Revista de Economia Contemporânea

REC

Revista de Economia Contemporânea (2022) 26: p. 1-14
(Journal of Contemporary Economics)
ISSN 1980-5527
http://dx.doi.org/10.1590/198055272620
elocation - e272620
https://revistas.ufrj.br/index.php/rec | www.scielo.br/rec

Special Section on Competition Policy Analysis

WHICH CHARACTERISTICS OF PRICE-FIXING AGREEMENTS ARE RELATED TO A GREATER DAMAGE TO CONSUMERS?*

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Recebido em 23/08/2020
Aceito em 15/02/2022
Received on 2020/8/23
Accepted on 2022/2/15

ABSTRACT: Despite the deterrence effect generated by antitrust laws, the fact is that many collusive agreements end up forming in the economy. An essential task is to understand how distinct market characteristics affect cartel profits and damages. This paper develops a theoretical model to assess which characteristics of price-fixing agreements are related to greater damage to consumers. Results indicate the following characteristics as responsible for greater damage: higher product similarity (substitutability); lower elasticity of demand (in absolute values); greater number of members; higher demand potential; and a higher velocity of transactions. Price-fixing agreements that have these characteristics more intensely, in comparison to others, tend to be more harmful to consumers, and, therefore, should be prioritized by antitrust authorities regarding detection, prosecution, and punishment.

KEYWORDS: price-fixing agreements; cartel damage; antitrust; cartels; antitrust authority.

JEL CODE: L41; D43.

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* Paper originally submitted to the II Rio International Workshop on Advances in Competition Policy Analysis, that took place on December 5-6, 2019, at the Institute of Economics, Federal University of Rio de Janeiro (IE-UFRJ), Rio de Janeiro, Brazil.

QUAIS CARACTERÍSTICAS DOS ACORDOS DE FIXAÇÃO DE PREÇOS ESTÃO RELACIONADAS A UM MAIOR DANO AOS CONSUMIDORES?

RESUMO: Apesar da atuação da lei antitruste no sentido de dissuadir carteis, muitos acordos acabam se formando na economia. É essencial compreender como as características dos mercados afetam os lucros e os danos causados pelos carteis. Este trabalho desenvolve um modelo teórico para verificar quais características de acordos de fixação de preços estão relacionadas a um maior dano aos consumidores. Os resultados indicam como responsáveis por um dano mais elevado: maior similaridade entre os produtos (substitutibilidade); demanda mais inelástica (em valores absolutos); maior número de membros; maior potencial de demanda; maior velocidade das transações. Acordos de fixação de preços que apresentem estas características de forma mais intensa em comparação a outros acordos tendem a ser mais prejudiciais aos consumidores e, portanto, deveriam ser priorizados pelas autoridades antitruste com relação à detecção, acusação e punição.

PALAVRAS-CHAVE: acordos de fixação de preços; danos dos carteis; antitruste; carteis; autoridade antitruste.

INTRODUCTION

Explicit cartels are recognized as the most harmful anticompetitive conduct. The damaging effects of collusive agreements are well known. Either by price fixing, quantity fixing, market division or other type of coordinated action, cartels reduce competition, which tends to increase prices, enhance profits, and harm consumers. It is not by coincidence that a major challenge of antitrust authorities around the world is assessing how to decrease the number of cartels in the economy, either by preventing cartel formation (deterrence aspect) or by punishing the discovered ones (punitive aspect).

A relevant issue in this context is that cartels form in industries with many different market characteristics. For example, demand behavior, sales potential, and number of competitors, among other aspects. It is presumable that incentives and outcomes vary among these cartels. It follows that it is essential to understand how these market characteristics affect cartel profits and damages, which allow us to check which ones tend to be more harmful to consumers.

The importance of this understanding lies on two main aspects. First, despite the deterrence effect of antitrust enforcement, the fact is that many cartels end up being formed and antitrust authorities must act the best way they can to detect and punish them. Considering budget and time constraints, it is presumable that for some cases to be investigated, others are neglected, which requires a criterion of choice.[1] Notwithstanding the importance of legal and political aspects, from the economic viewpoint, the chosen ones should be the most harmful.[2]

The second main aspect is related to fine setting. In many countries, the rules for fine setting are related to the illegal gains accrued (ICN, 2008). The first and best choice would be calculating the illegal gains for all cartels, but this is impracticable due to time and financial constraints, and also because data are not always available. Understanding which market characteristics are related to a greater profit and damage may help antitrust authorities in setting penalties for condemned cartels.

There are secondary aspects which are relevant for this analysis. For cartel members, before deciding to join a collusive agreement, they will have a notion of the magnitude of the damage inflicted based on market characteristics, which can also help to prevent cartel formation. For consumers, this may be relevant regarding an important side of

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[1] This is valid for all antitrust authorities around the world but possibly even more in developing countries.

[2] The analysis assumes that cartels stop their illegal activities after detection and punishment. In practice, it is possible for firms to return to an illegal activity a while after their judgment, known as recidivism. Connor (2010) highlights the importance of recidivism in some jurisdictions, such as Europe and Japan.

the antitrust enforcement discussed recently: private enforcement by damage claims. Knowing the market characteristics that result in greater damage may help consumers to claim more accurate refunds.

The objective of this paper is to assess which price-fixing characteristics are related to greater damage to consumers. The intention is providing apprehensible insights supported by a simple microeconomic model. Two points are worth noting: this paper focuses on one of the most common types of cartels: price-fixing agreements in regular markets;[3] and its focus is the damage to consumers, represented by the effects of each characteristic on market equilibrium.[4]

This article follows the stream of papers on collusive incentives in distinct industries. Rothschild (1992) analyzes collusion stability when firms produce horizontally differentiated products, while Häckner (1994) focuses on collusive pricing when products are vertically differentiated and Collie (2004) aims to understand how demand elasticity affects collusive agreements. However, this literature considers the deterrence aspect, that is, they find conditions in which cartels form or not, or how antitrust enforcement affects collusive behavior (HARRINGTON, 2004; HOUBA; MOTCHENKOVA; WEN, 2012). In this sense, the paper closest to this one is the one by Ivaldi et al. (2007), in which the authors analyze the relevant factors affecting the sustainability of cartel formation in simple models that involve market parameters and discount factors. But instead of focusing on cartel formation, I assume an amount of already existing price-fixing agreements in industries with different market characteristics and how these agreements distinctly harm consumers.

Lastly, this paper aims to contribute with the recent and ongoing literature on cartels and antitrust enforcement, which includes topics such as the following: the deterrent impact of anticartel enforcement (BOS et al., 2018); collusion with network externalities (SONG; WANG, 2017); cartel penalties (KATSOULACOS; MOTCHENKOVA; ULPH, 2015); and self-report and leniency policies (HARRINGTON; CHANG, 2015; BUCCIROSSI; MARVÃO; SPAGNOLO, 2020); among other topics. Important references to understand the state-of-the-art of the literature are Harrington (2017) and Marvão and Spagnolo (2018).

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[3] The analysis is not directly extended to other types of agreements, such as market division and cartels in procurement since distinct types of collusive agreements are strongly different in terms of incentives and price/quantity setting. Price-fixing agreements were chosen in this paper since they are one of the most common types of cartels.

[4] It is well known that other parties may also be injured by cartels, such as input sellers and governments. Moreover, in this paper, "damage" means the impact of the price-fixing agreement on consumer surplus in comparison to a competitive scenario, which is usually the definition applied in antitrust analyses.

This paper is organized as follows. Section 1 proposes a reflection on the damage caused by price-fixing agreements. Section 2 presents the main results, and Section 3 presents a brief discussion. The paper ends with Conclusions and References.

1. THE PRICE-FIXING DAMAGE: A REFLECTION

A price-fixing agreement occurs when two or more firms coordinate actions to act as one in pricing.[5] In the absence of an agreement, they would set competitive prices resulting from market conditions. I provide a simple linear model to illustrate this framework.

Suppose $n \geq 2$ firms are competing à la Bertrand.[6] The linear demand function[7] is given by $q_i = a - bp_i + \sum_{j \neq i}^{n} cp_j, i, j = 1, \ldots, n$, in which $q_i, i = 1, \ldots, n$ is the quantity; a, b, and c are positive coefficients; $p_i, i = 1, \ldots, n$ is the price of the represented firm; and $p_j, j = 1$ is the price set by rivals. Note that a is the maximum demand, b is the own-price effect, and c is the cross-price effect. Assume $q_i, p_i, p_j > 0, i, j = 1, \ldots, n$ and marginal costs equal to zero.[8] As firms are identical, it follows that $\sum_{j \neq i}^{n} cp_j = (n-1)cp_j, i, j = 1, \ldots, n$. Assume $b > (n-1)c$ to guarantee a negative slope.

The profit of a firm in a Bertrand equilibrium is $\pi_i = q_i p_i = [a - bp_i + (n-1)cp_i]p_i$, $i, j = 1, \ldots, n$. The first order condition and the system of equations for $p_i, i = 1, \ldots, n$ and $p_j, j = 1, \ldots, n$ results in $p_i = p_j = a/[2b - (n-1)c], i, j = 1, \ldots, n$. The quantity is $q_i = ab/[2b - (n-1)c], i = 1, \ldots, n$ and the profit is $\pi_i = a^2 b/[2b - (n-1)c]^2, i = 1, \ldots, n$. These are the competitive prices, quantities, and profits for all firms in Bertrand equilibrium. As values are the same, the subscript *i* is omitted from now on.

Consider now that all firms combine a collusive price p_c. The demand function per firm is now $q_c = a - bp_c + (n-1)cp_c$. Denoting the collusive profit of a firm by $\pi_c = [a - bp_c + (n-1)cp_c]p_c$, the first order condition defines $p_c = a/2[b - (n-1)c]$, then

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[5] If it involves all firms in a market, they act as a monopoly.

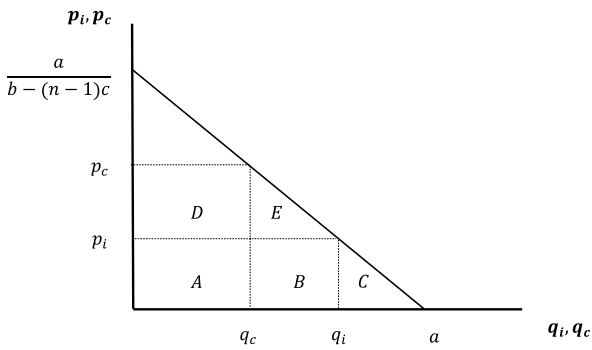
[6] The Bertrand-Nash framework is the most suitable because this paper focuses on price-fixing agreements. For quantity-fixing, bid-rigging, market division, and other types of cartels, there are other appropriated frameworks in Microeconomics (Cournot model, auction theory, and so on). However, further discussions are beyond the scope of this paper.

[7] Demand functions are usually illustrated in textbooks in linear or convex forms. Both would work in this paper but the linear function was selected to avoid unnecessary mathematical complexities. A more general functional form for the demand function could be $q_i = a - bp_i^{\alpha} + \sum_{j \neq i}^{n} cp_j^{\alpha}, i, j = 1, \ldots, n$, which is linear for $\alpha = 1$ and strictly convex for $\alpha > 1$.

[8] The analysis is the same in the presence of a strictly positive marginal cost, if the profit is positive.

$q_c = a/2$ and $\pi_c = a^2/4[b-(n-1)c]$. These are the collusive prices, quantities, and profits for all firms. Figure 1 below illustrates this scenario:

Figure 1 – Bertrand and collusive equilibria



Source: Own elaboration.

The area $A + B$ corresponds to the profit of a firm in Bertrand equilibrium, while C is the deadweight loss. The area $A + B + C$ is the damage caused by the oligopoly in comparison to the perfect competitive scenario which results from market conditions and does not constitute an infringement.[9] On the other hand, collusive equilibrium generates an additional profit represented by the rectangle D and an additional deadweight loss represented by the triangle E.[10] This additional area $D + ED + E$ comes from the price-fixing agreement and represents the impact of the infringement on consumer surplus.

It is easy to observe that price-fixing agreements are more harmful the greater $D + E$. This area is obtained by $(p_c - p_i)q_c + \frac{1}{2}[(p_c - p_i)(q_i - q_c)]$, which give us the following after replacing the variables (further details in the appendix):

$$D + E = \frac{a^2\{3bc(n-1) + c(n-1)[b - c(n-1)]\}}{8[b - c(n-1)][2b - c(n-1)]^2} \quad (1)$$

Observe that the model assumptions guarantee that (1) is strictly positive.[11] In the next section, I evaluate how the damage changes when parameters are modified.

[9] The perfect competition implies a price equal to the marginal cost, zero in this case.

[10] Note that B is part of the profit in Bertrand equilibrium but it is also part of the deadweight in collusion. In any case, both are damages to consumers.

[11] $n \geq 2$ and $b > (n-1)$

2. WHICH PRICE-FIXING AGREEMENT CHARACTERISTICS ARE RELATED TO GREATER DAMAGE TO CONSUMERS?

Overall, three observations are important here. The first one is the use of *everything else being equal* hereafter. When one characteristic is analyzed, I assume that everything else is unaltered. Secondly, only market characteristics that are directly observable by the model are analyzed. There are many other factors that distinguish industries. Ivaldi *et al.* (2007) mention market transparency and degree of innovation, for example, but these aspects are less observable and are not considered here. Third, greater damage, in this paper, means a greater $D + E$ in the same aspect analyzed, which means that only the ordinal aspect is considered. There is no attempt to evaluate a "high" or a "low" damage, since it would require other criteria of analysis, such as a "normal" level of damage for comparison. In the same way, there is no attempt to compare distinct characteristics given the difficulty in ordering terms of damage — besides involving the comparison of many parameters with no relevant results for our purposes.

2.1 THE DEGREE OF HORIZONTAL PRODUCT DIFFERENTIATION

The first aspect is the degree of horizontal product differentiation, represented by the coefficient c in (1). Horizontal product differentiation refers to differences in consumer perception regarding the degree of substitutability among products (products have the same quality but distinct degrees of substitutability).[12]

The smaller (greater) the c the greater (lower) the degree of horizontal product differentiation. The first derivative of (1) in relation to c is the following:

$$\frac{\partial(D+E)}{\partial c} = \frac{\begin{array}{c} 2a^2(n-1)[2b-c(n-1)][b-c(n-1)][2b-c(n-1)]^2 + \\ (n-1)\left\{[2b-c(n-1)]^2 + 2[2b-c(n-1)][b-c(n-1)]\right\} \\ \left\{3a^2bc(n-1) + a^2c(n-1)[b-c(n-1)]\right\}\big] \end{array}}{8\left\{[b-(n-1)c][2b-(n-1)c]^2\right\}^2} \tag{2}$$

Note that all parentheses, brackets, and braces are positive. Thus, $\frac{\partial(D+E)}{\partial c} > 0$. The first result of this paper is the following:

> Result 1: The higher the similarity (substitutability) among products, the greater the damage to consumers

[12] Vertical product differentiation refers to differences in consumer perception regarding product quality.

From this perspective, price-fixing agreements with more substitutable products tend to be more harmful to consumers than the ones with distinct products, and, therefore, should be prioritized regarding detection, prosecution, and punishment by antitrust authorities.

2.2 THE DEMAND BEHAVIOR

The demand behavior is represented by the coefficient b in (1). It measures the effect of a change in price on the quantity demanded. In absolute terms, the lower (greater) the b, the more inelastic (elastic) the demand. The first derivative of (1) regarding b is the following:

$$\frac{\partial(D+E)}{\partial b} = \frac{\begin{array}{c}-a^2c(n-1)\{20b^2[b-c(n-1)]+12b[b-c(n-1)]^2+\\ (n-1)^2[3bc^2+c^2(b-c(n-1))]\}\end{array}}{8[b-(n-1)c]^2[2b-(n-1)c]^4} \tag{3}$$

As all parentheses, brackets, and braces are positive, we have $\frac{\partial(D+E)}{\partial b} < 0$. The second result of this paper is the following:

> Result 2: The lower the price demand elasticity (in absolute value), the greater the damage to consumers

It follows that price-fixing agreements whose products are more inelastic tend to be more harmful to consumers than the ones with more elastic demands, and, therefore, should be prioritized regarding detection, prosecution, and punishment by antitrust authorities.

2.3 THE NUMBER OF MEMBERS

The number of cartel members is the term n in (1). It is clear that the higher the n, the greater the number of firms involved in the infringement. The first derivative of (1) regarding n is as follows:

$$\frac{\partial(D+E)}{\partial n} = \frac{\begin{array}{c}[4a^2bc-2a^2c^2(n-1)][b-c(n-1)][2b-c(n-1)]^2\\ +\{c^3(n-1)^2+[b-c(n-1)]\{6bc+2c[b-c(n-1)]\}\}\\ [4a^2bc(n-1)-a^2c^2(n-1)^2]\end{array}}{8[b-(n-1)c]^2[2b-(n-1)c]^4} \tag{4}$$

All parentheses, brackets, and braces are positive. Therefore, $\frac{\partial(D+E)}{\partial n} > 0$. The third result is the following:

> Result 3: The higher the number of cartel members, the greater the damage to consumers

By the result above, antitrust authorities should focus on price-fixing agreements with more members in comparison to the ones with less participants since they tend to be more harmful to consumers.

2.4 THE DEMAND POTENTIAL

I represent the demand potential by the parameter a in (1), that is, the maximum demand when prices tend to zero. The first derivative of (1) regarding a is the following:

$$\frac{\partial(D+E)}{\partial a} = \frac{ac(n-1)[4b-c(n-1)]}{4[b-(n-1)c][2b-(n-1)c]^2} \tag{5}$$

All terms inside parentheses and brackets are positive. Therefore, $\frac{\partial(D+E)}{\partial a} > 0$. The fourth result is stated below:

Result 4: The higher the demand potential, the greater the damage to consumers

Price-fixing agreements with higher demand potential, in comparison to the ones with lower demand potential, should be prioritized by antitrust authorities, particularly due to the greater harm caused to consumers.

2.5 THE VELOCITY OF TRANSACTIONS

The model represents a number of commercial transactions in a determined period. For example, cartel members sell p_c units at a collusive price p_c in a day, generating the damage in (1). To compare distinct velocity of transactions, I define a common period and state a parameter v representing the velocity, that is, the number of times the damage is caused in a determined period of time. The total damage in this period is the following:

$$v(D+E) = \frac{va^2\{3bc(n-1)+c(n-1)[b-c(n-1)]\}}{8[b-c(n-1)][2b-c(n-1)]^2} \tag{6}$$

The first derivative regarding v is simply the damage $D + E$:

$$\frac{\partial v(D+E)}{\partial v} = \frac{a^2\{3bc(n-1)+c(n-1)[b-c(n-1)]\}}{8[b-c(n-1)][2b-c(n-1)]^2} \tag{7}$$

The damage is strictly positive, which provides the fifth result of the paper:

Result 5: The higher the velocity of transactions, the greater the damage to consumers

As a consequence, antitrust authorities should focus on price-fixing agreements whose velocity of transactions is higher, in comparison to the ones with lower velocity of transactions, as the former tend to be more harmful to consumers than the latter.

3. A BRIEF DISCUSSION

Some industries tend to present the previous characteristics more intensely. Consider the price-fixing agreements among gas stations, very common worldwide. Based on the results above, we have:

- Fuels tend to show a low degree of product differentiation from the consumer's viewpoint, that is, tend to be highly substitutable.
- Fuels tend to have an inelastic demand for being a necessary product.
- Gas station cartels tend to involve many stations in the same region.
- Fuels tend to have a high demand potential.
- The velocity of transactions in gas stations tend to be extremely high.

It follows that gas station agreements should be prioritized by antitrust authorities and should be severely fined, as the damage caused to consumers tends to be greater than other cartels that do not fit the results.

It is important to highlight that the ideal scenario is being able to calculate the exact damage for all infringements. Then, authorities could prioritize the most harmful ones and impose a harsh penalty. This first best scenario is important especially because a case that apparently does not fit the results can be worse than the others. For example, suppose a national price-fixing agreement with a low c, a great b, a small n, and a small v in comparison to other cases. As firms sell to the whole country, the demand potential is extremely high. This case can be more harmful than a gas station cartel in a small region, for example. When calculating the harm to consumers is not possible, the above results are useful tools to understand which market characteristics are related to a greater damage to consumers.

CONCLUSIONS

Fighting cartels requires time and financial resources. Considering that both are limited, it is essential to understand which price-fixing characteristics are related to a greater damage to consumers. Based on linear demand functions, this paper aimed to develop

a simple model to represent the damage caused by price-fixing agreements and how the intensity of market characteristics interfere with this damage.

Everything else constant, the following price-fixing agreement characteristics are related to greater damage to consumers: higher product similarity (substitutability); lower elasticity of demand (in absolute value); greater number of members; higher demand potential; and higher velocity of transactions. The results provide relevant insights for economic and legal teams of antitrust authorities on the priority of investigation/punishment and on the severity of penalties for price-fixing agreements.

Lastly, a deterred cartel is better than a punished cartel, as the latter tends to demand more resources from antitrust authorities. Nevertheless, the fact is that many cartels exist. The results of this paper are essential in the sense of providing useful information to researchers and policy makers in fighting cartels.

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APPENDIX

This appendix aims to provide further details on the mathematical expressions (1) – (7). The analysis begins with Figure 1, where the geometric area $D + E$ represents the loss of consumer welfare caused by the collusive agreement (D is the collusive profit and E is the collusive deadweight loss). The greater the $D + E$ the higher the damage caused by the cartel. Geometrically, this area can be calculated by the expression $(p_c - p_i)q_c + \frac{1}{2}[(p_c - p_i)(q_i - q_c)]$, while $p_i = p_j = a/[2b - (n-1)c], i, j = 1, \ldots, n$ and $q_i = ab/[2b - (n-1)c], i = 1, \ldots, n$ are obtained through the Nash-Bertrand equilibrium and $p_c = a/2[b - (n-1)c]$ and $q_c = a/2$ are obtained through the collusive equilibrium. After replacing the variables, we reach the following:

$$D + E = \left[\frac{a}{2[b-(n-1)c]} - \frac{a}{[2b-(n-1)c]}\right]\frac{a}{2} + \frac{1}{2}\left\{\left[\frac{a}{2[b-(n-1)c]} - \frac{a}{[2b-(n-1)c]}\right]\left[\frac{ab}{[2b-(n-1)c]} - \frac{a}{2}\right]\right\} \quad \text{(A.1)}$$

Which can be expressed as the following, after some manipulation:

$$D + E = \frac{a^2\{3bc(n-1) + c(n-1)[b - c(n-1)]\}}{8[b - c(n-1)][2b - c(n-1)]^2} \quad \text{(A.2)}$$

Observe that (A.2) is precisely the expression (1) in this paper.

Result 1 is obtained through the first derivative of $D + E$ regarding c, which, by the product rule, gives us the following:

$$\frac{\partial(D+E)}{\partial c} = \frac{\begin{array}{c} a^2\left[4nc + 4bn - 4b - 2c - 2n^2c\right][b - c(n-1)][2b - c(n-1)]^2 \\ -a^2\{3bc(n-1) + c(n-1)[b - c(n-1)]\} \\ \left[9n^2c^2 - 9nc^2 + 3c^2 + 10bc + 10bn^2c - 20bnc + 8b^2 - 8b^2n - 3n^3c^2\right] \end{array}}{8\left\{[b-(n-1)c][2b-(n-1)c]^2\right\}^2} \quad \text{(A.3)}$$

After some algebraic manipulation, we reach the following:

$$\frac{\partial(D+E)}{\partial c} = \frac{\begin{array}{c} 2a^2(n-1)[2b - c(n-1)][b - c(n-1)][2b - c(n-1)]^2 + \\ (n-1)\left\{[2b - c(n-1)]^2 + 2[2b - c(n-1)][b - c(n-1)]\right\} \\ \left\{3a^2bc(n-1) + a^2c(n-1)[b - c(n-1)]\right\}\end{array}}{8\left\{[b-(n-1)c][2b-(n-1)c]^2\right\}^2} \quad \text{(A.4)}$$

Which is precisely expression (2).

Result 2 is obtained through the first derivative of $D + E$ regarding b, which, by the product rule, gives us the following:

$$\frac{\partial(D+E)}{\partial b} = \frac{\begin{array}{c} 4ca^2(n-1)[b - c(n-1)][2b - (n-1)c]^2 - \\ a^2\left(12b^2 + 16bc - 16bcn + 5c^2 + 5c^2n^2 - 10c^2n\right) \\ \{3bc(n-1) + c(n-1)[b - c(n-1)]\}\end{array}}{8[b-(n-1)c]^2[2b-(n-1)c]^4} \quad \text{(A.5)}$$

Which can be expressed as the following, after some manipulation:

$$\frac{\partial(D+E)}{\partial b} = \frac{\begin{array}{c} -a^2c(n-1)\{20b^2[b-c(n-1)]+12b[b-c(n-1)]^2+ \\ (n-1)^2[3bc^2+c^2(b-c(n-1))]\} \end{array}}{8[b-(n-1)c]^2[2b-(n-1)c]^4} \tag{A.6}$$

The expression above is precisely expression (3) in this paper.

The third result refers to the first derivative of $D + E$ regarding n:

$$\frac{\partial(D+E)}{\partial n} = \frac{\begin{array}{c} a^2[4bc-2c^2n+2c^2][b-c(n-1)][2b-c(n-1)]^2 \\ -a^2[6c^3n-3c^3n^2+10bc^2n-3c^3-10bc^2-8b^2c] \\ [3bc(n-1)+c(n-1)[b-c(n-1)] \end{array}}{8[b-(n-1)c]^2[2b-(n-1)c]^4} \tag{A.7}$$

Which gives us the following, after some manipulation:

$$\frac{\partial(D+E)}{\partial n} = \frac{\begin{array}{c} [4a^2bc-2a^2c^2(n-1)][b-c(n-1)][2b-c(n-1)]^2 \\ +\{c^3(n-1)^2+[b-c(n-1)]\{6bc+2c[b-c(n-1)]\}\} \\ [4a^2bc(n-1)-a^2c^2(n-1)^2] \end{array}}{8[b-(n-1)c]^2[2b-(n-1)c]^4} \tag{A.8}$$

Observe that (A.8) is precisely expression (4) in this paper.

The fourth result refers to the first derivative of $D + E$ regarding a:

$$\frac{\partial(D+E)}{\partial a} = \frac{a[2c^2n-c^2n^2-c^2+4bcn-4bc]}{4[b-(n-1)c][2b-(n-1)c]^2} \tag{A.9}$$

Which gives us the following, after some manipulation:

$$\frac{\partial(D+E)}{\partial a} = \frac{ac(n-1)[4b-c(n-1)]}{4[b-(n-1)c][2b-(n-1)c]^2} \tag{A.10}$$

Note that (A.10) matches expression (5) in the paper, precisely.

Lastly, the fifth result is based upon expression (7), which is precisely the area $D + A$.