3), (4), (9), (10) and (11) can be classified as the equations for flow balancing, while constraints (5), (6), (7) and (8) are equations for determining a logical path in such a way that LHD leaves the drift in a direction opposite to its entry.

3. Resolution algorithm and decision making process

Although, the solution of the IP model proposed can be obtained by using specialized software (e.g., LINGO®, AMPL®, GAMS®, among others), in practice, the use of such tools may be difficult given that skills and knowledge would be needed for the decision makers (e.g. LHD operator). On the other hand, a decision support system with an easy-to-use interface surely can help in this task, but an investment would be necessary (e.g., optimization engine, customized software, training, etc.).

Therefore, the study and definition of decision rules based on a resolution algorithm is a very interesting subject for approaching the real implementation. Ideally, this should be simple to execute for the workers and the resolution algorithm should lead to optimal solution in polynomial time. Two good examples are Johnson's rule (Johnson, 1954) for minimizing the makespan in flow shops with n jobs and two machines and McNaughton's rule (McNaughton, 1959) for minimizing the makespan in a parallel machine with n jobs and preemption. In both cases, the optimal result is found in a polynomial time $O(n)$.

Definition of resolution algorithm

In this work, an optimal algorithm for the LHD operation problem is presented. This algorithm yields the variable values of the IP model proposed and the optimal makespan of the drift workload defined by the plan-driven strategy.

ALGO 1

Input: Plan-driven strategy data, TL, LT, UT, TT, IT_i and FT_i $i \in STSD$

Step 1: Select *i*-th* drawpoint such that $(IT_{i^*} - FT_{i^*}) = \underset{i \in STSD}{Min}(IT_i - FT_i)$

Step 2: Define X_{2i^*}:=1; X_{2j} :=0; X_{3i^*}:=0; X_{3j}:=1 such that $j \in STSD \setminus \{i^*\}$

Step 3: *If i*-th* drawpoint is in left side of drift *then*

X_{1L}:=1; X_{4L} :=1; X_{5R}:=1; X_{1R}:=0; X_{4R}:=0; X_{5L}:=0

Else

X_{1L}:=0; X_{4L} :=0; X_{5R}:=0; X_{1R}:=1; X_{4R}:=1; X_{5L}:=1

Step 4: Compute the optimal makespan by equation (12)

Theorem 1. *The algorithm ALGO 1 gives an optimal solution for the LHD vehicle operation problem and polynomial time*.
Proof. Let M be the makespan for the workload assigned to the drift and expressed by equation (12). Then, it can be broken down into three parts: equation (13) represents the total time for the loading and unloading of ore bucketfuls and the trips from / to the drawpoints selected by the plan-driven strategy on both sides of the drift; equation (14) expresses the turning time and equation (15) the exit time, that is the travel time between the dumping site / turning point and the initial point.

$$\sum_{i \in SSDL} (2B_i - 1 + X_{3i})\, FT_i + X_{2i} IT_i + (LT + UT)B_i + \sum_{i \in SSDR} (2B_i - 1 + X_{3i})\, FT_i + X_{2i} IT_i + (LT + UT)B_i \quad (13)$$

$$\sum_{k \in \{L,R\}} X_{4k} TT \quad (14)$$

$$\sum_{k \in \{L,R\}} X_{5k} TL \quad (15)$$

Since B_i is the number of ore bucketfuls to be extracted from the *i-th* drawpoint, then the number of trips from / to the *i-th* drawpoint is $2B_i$. In (13) the value $2B_i$ - 1 is the number of trips from / to the *i-th* drawpoint without considering the initial trip (i.e., the first trip to the *i-th* drawpoint). The variables X_{2i} and X_{3i} are the options for the initial trip to the *i-th* drawpoint where only one must be selected (see constraint (4)). The total time of loading and unloading is $(LT + UT)B_i$ as shown in equation (13). Expressions (14) and (15) represent the time for the turning of the LHD and the time for the final trip, (i.e. the trip between the dumping site / turning point considered by plan-driven strategy to the initial point), respectively. By considering the binary definition of the variables, it can be shown that the value of the objective function given in equation (12) subjected to constraints (1), (2), (3), (4), (5), (6), (7) is equal to (16) and finally reduces to (17).

Therefore, to find *i*-th* drawpoint equal to arg $Min_{i \in STSD} (IT_i - FT_i)$ allows to encounter the variable X_{2i^*}=1 and the values of the other variables by using the constraints of the problem.

$$\underset{i \in STSD}{Min}(IT_i - FT_i) + \sum_{i \in STSD} (2FT_i + LT + UT)B_i + TT + TL \quad (16)$$

$$\underset{i \in STSD}{Min}(IT_i - FT_i) + C \qquad C \text{ constant} \quad (17)$$

Let n be the number of drawpoints considered by the plan-driven strategy. Step 1, Step 2 and Step 4 run in O(n) time while Step 3 runs in O(1) time.

Definition of the decision rules and decision-making process

The definition of the decision rules are obtained from ALGO 1 and they work in conjunction to develop a decision making process (DMP) for the LHD operators. The decision rules are defined as follows:

Decision rule 1 (R1): *"For a given plan-driven strategy, the i*-th chosen drawpoint to start with at the drift is the one for which the difference between the transfer time from the initial (entrance) point and the transfer time towards the dumping site/turning point, is minimum; that is, where* $(IT_i - FT_i)$ *is minimum".*

Decision rule 2 (R2): *"The minimum makespan of the drift workload defined by the plan-riven strategy is equal to equation (16).*

Decision rule 3 (R3): *"The drift workload determined by the plan-driven strategy must be objected (e.g., negotiated, modified, or rejected) if the available time of a working shift TWS is less than the minimum makespan of the drift workload defined by the plan-driven strategy".*

Decision rule 4 (R4): *"The total number of ore bucketfuls assigned to a drawpoint by the plan-driven strategy must be extracted iteratively as many times as defined by the plan".*

Decision rule 5 (R5): *"The workload assigned to a side of the drift must be fully completed before changing to the remaining side, following any order to visit the remaining drawpoints".*

R1 and R2 are obtained from ALGO 1. They show the first drawpoint to visit in the work path and the minimum makespan value for drift workload within a working shift, given the plan-driven strategy data and the average for operation and transfer times(i.e., times for loading, humping, moving, dumping, turning).

R3 gives a feasibility condition for the fulfillment of drift workload within the working shift.

R4 and R5 are the result of two simplifications introduced into the IP model to reduce the number of variables. R4 offers a feasible way to complete the total workload assigned to the *i-th* drawpoint by minimizing the travel time of the LHD vehicle between the drawpoint and the dumping site, while R5 offers a feasible way to complete the workload assigned to a drift side by minimizing the LHD vehicle's turning time. In R5, an important remark is that the remaining drawpoints at the same drift side can be visited in any order, without affecting the minimum makespan value.

The five decision rules above are integrated into a DMP in a sequential way as shown in Figure 2.

A complete example of DMP use is given in Figure 3.

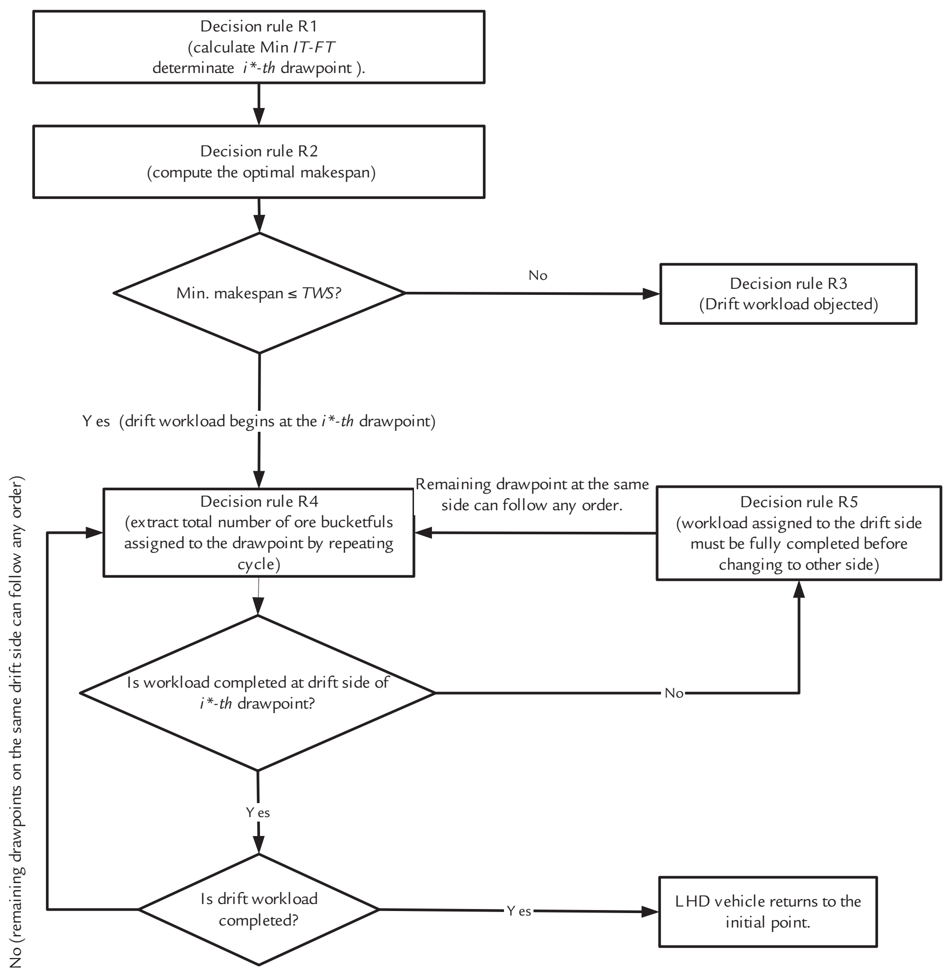


Figure 2
Flow diagram of the DMP proposed for the LHD operation problem.

4. A comparative analysis of DMP: *El Teniente* underground mine case

A comparative analysis between the current and the proposed DMP is realized. The objective is compute the improvement of makespan due to the DMP proposed. For this, the DMP used in *El Teniente* copper Chilean underground mine is described and numerical cases are studied, by using data available from Dubos (2006). The total number of cases (plan-driven strategies) are 90 (three months), which consider different topologies of drift; and the production goals for each drawpoint are between 29 and 280 ore tons.

Date				1/10/2007	
Drift				1	
Working shift				2	
LHD vehicle number				1	
LT (s)	60	*TT (s)*		120	
UT (s)	10	*TL (s)*		360	
	Left side				
Drawpoint	**1**	**2**	**3**	**4**	**5**
Bucketfuls	10	13	13	11	9
IT (s)	240	260	280	305	330
FT (s)	120	100	80	55	30
	Right side				
Drawpoint	**6**	**7**	**8**	**9**	**10**
Bucketfuls	7	10	13	14	10
IT (s)	250	270	295	325	350
FT (s)	110	90	65	35	10
Dumping site /turning point					1
TWS (s)					25,200

(a) The data of plan-driven strategy and average time of operation and transfers are given to the LHD operator.

	Left side				
Drawpoint	**1**	**2**	**3**	**4**	**5**
IT - FT (s)	120	160	200	250	300
	Right side				
Drawpoint	**6**	**7**	**8**	**9**	**10**
IT - FT (s)	140	180	230	290	340
Min IT - FT (s)					120
i-th* drawpoint					1
OPT Makespan (s)					23,340
Feasibility condition (OPT Makespan < *TWS*)					Yes

(b) The LHD operator uses R1 to calculate Min *IT - FT* and to determinate *i*-th* drawpoint (R1 used). Next, he uses R2 to compute to optimal makespan and decides if the fulfillment of drift workload is feasible by using R3.

Drawpoint	Arrival time(s)	Working time (s)	Predecessors
1	240	2,980	-
2	3,320	3,410	1
3	6,810	2,910	2
4	9,775	1,925	3
5	11,730	1,140	4
6	13,100	1,920	5
7	15,110	2,410	6
8	17,585	2,535	7
9	20,155	1,925	8
10	22,090	890	9
Depart time from initial point (s)			0
Arrival time from initial point (s)			23.340

(c) Given, the fulfillment feasibility of the drift workload (by R3) and the selection of number one as first drawpoint to visit from the initial point (by R1), the LHD operator decides the work sequence. Thus, he extracts the total of ore bucketfuls from one number drawpoint (R4 is used). Next, he arrives to another drawpoint on the same side of the drift following any order (R5 is used). In this new drawpoint, he extracts the total of ore bucketfuls before changing to another drawpoint on same side (R4 is used). The two actions above are repeated before changing to the remaining side of drift (Recurrently, R4 and R5 are used). Once completing one drift side, he turns and arrives to one drawpoint at the other side of the drift (R5 is used). On this side, he repeats the two actions used before changing sides, i.e. the total of ore bucketfuls is extracted before changing to another drawpoint on the same side and the visits to the drawpoints on the same side can follow any order (Recurrently, R4 and R5 are used). Finally, once completing the drift workload, the LHD operator travels to the initial point.

Figure 3
An example of DMP use.

DMP in *El Teniente* underground mine. When is it good?

The DMP used by LHD operators is a set of historically-employed, simple-to-execute rules. In this DMP, it is possible to identify two types of decision rules (Sepúlveda et al., 2005): 1) to meet the operational constraints and, 2) to define the work sequence. Surprisingly, the two decision rules of type 1 are equivalent to decision rules R4 and R5, while the decision rule of type 2, denominated HPF (after Highest Production First), defines the visit to drawpoints in decreasing order according to the number of ore bucketfuls that must be extracted. The decision rules above are integrated into a DMP as shown in Figure 4.

It is obvious that the current decision rules of type one are necessary conditions to reach the optimal makespan and that the HPF rule yields the optimal solution, if and only if the *i*-th* drawpoint is the greatest number of assigned ore bucketfuls.

The study of numerical cases shows that DMP leads to satisfactory results, since they help to meet the production goal as imposed by the plan-driven strategy. Nevertheless, a priori the feasibility of finishing the drift workload within a working shift is not assured. In practice, 62.5% of the cases analyzed matched the optimal solution with the HPF rule; the average improvement of makespan due to DMP proposed is 5.6 % and three infeasibility cases are found.

5. Conclusion

The main results of this work are two: First, a polynomial time optimal algorithm for the LHD operation problem, which gives the variable values for the IP model proposed. Second, a decision making process (DMP), which integrates a set of decision rules obtained from the above algorithm. This DMP is simple to execute for the LHD operators and allows to determine the optimal makespan and the feasibility of drift workload fulfillment within a working shift.

The comparative analysis between the DMP proposed and the DMP used in the *El Teniente* copper Chilean underground mine showed three interesting results. First, the current DMP has rules that meet the operational constraints, which are necessary conditions for finding the optimal makespan. Second, the HPF rule to define the work sequence matched the optimal solution in more than half of the cases studied (62.5%). Third, although the current DMP cannot assure the drift workload fulfillment within a working shift, in practice only three cases were infeasible. These above results show an important conclusion: cumulative operator experience has converged to solutions near to optimal makespan.

Further research is proposed regarding aspects such as: a) multiple dumping sites / turning points, b) objective functions other than makespan, such as the ore quality, c) on-line LHD operation problem and d) decision integration with transportation and reduction levels.

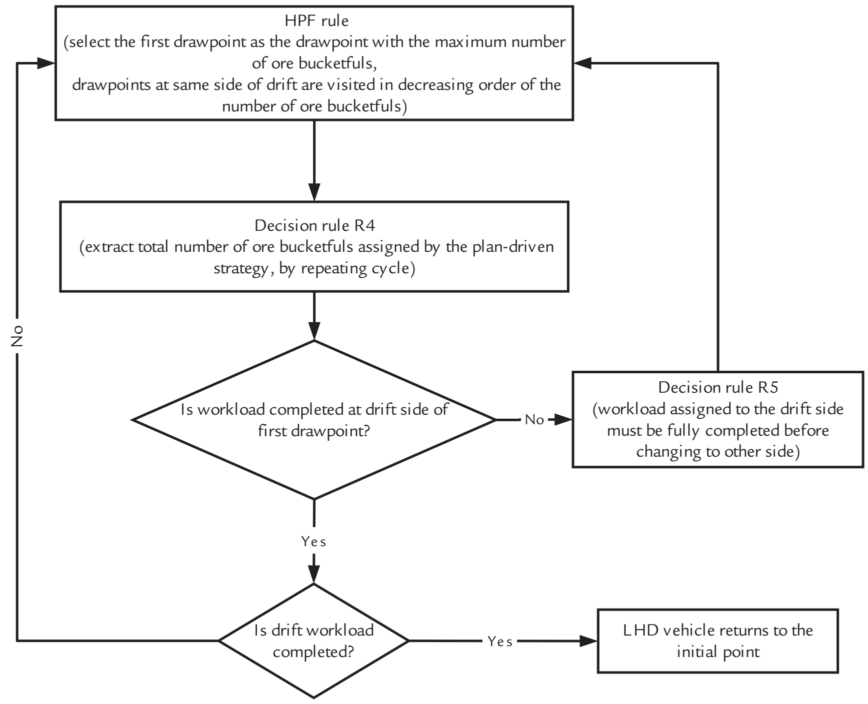


Figure 4
Flow diagram of the DMP used in *El Teniente* copper Chilean underground mine for the LHD operation problem.

6. Acknowledgements

This work has been supported by the University of Santiago of Chile, DICYT N° 205-0258 and FONDEF D01I1091.

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Artigo recebido em 23 de novembro de 2010. Aprovado em 21 de junho de 2011.