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## Mathematical modeling of the foam-mat drying curves of guava pulp

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**Key words:**

*Psidium guajava*  
tray dryer  
Midilli

**A B S T R A C T**

In foam-mat drying, the liquid food is foamed by the addition of a foaming agent, for example, albumin. The aim of this study was to evaluate the adjustment of mathematical models to foam mat drying curves of guava pulp. The fits were evaluated using samples with 4 and 8% albumin ( $\text{m m}^{-1}$ ) and drying temperatures of 75, 80 and 85 °C. The samples were placed on aluminum trays. Drying was carried out in a tray dryer. The Lewis, Page, Midilli and Logarithmic models were fitted to the experimental data and evaluated based on the coefficient of determination ( $R^2$ ), root-mean-square error (RMSE) and chi-square test. All models fitted well to experimental data and Midilli was the best. Albumin concentration and temperature influenced the drying rate; the lowest drying times occurred for the highest albumin concentration and highest drying temperature.

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**Palavras-chave:**

*Psidium guajava*  
secador de bandeja  
Midilli

**Modelagem matemática das curvas  
de secagem em camada de espuma de polpa de goiaba****R E S U M O**

Na secagem em camada de espuma o alimento líquido é transformado em espuma, através da adição de um agente espumante, por exemplo, a albumina. Objetivou-se, neste trabalho, avaliar o ajuste de modelos matemáticos das curvas de secagem por camada de espuma de polpa de goiaba. Na avaliação dos ajustes utilizaram-se concentrações de 4 e 8% ( $\text{m m}^{-1}$ ) de albumina e temperaturas de secagem de 75, 80 e 85 °C. As amostras foram colocadas em bandejas de alumínio. A secagem foi realizada em secador de bandejas. Os modelos de Lewis, Page, Midilli e Logarítmico foram ajustados aos dados experimentais e avaliados segundo os coeficientes de determinação ( $R^2$ ), os desvios médios estimados e os testes qui-quadrados. Os modelos propostos se ajustaram bem aos dados experimentais, porém o melhor foi o de Midilli. A concentração de albumina e a temperatura influenciaram na velocidade de secagem, em que os menores tempos de secagem foram para a maior concentração de albumina e a maior temperatura de secagem.

## INTRODUCTION

Guava (*Psidium guajava* L.) is a fruit with high contents of vitamins A, B1 and C, besides calcium, phosphorus, iron and soluble fibers (Oliveira et al., 2011). Drying is a process widely used in the industry to preserve the quality of agricultural products (Sousa et al., 2011b). Among the drying processes to obtain fruit pulp powders, foam mat drying stands out. This is a method in which the liquid or semi-liquid food is mixed with foam agents and transformed into a foam through agitation, and then subjected to drying with hot air (Baptistini et al., 2015).

The advantages of this method include lower temperatures and shorter drying times, because of the increase in the surface area in contact with the air, which increases the water removal speed; in addition, it also stands out for maintaining the nutritional and sensory quality of the product (Kadam et al., 2010).

During the drying, it is possible to determine the behavior of the solid material based on the drying kinetics. The graphical representation is made from the drying and drying rate curves (Menezes et al., 2013). The mathematical representation of the drying process through mathematical models has aroused the interest of various researchers in the most varied agricultural products, such as apple (Rayaguru & Routray, 2012), coffee (Corrêa et al., 2010), 'mulungu' husks (Martins et al., 2014), basil leaves (Reis et al., 2012), strawberry (Sousa et al., 2014).

The present study aimed to evaluate the fit of mathematical models to foam mat drying curves of guava pulp, besides evaluating the influence of albumin and dry temperature on the process.

## MATERIAL AND METHODS

The drying processes used three temperatures: 75, 80 and 85 °C, and two albumin concentrations: 4 and 8% (m m<sup>-1</sup>), mixed with guava pulp using a domestic food mixer for 5 min to form the foam. Subsequently, they were spread on aluminum trays (diameter of 12 cm, height of 0.5 cm) and placed in natural-convection tray dryer. Weighing was conducted on analytical scale at regular intervals of 15 min. The moisture ratio was calculated according to Eq. 1.

$$RX = \frac{X - X_e}{X_0 - X_e} \quad (1)$$

where:

$RX$  - moisture ratio;

$X$  - moisture over time (g g<sup>-1</sup> d.b.);

$X_e$  - equilibrium moisture (g g<sup>-1</sup> d.b.); and,

$X_o$  - initial moisture (g g<sup>-1</sup> d.b.).

The models fitted to the experimental data are presented in Table 1. The fits were performed using the program Statistica 10.0.

The fit of each model was evaluated based on the coefficient of determination, root-mean-square error (RMSE) and chi-square test ( $\chi^2$ ). RMSE was calculated according to Eq. 6 (Resende et al., 2010) and chi-square according to Eq. 7.

Table 1. Mathematical models used to fit the drying curves

Model	Source	Formula	
Lewis	Andrade et al. (2006)	$RX = \exp(-kt)$	(2)
Page	Corrêa et al. (2010)	$RX = \exp(-kt^n)$	(3)
Midilli	Midilli et al. (2002)	$RX = a \exp(-kt^n) + bt$	(4)
Logarithmic	Sousa et al. (2011a)	$RX = a \exp(-kt) + C$	(5)

RX - Moisture ratio; k - Drying constant (h<sup>-1</sup>); n - Internal resistance; t - Time (min), a, b, c - Constants

$$RMSE = \sqrt{\frac{\sum (V_o - V_c)^2}{DF}} \quad (6)$$

$$\chi^2 = \frac{\sum (V_o - V_c)^2}{DF} \quad (7)$$

where:

$V_o$  - experimentally observed value;

$V_c$  - value calculated by the model; and,

DF - degree of freedom (observations minus the number of parameters of the model).

## RESULTS AND DISCUSSION

All values of chi-square were low, demonstrating the good fit of the models (Table 2). Although the chi-square values of the Lewis model were low, they were superior to those of the other models, which also showed the highest RMSE and lowest R<sup>2</sup>. In the present study, the Midilli model showed the best fit to the experimental data.

Madureira et al. (2011) and Reis et al. (2011) found R<sup>2</sup> values around 0.99 for the models Page, Midilli and Logarithmic, during the mathematical modeling of the drying of cactus pear and 'Cumari do Pará' pepper, respectively. The results in

Table 2. Evaluation of the models to fit the foam mat drying kinetics of guava pulp

Models	Temperature (°C)	Albumin concentration (%)	R <sup>2</sup>	RMSE	$\chi^2$
Lewis	75	4	0.9413	0.0802	0.0067
		8	0.9406	0.0814	0.0070
	80	4	0.9291	0.0881	0.0081
		8	0.9357	0.0843	0.0074
	85	4	0.9224	0.0950	0.0095
Page	75	8	0.9433	0.0801	0.0067
		4	0.9940	0.0255	0.0007
	80	8	0.9926	0.0287	0.0009
		4	0.9926	0.0285	0.0008
	85	4	0.9904	0.0326	0.0011
Midilli	75	8	0.9893	0.0353	0.0013
		4	0.9955	0.0225	0.0005
	80	4	0.9982	0.0144	0.0002
		8	0.9967	0.0191	0.0004
	85	4	0.9976	0.0162	0.0003
Logarithmic	80	4	0.9964	0.0198	0.0004
		8	0.9970	0.0185	0.0004
	85	8	0.9972	0.0179	0.0003
		4	0.9939	0.0259	0.0007
	75	8	0.9908	0.0320	0.0019
	80	4	0.9940	0.0256	0.0007
		8	0.9918	0.0301	0.0009
	85	4	0.9935	0.0275	0.0008
	85	8	0.9856	0.0403	0.0017

R<sup>2</sup> - Coefficient of determination; RMSE - Root-mean-square error;  $\chi^2$  - Chi-square

which the Midilli model was the one that best described the drying behavior were found for 'ceriguela' through foam mat drying (Furtado et al., 2010) and 'Cumari do Pará' pepper (Reis et al., 2011).

Table 3 shows the values of the parameters of the fitted models. The parameter "k" represents the drying constant and the parameter "n" refers to the internal resistance to drying (Perez et al., 2013).

It is observed that the parameter "k" of the Lewis model increases with the increment of temperature (Table 3). The same occurred with the results of Santos et al. (2012), studying the drying kinetics of Malagueta pepper, André et al. (2013) in the drying of mango peel.

In the Page model, the increment of temperature leads to reduction in the parameter "k" and increase in the parameter "n" at the albumin concentration of 4%. Venturini et al. (2012) found similar behavior in the drying of papaya seeds. In the present study, for the Page model and albumin concentration of 4%, there was not a defined behavior, i.e., similar to the study of Alexandre et al. (2013), in the drying of pineapple residue.

In the fit of the logarithmic model, the parameter "k" tended to increase, whereas "n" tended to decrease with the increment of temperature for the albumin concentration of 8%; for albumin concentration of 4%, there was not a defined trend.

For the Midilli model, there was an increment in the values of "k" and "n" between the temperatures of 75 and 80 °C. According to Corrêa et al. (2010), this behavior of the parameter "k" is expected, because higher temperatures lead to higher drying rates. Still according to these authors, equilibrium moisture is obtained faster with shorter time of exposure of the product to drying. Martins et al. (2014) found similar behavior for the parameter "k" studying the drying of 'mulungu' husks.

The parameters "a" and "b" of the Midilli model did not show a clear behavior as temperature increased. Reis

et al. (2012), Resende et al. (2010) and Sousa et al. (2011b) observed a similar behavior for the same model, studying the drying kinetics of basil leaves, Adzuki bean and forage turnip, respectively. The negative values for the parameter "b" of the Midilli model (Table 3) have been observed by various authors in the literature and accepted in the validation of the model, such as Darvishi et al. (2014), Çakmak & Yıldız (2011) and Shi et al. (2013) in the drying of pepper, grape and yacon, respectively.

According to Figures 1A, B and C, moisture loss was faster in samples containing higher albumin concentration. In the drying at 75 °C (Figure 1A), the times necessary to reach the moisture ratio of 0.50 were 108.9 and 94.7 min for the albumin concentrations of 4 and 8%, respectively, i.e., at the albumin concentration of 8% there was a reduction of 13.04% in the drying time, in comparison to the sample with 4%. Similarly, in the samples containing albumin concentration of 8%, there were reductions in the drying time of 13.22 and 21.85% in the drying at 80 and 85 °C, respectively. (Figures 1B and C). Soares (2009) observed similar results, comparing the drying kinetic curves relative to different albumin concentrations (5, 10 and 15%) added to the 'araçá-boi' pulp, in which the albumin concentration of 15% required the shortest time to reach equilibrium moisture, i.e., a reduction in the drying time was obtained. The same author mentions that the use of the foam agent and albumin reduced in half the drying time of 'araçá-boi' pulp.

According to the fit of the Midilli model, it was observed that albumin concentration influenced drying speed, because at the highest albumin concentration the drying times were shorter (Figures 1A, B and C). This occurs because albumin at higher concentration causes greater incorporation of air and, thus, favors the formation of foam, facilitating the efflux of water from the product. Madureira et al. (2011), in the

Table 3. Parameters of the mathematical models to describe the foam mat drying kinetics of guava pulp

Model	Albumin (%)	Temperature (°C)	Parameters of the models			
			k	n	a	b
Lewis	4	75	7.932E-3	-	-	-
		80	9.970E-3	-	-	-
		85	1.049E-2	-	-	-
	8	75	8.925E-3	-	-	-
		80	1.136E-2	-	-	-
		85	1.306E-2	-	-	-
Page	4	75	5.580E-4	1.535	-	-
		80	5.000E-4	1.612	-	-
		85	4.970E-4	1.635	-	-
	8	75	5.320E-4	1.579	-	-
		80	7.690E-4	1.564	-	-
		85	7.500E-4	1.625	-	-
Midilli	4	75	1.089E-3	1.351	0.9968	-3.560E-4
		80	1.330E-3	1.359	1.0020	-4.870E-4
		85	1.222E-3	1.386	0.9955	-5.840E-4
	8	75	1.063E-3	1.400	0.9867	-3.150E-4
		80	1.338E-3	1.406	0.9853	-3.720E-4
		85	0.981E-3	1.546	0.9886	-1.430E-4
Logarithmic	4	75	4.210E-3	-	1.531	-
		80	4.637E-3	-	1.601	-
		85	4.445E-3	-	1.703	-
	8	75	5.033E-3	-	1.452	-
		80	6.203E-3	-	1.420	-
		85	9.199E-3	-	1.272	-

k - Drying constant ( $\text{min}^{-1}$ ); a, b, c - Constants of the models; n - Internal resistance

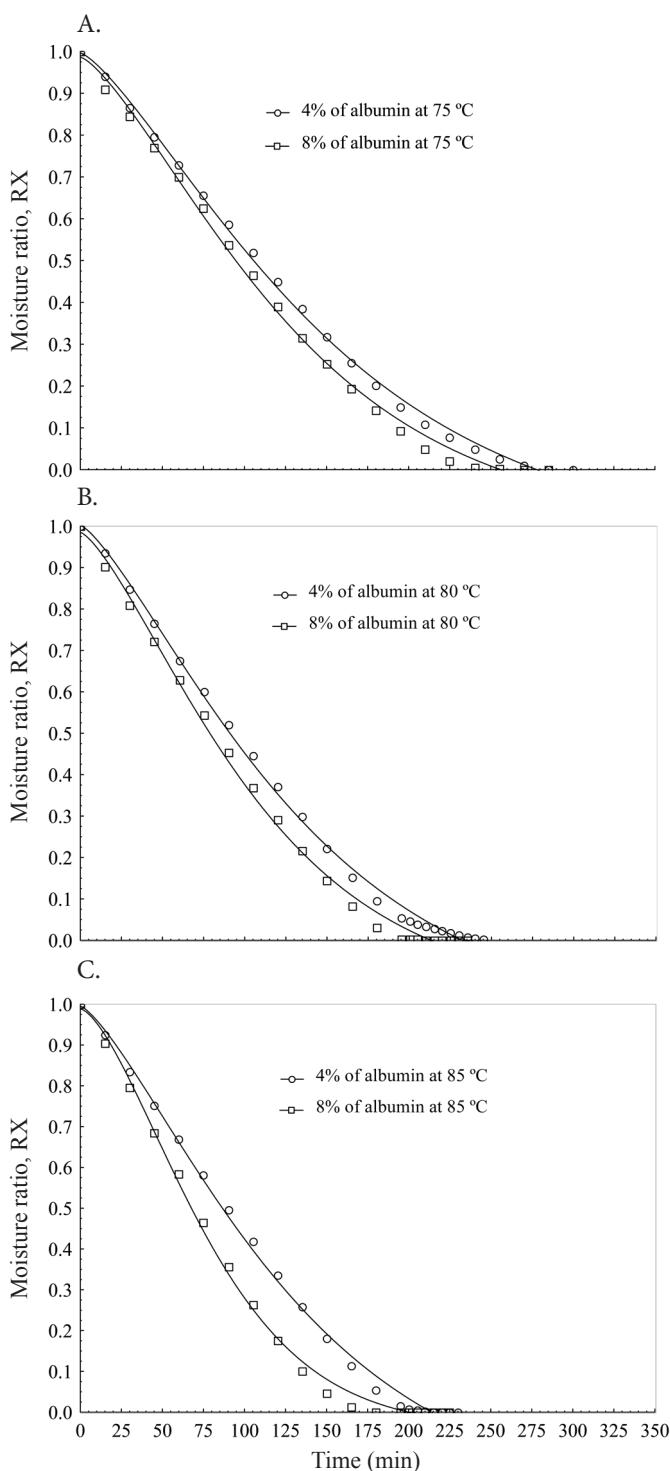


Figure 1. Foam mat drying curves of guava pulp containing 4 and 8% of albumin fitted to the Midilli model: 75 (A), 80 (B) and 85 °C (C)

drying of cactus pear at 70 °C, observed that the increase in the concentration of modified starch, from 20 to 30%, reduced the drying time by 10.48%. Fernandes et al. (2014) observed that the foam obtained using albumin reduced the time to reach equilibrium moisture in the drying of tomato pulp.

Figures 2A and B show the drying curves of the samples containing 4 and 8% of albumin, respectively, fitted to the Midilli model.

In the results shown in Figure 2, it is possible to observe that the increase of drying temperature reduced the drying time. Fernandes et al. (2014), studying the influence of albumin

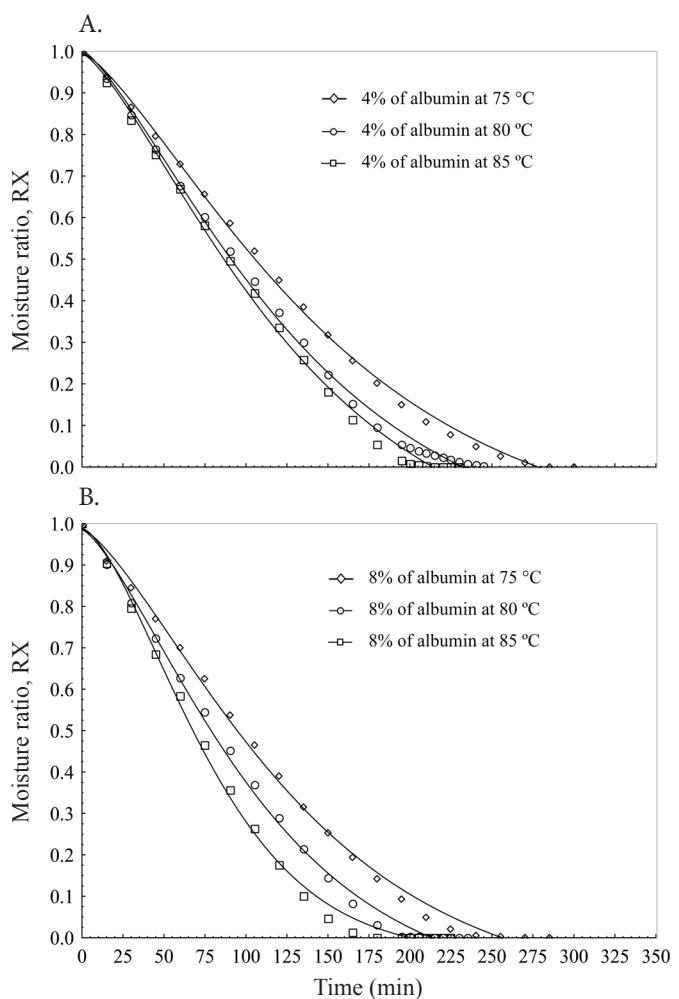


Figure 2. Foam mat drying curves of guava pulp at 75, 80 and 85 °C fitted to the Midilli model at two albumin concentrations: 4 (A) and 8% (B)

and drying temperature on tomato pulp powder, observed that the time to reach equilibrium moisture decreased by 55% with the increment of temperature from 60 to 80 °C. In the samples containing 4% of albumin (Figure 2A), the drying times for the samples to reach the moisture ratio of 0.5 were 108.9, 90.8 and 86.5 min at temperatures of 75, 80 and 85 °C, respectively. Thus, there was a reduction of 20.57% in the drying time at 85 °C, compared with the temperature of 75 °C. In the samples containing 8% of albumin (Figure 2B), there was a reduction of 28.62% of the drying time for the samples to reach the moisture ratio of 0.5.

Baptistini et al. (2015) subjected soursop pulp mixed with 7.43% of albumin to the temperature range of 40 to 80 °C and observed that the drying time at 80 °C was 4.6 times shorter than that for 40 °C. Madureira et al. (2011) observed that the increase of 10 °C in the temperature for the drying of cactus pear in forced-air oven led to reduction in the drying time. Sousa et al. (2014) found that the drying time and temperature are inversely proportional, i.e., the increment in drying temperature caused a reduction in the drying time for strawberry fruits.

## CONCLUSIONS

1. The Midilli model showed the best fit to the experimental data of drying kinetics of guava pulp through the foam mat drying method.

2. The increase in albumin concentration and drying temperature led to reduction in the time necessary to remove water from the product, during the drying.

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