Mining
Mineração

The effect of accumulation in 2D estimates in phosphatic ore

O efeito da acumulação em 2D em estimativas de minério fosfático

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Abstract

The geological modeling of stratiform deposits can become very complex, often making use of geological envelopes of small thickness and requiring the use of sub-blocks (based on Cartesian coordinates) to produce a coherent block model. However, geological events after the formation of the deposit (folds, faults, etc.) can change the direction of spatial continuity of certain attributes, with the mixing of samples belonging to different formation eras (in the case of stratiform deposits) in the same elevation. This study presents a solution for deposits with stratigraphic grades combined with samples of different origins. The solution is a two-dimensional estimate obtained by accumulating the thicknesses of P_2O_5 in a phosphate deposit (as compared to traditional statistical analysis in three dimensions).

Keywords: accumulation, ordinary kriging, phosphate.

Resumo

A modelagem geológica de depósitos estratiformes pode-se tornar muito complexa, muitas vezes fazendo uso de envelopes geológicos de pequena espessura e exigindo o uso de sub-blocos (com base em coordenadas cartesianas) para produzir um modelo de blocos coerente. No entanto, eventos geológicos após a formação do depósito (dobras, falhas, etc.) podem mudar a direção da continuidade espacial de certos atributos, com mistura de amostras pertencentes a épocas de formação geológicas diferentes (no caso de depósitos estratiformes) na mesma elevação. Esse estudo apresenta uma solução para depósitos com teores estratigráficos combinados com amostras de diferentes origens. A solução é uma estimativa bidimensional obtida pela acumulação da espessura de P_2O_5 em um depósito de fosfato (em comparação com a análise estatística tradicional em três dimensões).

Palavras-chave: *acumulação, krigagem ordinária, fosfato.*

1. Introduction

The estimate of an attribute (grade, thickness, etc.) using geostatistical data requires certain assumptions, and among them is the assumption that the attribute presents autocorrelation in time and/or space. These estimates are based on temporal/spatial continuity for the attribute, which in turn is based on the sample values of the attributed model. In many cases, the database is geo-positioned using a Cartesian coordinate system. However, the continuity of the phenomenon may not be compatible with this type of coordinate. At the time of formation of mineral deposits, the minerals are crystallized or deposited in positions consistent with an active geological process, and this process determines the continuity of attributes related to the constitution of the rocks (e.g. mineral grades). Geological events subsequent to the formation of the deposit, such as folding, can change the direction of continuity of certain attributes (Koppe et al., 2006). There are some ways around this issue, such as coordinate transformations (McArthur, 1987; Deutsch, 2002) and accumulation in two dimensions (2D) (Krige, 1981).

In cases where one dimension is much smaller than the others (stratiform deposits), it is usual to use the accumulation method of grades by the layer thickness, where the estimate of the grades is obtained indirectly. This process cannot be considered a simplification because it adds complexity to the estimation process. Besides requiring additional steps to obtain the final grades, it can generate inconsistencies if not executed properly, with, for example, extreme values exceeding the minimum and maximum grades of the original data.

This article aims to present the steps in the estimation of a phosphatic deposit (P_2O_5) through the accumulation process in 2D, showing all the necessary steps for its implementation. In addition, the reasons for choosing to perform the estimation in 2D rather than the conventional three dimensions (3D) are shown. The deposit studied had several layers of phosphatic ore interspersed with layers of waste rock.

The positions of the Highwall and the Footwall were determined using data from drillholes. The results of the modeling showed almost horizontal layers, with a slight folding, and large variations in thickness

The determination of the spatial continuity of the deposit could be undermined by mixing samples of the same level (elevation), Z, belonging to a different geological formation era (distinct stratigraphic levels), correlating samples that may be on the Highwall in a drillhole at one given Z coordinate and on the Footwall in another drillhole at the same Z coordinate, as shown in Figure 2(a) (holes A, B, C and D). In view of this, the unfolding (transformation of Cartesian coordinates to stratigraphic coordinates) of the deposit could be a way of improving the results of the analysis of spatial continuity, given that in deposits of sedimentary origin, the values from sediments of the same geological age (stratigraphically on the same horizon) tend to have high spatial correlation (Figure 1 (b)).

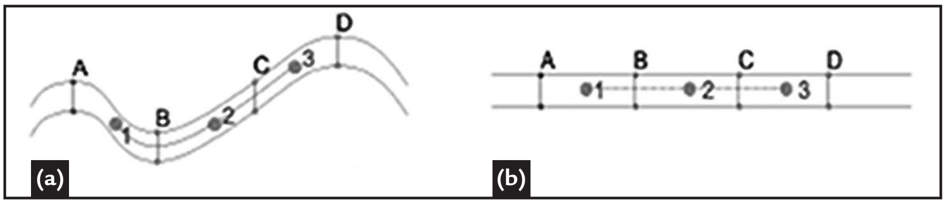

Figure 1

Figure 1. Theoretical example of how folds in the geological phenomenon (a) may affect the analysis of spatial continuity, and (b) how unfolding of the layer can reduce the problem. It can be noted in (a) that, independent of the orientation of the ellipsoid search of samples, those of the same geological era (1, 2 and 3) will be the same, if properly analyzed. After the transformation (b), samples of the same geological era (1, 2 and 3) have the same orientation and coordinates in the search plan.

Finally, in certain cases, the Euclidean distance is an inadequate measure (as in the Figure 2) and should be replaced by, for example, a "curved" distance measurement in layers with folds (Dagbert *et al*. 1984).

To determine whether the study should be performed in 3D with support, or in 2D, accumulating grades by the thickness, the series of tests presented below were performed.

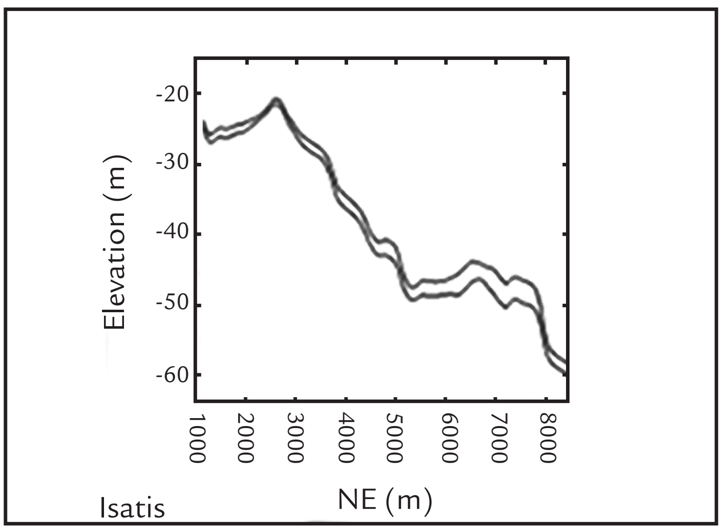

Figure 2
Cross section of interest layer (north–south)

2. Methodology

The sample data from a drilling campaign were obtained from drill cores of different lengths (different bases).

These data from distinct bases can be used for accurate geological modeling, but to estimate grades (from a statistical point of view) using them may create bias in the results.

The database provided for geostatistical modeling had a total of 1643 samples in the layer of interest. Because of the characteristics of the mineral deposit, it was impossible to regularize the bases of the samples, i.e. setting all the samples to a constant length for geostatistical analysis. In addition, the database was composed of data from four drilling campaigns, each with specific goals and its own protocol for sample preparation, taking no account of a standard sample size, varying between 0.1 and 2.12 m.

The bias created by the use of that data unmodified is shown in Table 1, where data for a drillhole obtained by the fourth drilling campaign is shown. Note that the layers adjoining the contact region show a small sample thickness (some with high grade).

Table 1
Example of statistical bias created by using samples with different bases.

Drillhole	Length of sample (m)	Grade of sample P_2O_5 (%)	Total length (m)	Arithmetic mean of drillhole P_2O_5 (%)	Weighted mean of drillhole P_2O_5 (%)	Relative error (%)
FS4101	0.1	14.2	1.87	12.3	19.9	38.3
	0.1	6.7				
	0.1	9.8				
	1.37	23.5				
	0.1	9.5				
	0.1	9.9				

Sample regularization usually involves combining existing sample values; i.e. it is a procedure that involves numerical calculation of the weighted means of the grades in larger volumes than the original samples. Typically, such a composition is linear in nature, involving calculation of a weighted mean of adjacent samples uniformly over a greater length than a single sample length (Sinclair and Blackwell, 2002). Some of the common reasons for sample regularization are: to provide an equal basis (support) for geostatistical analysis, and reduce the number of extreme values and the variability that these values may cause.

There are several different methods for the regularization that can be used, depending on the nature of the mineralization and type of mining process. Common regularization methods are: regularization by the height of the bench, regularization by constant length, and regularization by ore zone.

The only method applicable (without breaking the samples into smaller intervals) is the regularization method by ore zone. The result of this adjustment may be seen in Figure 3, where each sample represents a drillhole.

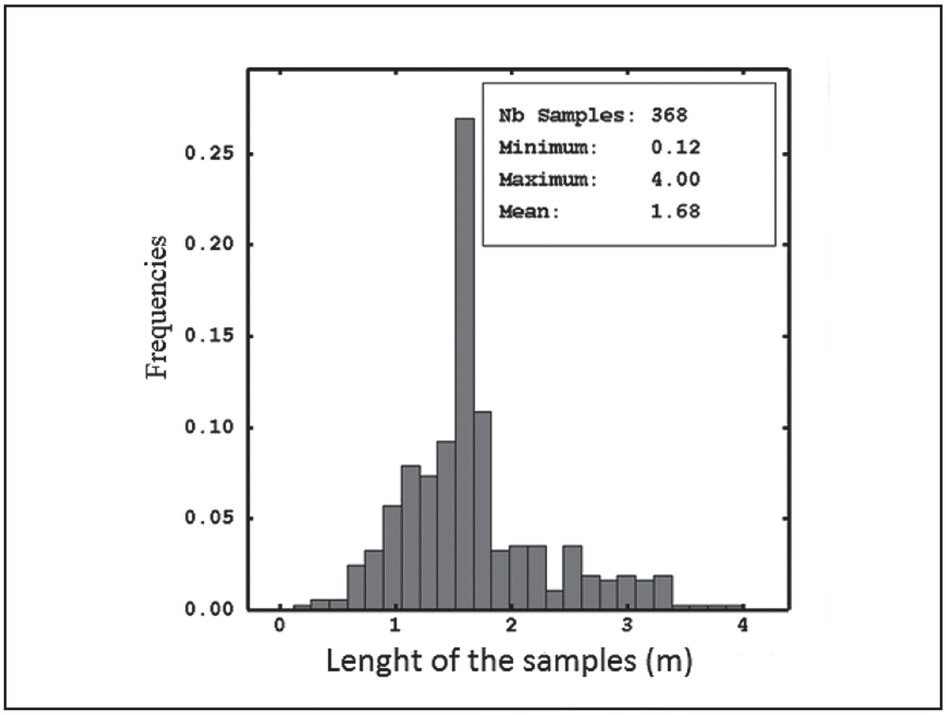

Figure 3
Histogram of the length of the sample of interest after the regularization by ore zone.

After sampling regularization, it can be seen that there is a great variability in thickness. Thus, a sample set does not exist on the same basis, preventing a study in 3D without bias, as seen in the theoretical example shown in Figure 4 and Table 2.

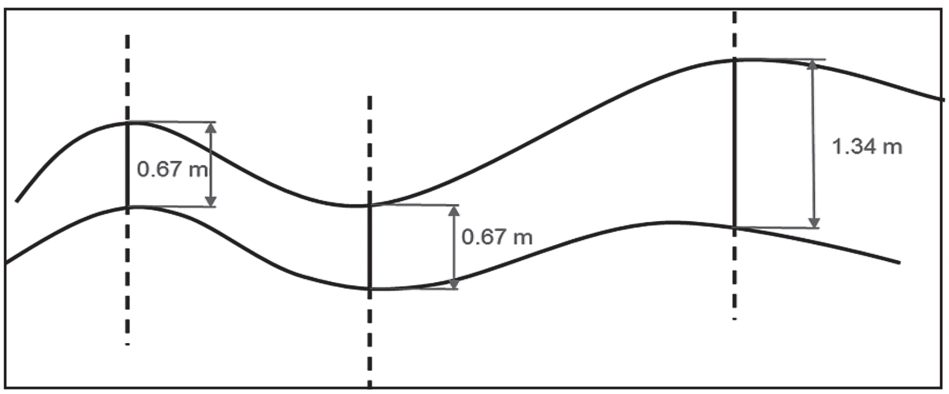

Figure 4
Theoretical example of regularization by ore zone.

Table 2 summarizes the mean calculations ignoring the differences in basis between samples, and compares them with results considering the grades by thickness in the calculation of the mean.

Length of drillhole (m)	Grade of drillhole P_2O_5 (%)	Arithmetic mean P_2O_5 (%)	Weight mean P_2O_5 (%)	Relative error P_2O_5 (%)
0.67	12	14.33	15.25	6.03
0.67	13			
1.34	18			

Table 2
Summary statistics for the theoretical example of regularization by ore zone.

All possibilities were analyzed for correction of basis, to try to determine a method without bias, and the only possible solution found by this study was the accumulation of the database in 2D. The following steps were performed:

- accumulation of all data by the length of the sample;
- analysis of spatial continuity of the accumulated variables and the length of the sample (thickness);
- definition of the search strategy for estimation;
- estimation of cumulative and variable thickness;
- validation of estimates;
- de-accumulation;
- validation of the results of the de-accumulation.

3. Accumulation

The regionalized variables (VRs) must be such that all linear combinations of their values keep the same mean. If f(x) represents the grade of a sample, the sum of samples multiplied by their weight (in this example, the length of the sample), Σiλi*f(xi), is the arithmetic mean (1/n)*Σif(xi), which must have the same mean grade (Journel and Huijbregts, 1978).

The process of regularization ensures that the sample regionalized variables have the same length and therefore the same basis, which is necessary for the implementation and use of geostatistical analysis. But in some cases, this is not enough (e.g. chemical variables associated with particle size fractions) or not applicable (e.g. small, thick layers of ore). In the case of a thin layer of iron ore, one usual method of estimation is the use of an auxiliary variable obtained by multiplying the grade by the thickness (i.e. accumulation). The estimated grades are obtained indirectly by the relationship between the estimated cumulative variable and the estimated thickness. This practice stems from variations in the basis (thickness) and the problem of the non-additive nature of the content of the resulting variable (Marcotte and Boucher, 2001).

An accumulation process for transforming a set of 3D data to a 2D data set is divided into two steps. In the first step, for each sample, the chemical variable is multiplied by its length, as in Equation 1.

$$Z^c_i = Z_i + L_i \qquad i= 1,2,\ldots,n \tag{1}$$

where:

- Z^c_i = accumulated sample value;
- Z_i= grade of the sample;
- L_i = length of the sample.

In the second step, the sum of the accumulations is divided by the total length after providing the weighted mean grade of the drillhole. At the end of the process, there is a set of 2D data. A simple example illustrates the procedure – see Table 3.

Drillhole	Length of sample (m)	Grade of sample of P_2O_5 (%)	Accumulated grade of P_2O_5 (%) for each sample	Total length of drillhole (m)	Accumulated grade of drillhole P_2O_5 (%)	Weight mean of drillhole P_2O_5 (%)
FS4101	0.1	14.2	1.42	1.87	37.21	19.9
	0.1	6.7	0.67			
	0.1	9.8	0.98			
	1.37	23.5	32.195			
	0.1	9.5	0.95			
	0.1	9.9	0.99			

Table 3
Example of the accumulation process used in this study.

Accumulation creates a new set of 2D data, where each value represents the grade of a drillhole from accumulated survey data (multiplied by the total thickness of the mineralized layer). Therefore, all the calculations in this study will be conducted using the accumulated variables (auxiliary variables) and the total length of mineralization in each drillhole. The grade of the variable of interest is obtained by an indirect method, which divides the estimated auxiliary variable by the estimated thickness of the layer on the same node of the grid or block.

4. Results

4.1. Exploratory data analysis

Statistical analysis of the data is useful in helping to understand natural phenomena and especially the grades of the mineral deposits. Statistical analyses were performed with the accumulated data and the data was weighted. The weighted data was used as reference for validation of the final results. For each survey, the cumulative variable was divided by the length, resulting in a weighted average for the survey. This weighted variable (as it will be called from now on) is extremely important because it serves as a basis for the validation of the estimated model, as well as a reference for possible post-processing of the results (if necessary).

The layer of interest has 368 drillholes with information on P_2O_5. Histograms with the accumulated data and weighted data on P_2O_5 can be seen in Figure 5 (a) and (b), respectively. Note that there are no multiple populations in the histograms, validating the geological model for this domain. Also, there are extreme values.

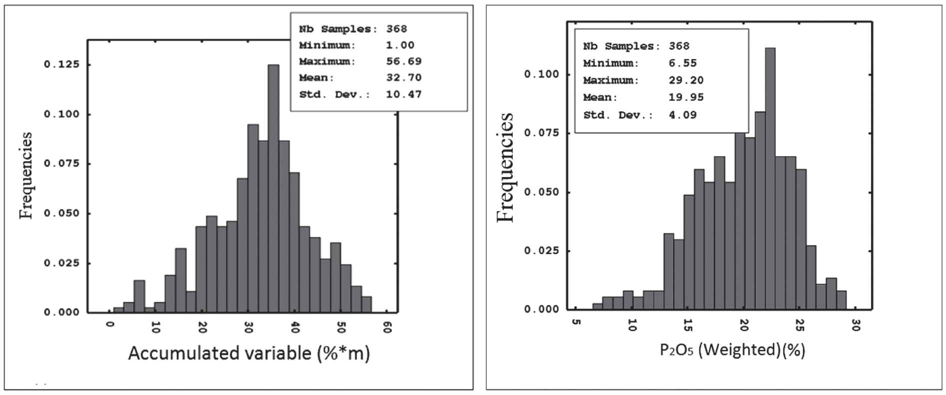

Figure 5
Histogram of (a) the accumulated variable and (b) the variable of interest, the P_2O_5 layer.

To obtain a representative statistic for the deposit, an unbinding method was applied, namely the method of Polygonal Declustering (Isaaks and Srivastava, 1989). The weighted and declustering mean of the grades of P_2O_5 was 18.39%. This mean grade will serve as a basis for validating the final model, after the estimation process de-accumulation.

4.2. Structural analysis

The analysis of the spatial characteristics (similarity patterns) of regionalized variables relating to a mineral deposit is usually preceded by a critical examination of the geology of the deposit and a full analysis of the data. There are several mathematical tools to measure the spatial continuity of a mineral deposit, including madograms, covariograms, correlograms, etc. Studies of autocorrelation in geostatistics are often referred to as variography because of traditional emphasis on the variogram or semivariogram (Sinclair and Blackwell, 2002).

There is a strong anisotropy in the major directions of continuity (for both variables), as can be seen in the Figure 6. The result is summarized in Equation 2 and 3 (sph refers to a spherical shape).

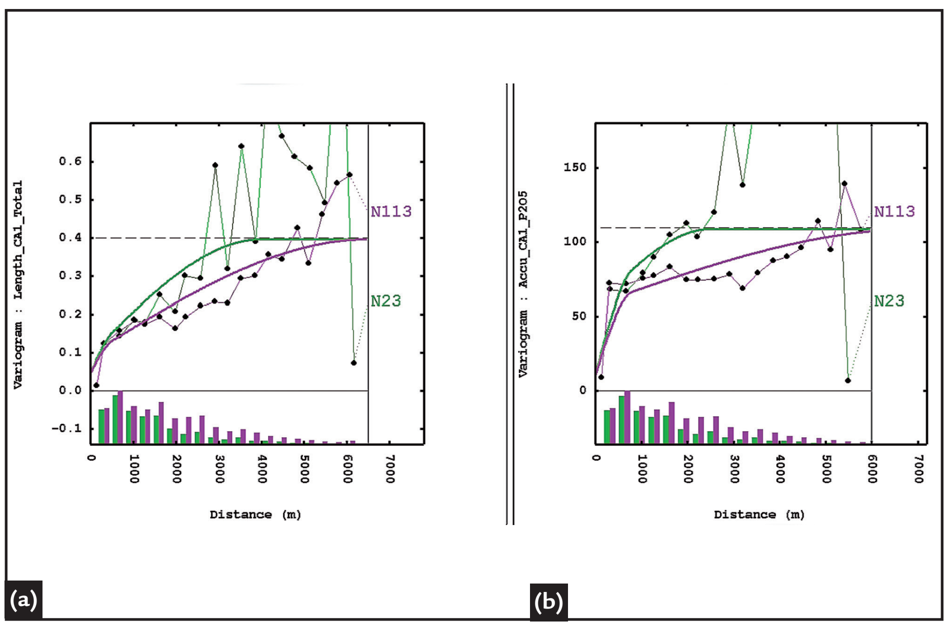

Figure 6
(a) Directional variogram of variable thickness and (b) directional variogram of the accumulated variable of P_2O_5.

$$\gamma(h) = 0.046 + 0.05 * sph\left(\frac{N23}{500m}, \frac{N113}{500m}\right) + 0.3 * sph\left(\frac{N23}{4000m}, \frac{N113}{6500m}\right) \tag{1}$$

$$\gamma(h) = 10.72 + 47 * sph\left(\frac{N23}{800m}, \frac{N113}{800m}\right) + 0.3 * sph\left(\frac{N23}{2500m}, \frac{N113}{7000m}\right) \tag{2}$$

Importantly, both variables show the same directions of continuity, which was expected. This does not mean that the direction of major continuity of variable P_2O_5 is the same as that presented in the cumulative variable, although it is very likely.

4.3. Estimates

To estimate the accumulated P_2O_5 variable and the variable thickness, the ordinary kriging method was used (Matheron, 1963). The estimation was performed using a 2D block model. The blocks had X and Y dimensions of 50 m, and after the process of de-accumulation, had variable heights (Z-coordinates of the blocks in the geologic modeling procedure).

To perform the estimation by ordinary kriging, the search ellipsoid was aligned with the variogram of the corresponding variable. A minimum of six and a maximum of 16 samples at a distance of 3000–2000 m in N135 and N45 were required. The search ellipsoid was divided into eight angular sectors. To avoid problems of de-accumulation after the estimates, it is important that the search strategy of the accumulated variable and the accumulator (in this case, the thickness variable) is the same, and if necessary, the same variogram should be used for the estimation. In this case, it was possible to estimate each variable with its own variogram without further problems when making the de-accumulation. Figure 7 (a) shows the results of estimation for the variable thickness. Figure 7 (b) shows the estimation results for the cumulative variable P_2O_5.

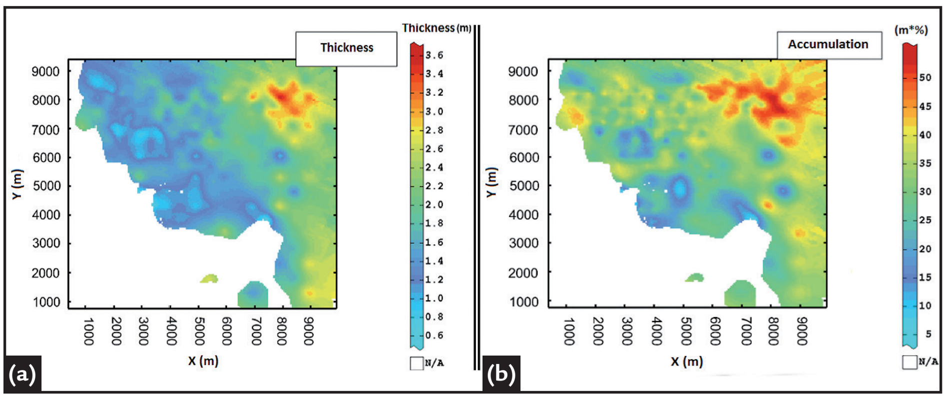

Figure 7
Location map with the results of the estimates for (a) variable thickness and (b) accumulated variable P_2O_5.

For each block of the estimated layer, the result of the accumulated variable P_2O_5 was divided by the estimated value of the thickness. The results are shown in Figure 14 (b). A quick comparison of the measured variable P_2O_5 (Figure 8 (a)) and the de-accumulated variable P_2O_5 (Figure 8 (b)) shows a good visual validation.

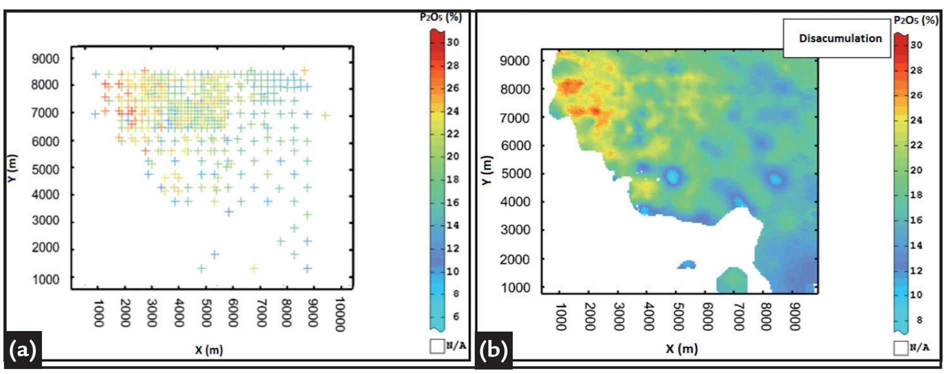

Figure 8
Location map of the (a) weighted variable of P_2O_5 (data) and (b) variable de-accumulation of P_2O_5.

Besides visual validation, a validation of the global mean was undertaken. The mean of the declustering weighted data was 18.39% and the estimate gave a result of 18.11% P_2O_5. Another way to validate the estimation is through the analysis of derivation (Swath Plot). The analysis derivation performs a local validation, where the average block is compared with the average of the data in narrow bands in various directions of the block model. The validation was performed in two directions: along the east (slices every 500 m) and along the north (slices every 500 m). The results can be seen in Figure 9 (a) and (b). In these figures, the red line represents the values read from ungrouped data by 'nearest neighbor' and the black line represents the values from the model obtained by kriging.

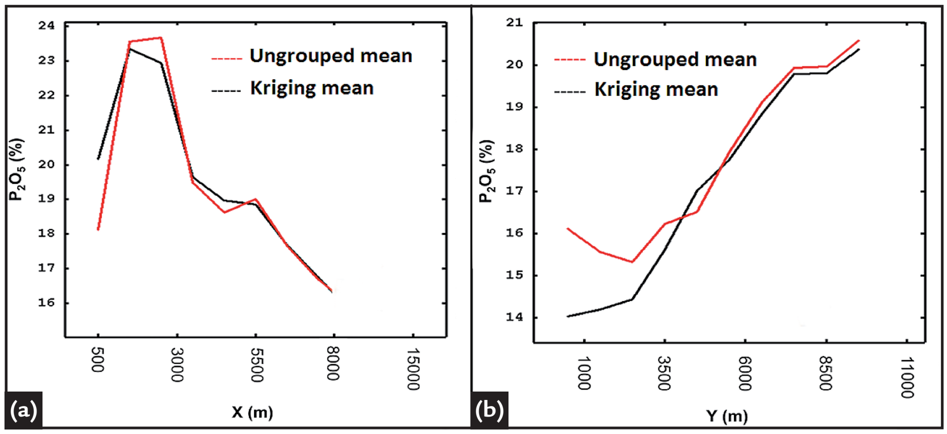

Figure 9
Analysis of drift (a) east and (b) north. The red line represents the contents obtained by 'nearest neighbor' ('declustering mean') and the black line represents the contents obtained by kriging.

5. Conclusions

Estimates will only be adequate if the initial data receives the correct treatment. The correction of the samples using regularization is essential to the practice of geostatistics, and the wrong choice of method can lead to a significant bias in the estimates. As demonstrated in this case study, the use of samples with different bases causes bias of the data mean.

Furthermore, regardless of the adjustment of the sampling method, this bias can be propagated by spatial continuity analysis, leading to inappropriate calculation of the weights of the samples in the estimation process.

As the case study with a stratiform deposit shows, with low thickness, there is the possibility of using unfolding techniques, a dynamic anisotropy search (in the estimate) and/or sub-blocks.

However, as no adequate basis correction method was found to permit analysis of spatial continuity in 3D, these methods were discarded.

Although being an indirect estimation method, the analysis and estimation of spatial continuity of the variables in 2D showed satisfactory results. Estimates made by this method require additional steps and more time to be realized than conventional estimates.

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