

Analysis of the Stresses in Corrugated Sheets under Bending

Julio Cesar Molina^{a*}, Juliano Fiorelli^b, Holmer Savastano Jr.^b

^aUniversity of the Wood Industrial Engineering, Universidade Estadual Paulista – UNESP, Rua Geraldo Alckmin, 519, Nossa Senhora de Fátima, CEP 18409-010 Itapeva, SP, Brazil

^bCollege of Zootechny and Food Engineering, Universidade de São Paulo – USP, Av. Duque de Caxias Norte, 225, Centro, CEP 13635-900 Pirassununga, SP, Brazil

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This paper presents three different numerical models for the evaluation of the stresses in corrugated sheets under bending. Regarding the numerical simulations different approaches can be considered, i.e., a elastic linear analysis or a physical nonlinear analysis, that considers criteria to fail for the sheet material. Moreover, the construction of the finite element mesh can be used shell elements or solid elements. The choice of each finite element must be made from the consideration of their representativity before behavior to be simulated. Thus, the numerical modelling in this manuscript was performed from the three-dimensional models using the SAP2000Nonlinear software, version 7.42, which has as base the finite elements method (FEM). It was considered shell elements in the build the mesh of finite elements and an analysis of type elastic linear in this case. Five mm thick sheets were evaluated considering three different longitudinal dimensions (spans), i.e., 1100 mm, 1530 mm and 1830 mm. The applied load to the models was 2500 N/m and it was verified that the spans of support of sheets have a significant influence on the results of stresses. The sheets with larger spans present larger stresses for the same applied load. The most intense values of tension occur in the troughs (low waves) of the sheets, on the lower surface, while the most intense values of compression occur in the crests (high waves), on the upper surface of the sheet. The flanks, which are the parts among the troughs and crests of the sheets, are submitted to low levels of stresses. The numeric results of the stresses showed a good agreement with the results obtained from other researchers³ and these results can be used to predict the behavior of corrugated sheets under bending.

Keywords: *corrugated sheet, numerical modelling, stress analysis*

1. Introduction

In general, the test load of corrugated sheets is of large interest for fiber-cement industries. This value serves as parameter of acceptance or rejection of the lots of sheets produced. The asbestos cement is known to be a very durable material and is used in the composition of corrugated sheets¹. However, nowadays most published studies about fiber-cement sheets discuss the use of materials or compositions for sheets free of asbestos². Thus, fiber-cement industries are migrating for new technologies that use synthetic fibers in the production of components for civil construction. However, technology of the production of fiber cement without asbestos has presented unsatisfactory results due to high production costs and to the low mechanical performance of the components compared to the conventional products. An alternative to solve these problems is the use of fiber cement with functionally graded, which consists in cement composites reinforced with fibers presenting varying properties in a local way³. Within this context, the knowledge of the stress distribution in corrugated sheets becomes important. The numerical simulation is an important tool in this case and

allows the analysis of the stresses in the corrugated sheet, no matter of the material used in their composition, since the experimental tests do not allow this type of analysis of a direct way.

Different approaches for the numerical simulations can be considered, i. e., an linear elastic analysis with geometric nonlinearity or a physical nonlinear analysis, that considers criteria to fail for the sheet material. The linear elastic analysis is simple to be performed, and can be used for checking the global behavior of a structural element. For this purpose can be used the SAP2000NonLinear software, which has in its internal library finite elements and other tools that facilitate the construction and analysis of numerical models. On the other hand, a physical non-linear analysis is often used in the verification of located points of interest as, for example, the concentration of stress at connections. Besides, with respect to the discretization of the finite element mesh can be used shell elements or solid elements. In this case, can be used other commercial softwares as, for example, ANSYS, ABAQUS, DIANA and others that have finite elements with nonlinear properties. However, the utilization of these softwares is more complex. For both types of analysis presented, linear and nonlinear,

*e-mail: juliocesarmolina@yahoo.com.br

the degree of refinement of finite element mesh also is important. Verification of localized aspects of interest such as the areas of connections usually requires more discretization for the mesh, while the verification of global aspects does not require meshes very discretized.

This work is inserted in the particular scenery of the Brazilian section of fiber cement in which there is social motivation for banning fibers of asbestos. Thus, no matter the material used in the composition of the corrugated sheets it is important the knowledge of the behavior of final product under bending, since the corrugated sheets have different distribution of stresses in the transverse and longitudinal directions of the component. There are no many studies involving the numerical analysis of corrugated sheets about the analysis of transverse and longitudinal stresses.

This manuscript presents the numerical results of stresses distribution in corrugated sheets under bending considering a load of 2500 N/m (load/width of sheet). The behavior of the sheets in the longitudinal and transverse directions are presented in terms of stresses as well as the influence of span of sheet on the longitudinal stresses. These results obtained in this study can help the industries and researchers to solve the main problems related to this product. Besides, the strategy of the numerical modelling from the use of SAP2000NonLinear software is also presented. Therefore, the main contributions of this work are the presentation of the distribution of stresses in corrugated sheets in transverse and longitudinal directions as well as show how these stresses behave due the increase of the size longitudinal of span support of the sheet. Furthermore, it is worth saying that the method of analysis used in this work from the use of software SAP2000NonLinear is simple when compared with other programs of numerical analysis as the

ANSYS software and leads to satisfactory results in terms of stresses and strains.

2. Material and Methods

The space among supports (span) has an important role in the acting of fiber-cement corrugated sheets. The recommended span for tests according to national standard⁴ is of 1100 mm for sheets of up to 6 mm of nominal thickness. In this work, three different spans for the corrugated sheets: 1100 mm, 1530 mm and 1830 mm were considered.

2.1. Analyzed sheets

Corrugated sheets with the following configuration were analyzed:

- Height: 51 mm;
- Transverse width: 1100 mm;
- Thickness: 5 mm.

Figure 1 shows the dimensions of the cross section of the sheets and Figure 2 shows the nomenclature used for the corrugated sheets.

The load was distributed in the transverse width of the sheets in a central strip of 230 mm. Figure 3 shows the load and support conditions for the models analyzed.

For simplicity of the models, the sheets were supported without lateral balances, as shown in Figure 4. The width of the sheets as well as the spans admitted for the models are indicated in Table 1.

2.2. Load conditions

The load (2500 N/m) was applied on the high wave (troughs) of the sheets in a central strip of 230 mm in fractions of load as indicated in Figure 5. The fractions of load were determined by experimental tests starting from

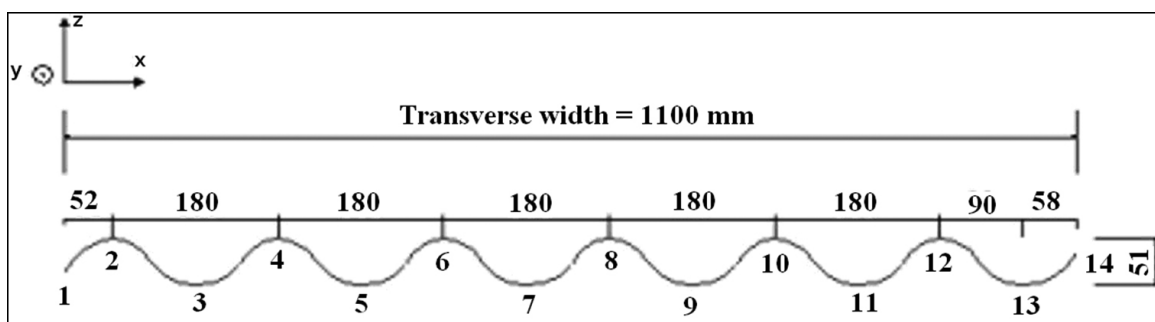


Figure 1. Cross section of the sheets considered in the numerical models.

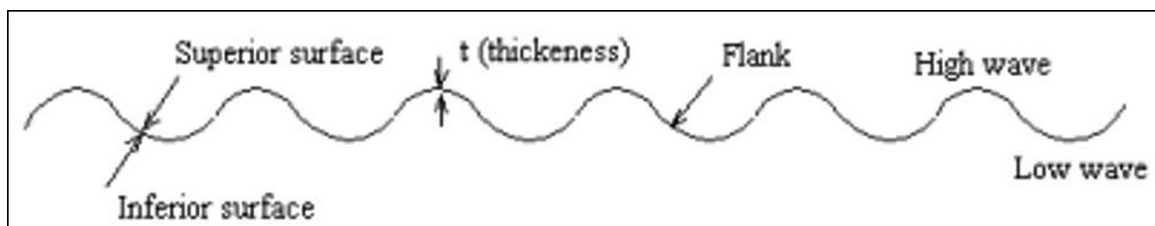


Figure 2. Nomenclature used for the corrugated sheets.

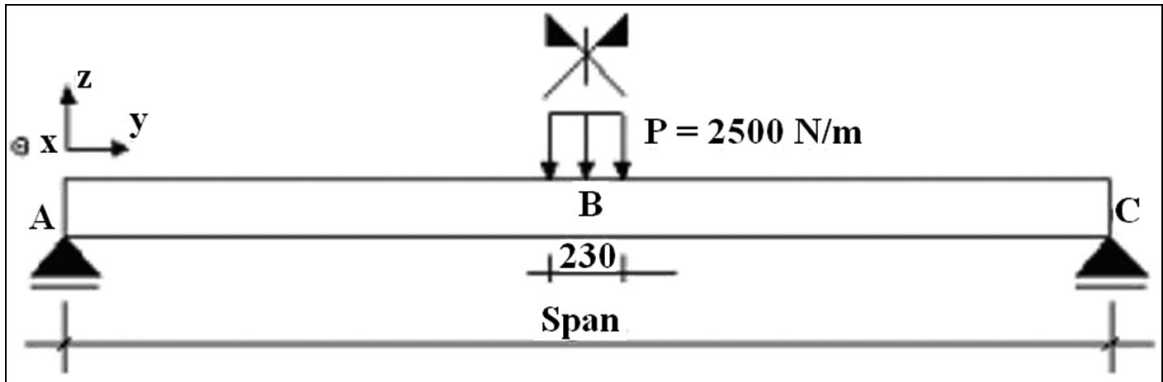


Figure 3. Support and load conditions for the models analyzed under bending.

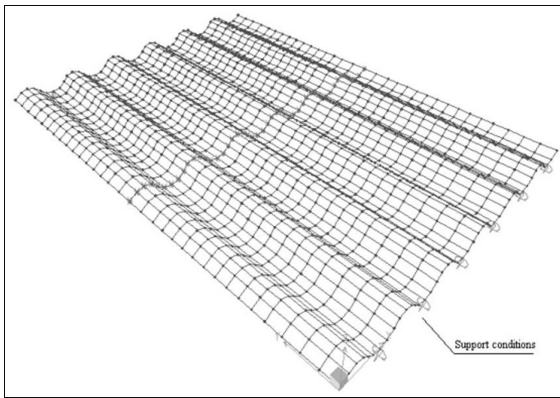


Figure 4. Support conditions admitted in the models.

Table 1. Support conditions and lengths admitted for the sheets under bending.

Length of the sheets (mm)	Support spans (mm)
1100	1100
1530	1530
1830	1830

the use of a carbon paper located between the cleaver and the high wave of the sheets³. Starting from this experimental tests it was observed that the load is distributed in lines from the end of the cleaver until the points away from 50 mm of the extremities³. The load of the cleaver is distributed in the waves in fractions of the total load³. It is important to mention that in a normalized test of bending, for the verification of the performance of sheets, the transverse load in the wave high is applied by the use of a cleaver (test apparatus)⁴.

The calculation of the fractions of load applied in each high wave is presented in the sequence. Figure 5 shows the calculated values.

$$\text{wave 2: } (2500 \text{ N/m} \times 1.10 \text{ m}) \times 0.0741 = 203.775 \text{ N/4} = 50.94\text{N}$$

$$\text{wave 4: } (2500 \text{ N/m} \times 1.10 \text{ m}) \times 0.2098 = 576.950 \text{ N/4} = 144.24\text{N}$$

$$\text{wave 6: } (2500 \text{ N/m} \times 1.10 \text{ m}) \times 0.1640 = 451.000 \text{ N/4} = 112.75\text{N}$$

$$\text{wave 8: } (2500 \text{ N/m} \times 1.10 \text{ m}) \times 0.1768 = 486.200 \text{ N/4} = 121.55\text{N}$$

$$\text{wave 10: } (2500 \text{ N/m} \times 1.10 \text{ m}) \times 0.1590 = 437.250 \text{ N/4} = 109.31\text{N}$$

$$\text{wave 12: } (2500 \text{ N/m} \times 1.10 \text{ m}) \times 0.2163 = 594.825 \text{ N/4} = 148.71\text{N}$$

However, for the simulation of the effect of load below the region of cleaver, the load was applied in line, in the high waves up to the distance of 50 mm from the extremities of cleaver as shows Figure 6.

Some Brazilian standards recommend for the tests of fiber-cement corrugated sheets the load of bending equal to 5 kN/m². Figure 7 shows the test apparatus used in experimental tests⁵ for the determination of the strength in the corrugated sheets under bending. For the corrugated sheets without asbestos, there are no standard recommendations about the limit value of load to be used in the tests under bending.

2.3. Boundary conditions

To simulate the bending tests of the corrugated sheets considering the lengths indicated in Table 1 with an applied load of 2500 N/m, in the strip of 230 mm, the following boundary conditions were considered:

- Restrain of the displacements in the x and z directions in the sections of support;
- Restrain of the displacement y at points located in the central section.

Figure 8 shows the details of boundary conditions and the mesh used for the corrugated sheets in the models.

2.4. Mesh of finite elements

The mesh of the sheet was discretized in a total of 1152 elements to the sheet with span of 1100 mm and of 1536 elements for the sheets with spans of 1530 mm and 1830 mm. The meshes of the numerical models were analyzed from different levels of refinement, until the results would lead to satisfactory answers in terms of displacements and strains. For the mesh m_1 (sheet with span of 1100 mm) were considered initially 768 finite elements and for meshes m_2 , m_3 and m_4 were considered 864, 1152 and 1728 finite elements, respectively. The results obtained for the

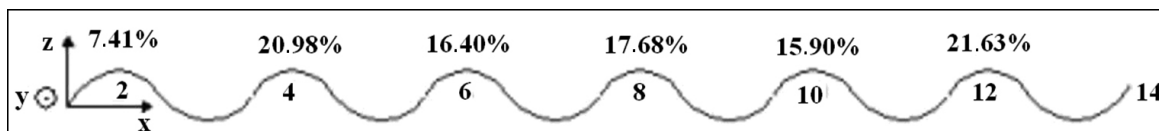


Figure 5. Load of 2500 N/m applied in fractions of the total load in the high waves 2, 4, 6, 8, 10 and 12 in the central strip of width equal 230 mm.

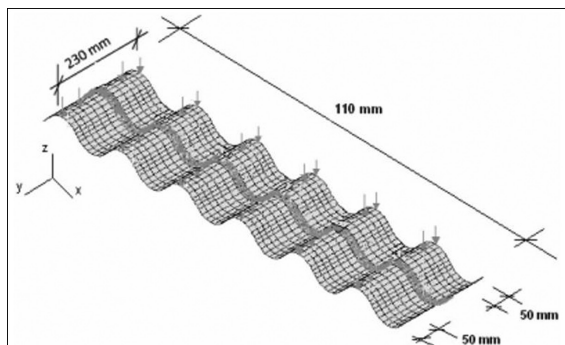


Figure 6. Fractions of load applied in line, in the high waves 2, 4, 6, 8, 10 and 12, in the central area of width 230 mm to the 50 mm of the extremities of cleaver.

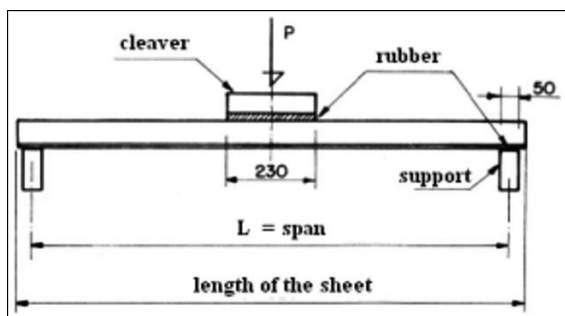


Figure 7. Test apparatus used for determination of the strength of the corrugated sheets⁶.

configuration m_1 were different from the results obtained for configurations m_2 , m_3 and m_4 , which were more discretized. However, the results obtained for the configuration m_1 were similar to those obtained to m_2 configuration. It was observed greater numerical stability for m_3 and m_4 configurations with respect to m_1 and m_2 . The results for configurations m_3 and m_4 were about the same but, however, the processing time for m_4 was three times greater than the processing time for m_3 . For these reasons, was chose the configuration m_3 for the modeling of the sheets with span of 1100 mm. For the spans of 1530 mm and 1830 mm, with base on what was presented, the same study was made, and the number of finite elements considered for the mesh m_3 for these cases was 1536. There are specific computer programs for construction of complex meshes as, for example, the TrueGrid software.

The discretization of the mesh was made using the software SAP2000Nonlinear⁶, version 7.12, which has as base the finite element method (MEF). Shell elements in the composition of the mesh were used. The shell element has four nodes with six degrees of freedom per node, being three

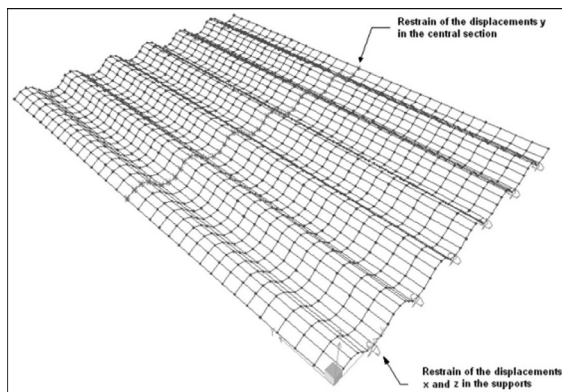


Figure 8. Boundary conditions: restrain of the displacements in the supports and in the central section of the sheet.

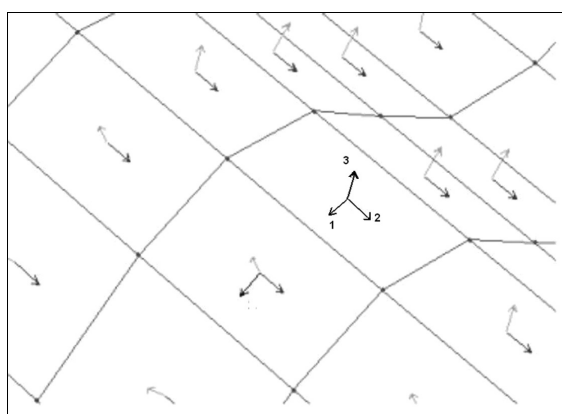


Figure 9. Orientation of the local axes of shell elements and mesh of finite elements of the model.

translations and three rotations. Each shell element was built having the same orientation for the quadrilaterals that formed each one of the shell elements. In the SAP2000Nonlinear, the local axe 3 is oriented in perpendicular direction to the surface of the element. This way the local axes 1, 2 and 3 from the shell elements were admitted with the same orientation facilitating the discretization of the mesh of finite elements and subsequent the analysis of the results. Figure 9 shows the orientation of the local axes in the sheets and the mesh of finite elements.

2.5. Analysis considered in the models

The models were developed considering the physical linearity of the material (stress is proportional to strain) and geometric non-linearity of the sheets (deformed configuration)⁷. Isotropic behavior for the material used

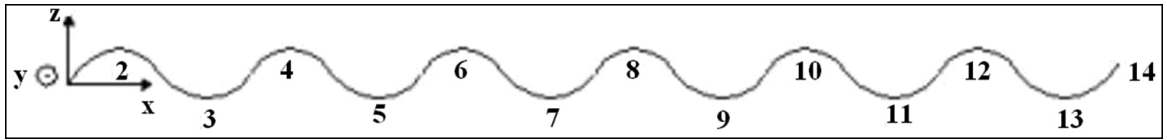


Figure 10. Points in the sheets where verified the maximum stresses: σ_{\max} (tension) and σ_{\min} (compression).

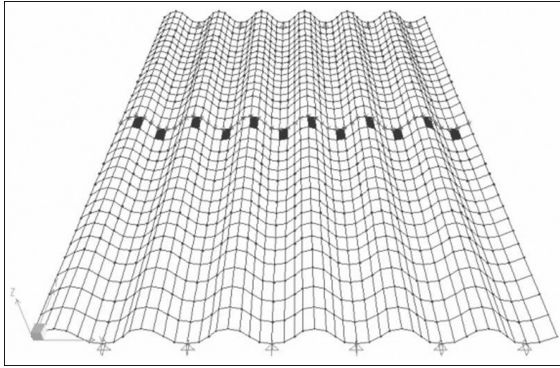


Figure 11. Finite elements in the central section of the sheet where the maximum stresses σ_{\max} and σ_{\min} were verified.

in the composition of the sheets was admitted. The elastic properties of the sheet admitted for the models were:

- Module of elasticity (E): 9000 N/mm² (9000 MPa);
- Poisson ratio (ν): 0.2 (typical value for the fiber cement used in the corrugated sheets).

2.6. Stresses and strains evaluated in the model

To the considered load of 2500 N/m the analysis of the stresses with larger potential to cause damage to the component was made. The stresses with this potential are the followings:

- Longitudinal stress σ_{\max} (maximum value of tension in the lower surface of the low waves);
- Longitudinal stress σ_{\min} (maximum value of compression in the upper surface of the high waves).

The values of the main stresses, σ_{\max} and σ_{\min} , were verified in the finite elements localized in the central section of the sheet, as indicated in Figures 10 and 11. The validation of the numerical results presented in this manuscript was made from the comparison with the numerical results presented by other authors³. The comparison between results in this case presented a good agreement. The numerical results³ used to validate the model of this manuscript were validated through experimental bending tests conducted in sheets of fiber-cement.

The maximum stresses of tension (σ_{\max}) were verified in the low waves 3, 5, 7, 9, 11 and 13 while the maximum stresses of compression (σ_{\min}) were verified in the high waves 2, 4, 6, 8, 10 and 12. In this case, the values of stress correspond to the stress σ_{11} (normal stress caused by bending moment acting around the neutral line in the x direction).

2.7. Material properties

The conventional fiber cement without asbestos when subjected to stresses above the stress in the elastic limit start

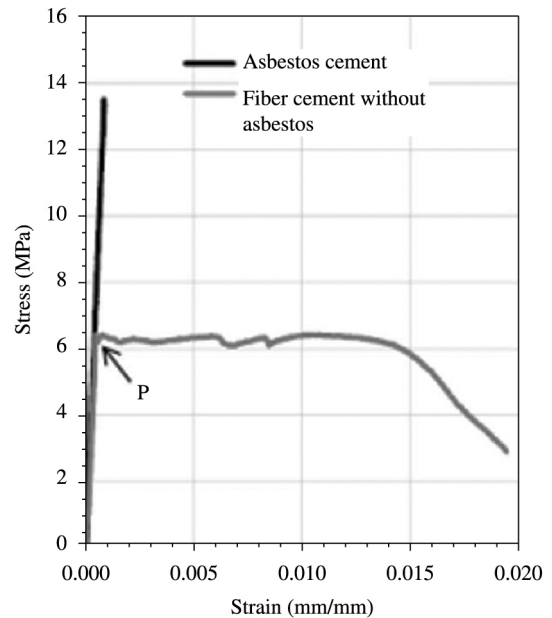


Figure 12. Curves of stress versus strain obtained from tension tests in specimens extracted of sheets available in Brazil³.

to present cracking and plastic deformations³. The tension strength of the asbestos cement is from 15 MPa to 20 MPa reaching up to 25 MPa. However, the average strength of the asbestos cement produced in Brazil is 13.2 MPa³.

For the tests of corrugated sheets the minimum load required by standard equates the maximum load that the sheet supports under bending and not to the load of first crack from which the tightness of sheet becomes compromised³. Point P in Figure 12 corresponds to the point of first crack of the matrix of fiber cement without asbestos, i.e., the matrix begins to crack when the tensile stress reaches values close to 6 MPa³. These values are equivalent about half of failure stress of the asbestos cement³.

The elasticity module suitable to the use in numerical models is that obtained from the direct tests of tension instead of bending in four points⁸.

3. Results and Discussion

The results of longitudinal stresses obtained from the numerical simulations are shown in Figures 13, 14 and 15. The maximum stress values (compression and tension) for the sheets in the points showed in Figure 10 are shown in Tables 2, 3 and 4.

3.1. Longitudinal stresses

The maximum values of tension of stress (σ_{\max}) in the lower surface of the sheets are summarized in Table 5.

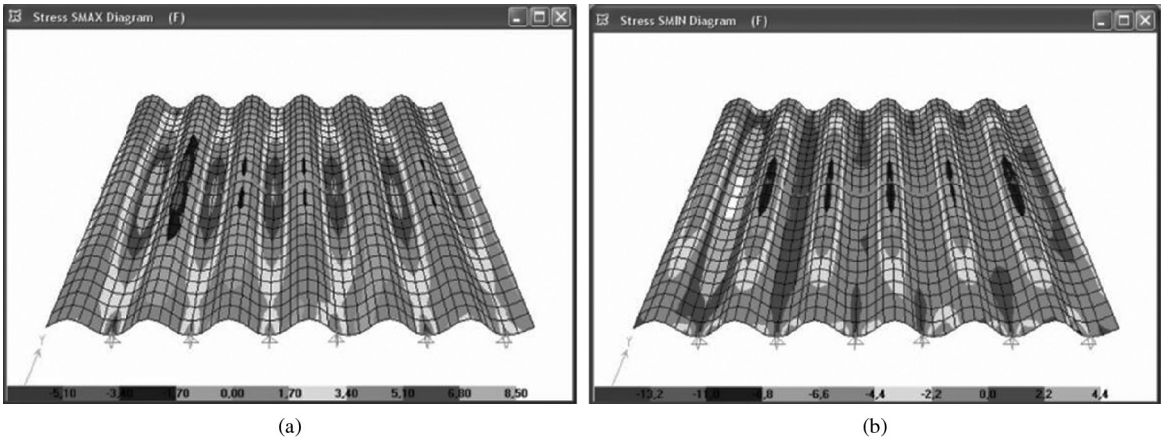


Figure 13. Stresses in the model with span of 1100 mm: (a) Maximum value of tension (σ_{max}) in the lower surface of the low wave, (b) Maximum value of compression (σ_{min}) in the upper surface of the high wave.

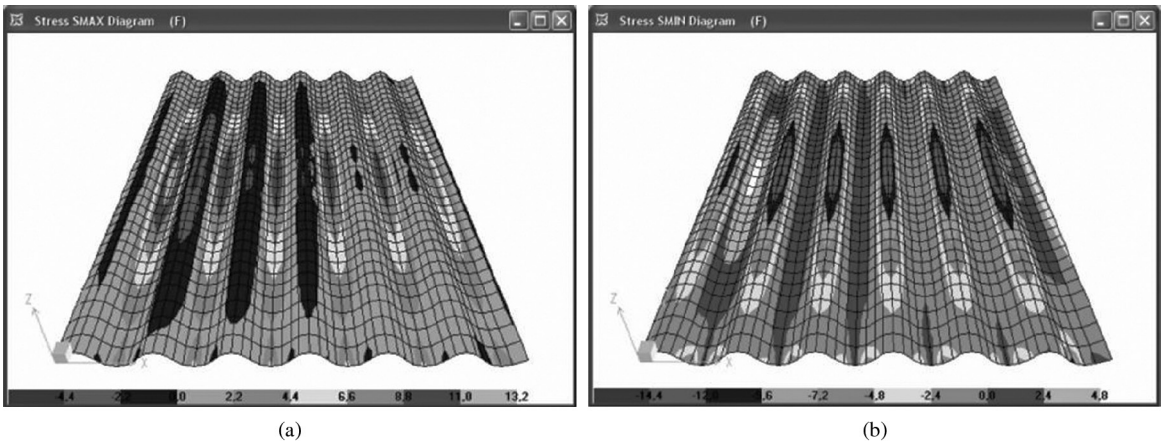


Figure 14. Stresses in the model with span of 1530 mm: (a) Maximum value of tension (σ_{max}) in the lower surface of the low wave, (b) Maximum value of compression (σ_{min}) in the upper surface of the high wave.

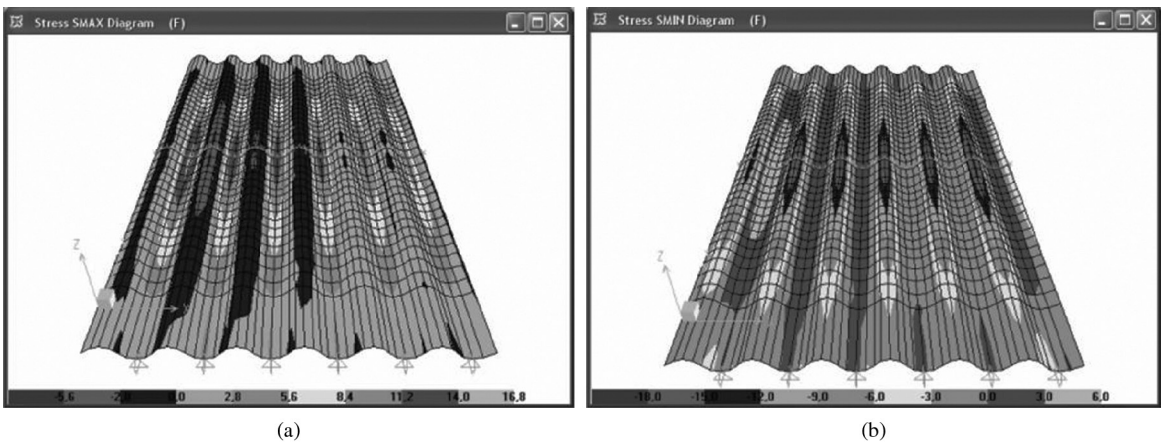


Figure 15. Stresses in the model with span of 1830 mm: (a) Maximum value of tension (σ_{max}) in the lower surface of the low wave, (b) Maximum value of compression (σ_{min}) in the upper surface of the high wave.

Table 2. Values of maximum longitudinal stresses in the model with span of 1100 mm.

σ_{min} (MPa) in the high wave (compression)						σ_{max} (MPa) in the low wave (tension)					
2	4	6	8	10	12	3	5	7	9	11	13
7.79	10.64	9.85	10.07	9.02	10.68	8.51	9.81	8.90	8.71	9.94	7.63

Table 3. Values of maximum longitudinal stresses in the model with span of 1530 mm.

σ_{\min} (MPa) in the high wave (compression)						σ_{\max} (MPa) in the low wave (tension)					
2	4	6	8	10	12	3	5	7	9	11	13
11.38	15.04	14.56	14.28	13.99	15.67	12.88	13.10	13.12	13.36	13.78	11.11

Table 4. Values of maximum longitudinal stresses in the model with span of 1830 mm.

σ_{\min} (MPa) in the high wave (compression)						σ_{\max} (MPa) in the low wave (tension)					
2	4	6	8	10	12	3	5	7	9	11	13
13.92	17.88	17.73	17.48	17.26	19.13	15.95	16.02	16.03	16.28	16.46	13.64

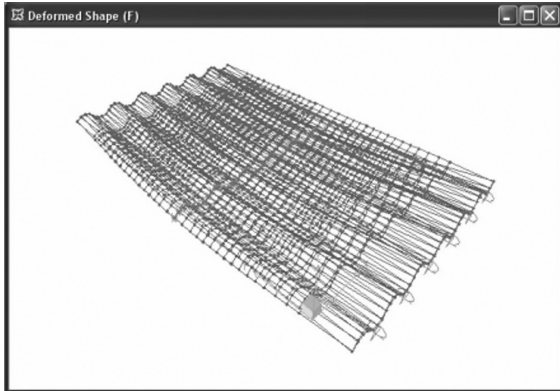
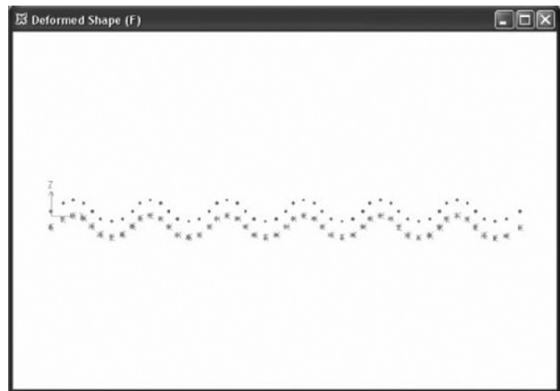
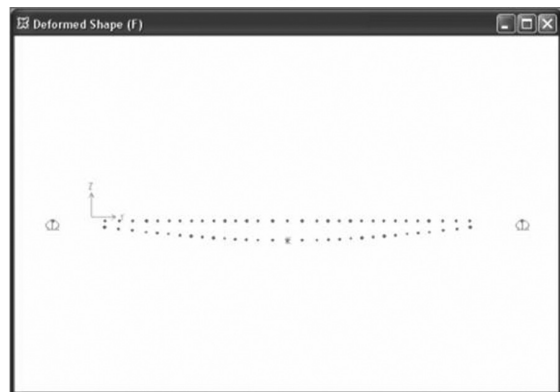


Figure 16. Global view of the deformed configuration of the sheets considering a vertical load of 2500 N/m under bending.



(a)



(b)

Figure 17. Deformed configuration of the sheets: (a) Transverse deformation in the center of span, (b) Longitudinal deformation between supports.

Table 5. Values for the fiber-cement corrugated sheets with thickness of 5 mm.

Span (mm)	Load (N/m)	σ_{\max} (MPa)
1100	2500	9.94
1530	2500	13.78
1830	2500	16.46

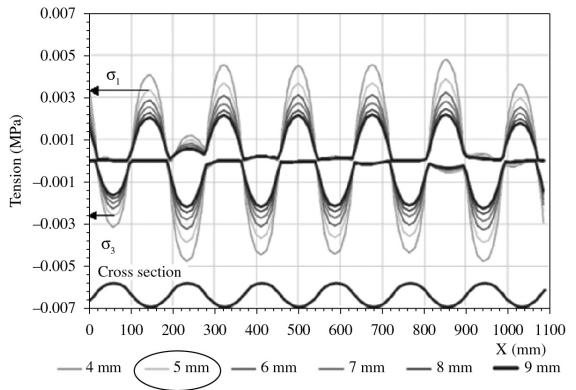


Figure 18. Simulation of the bending test of ABNT NBR 15210-2: Distribution of stresses σ_1 (bottom surface) and σ_3 (upper surface) in the central section (span of 1100mm) for bending load of 1 N/m.

3.2. Strain of the sheets

The global, longitudinal and transverse strains of the sheets are presented in Figures 16 and 17 in the follow.

3.3. Comparison of the results obtained with the other author³

Figure 18 shows the results of stresses in the central section obtained by the other author³ considering a 3D model of corrugated sheet with 1100 mm for the comparison of the numerical results. The comparison between the results show that the value of stress (σ_{\min}) obtained in this manuscript for the wave 2 (maximum value of compression in the upper surface of the high waves) was about -0.0026 MPa (see Figure 1). On the other hand, the value of stress (σ_{\max}) obtained for wave 3 (maximum value of tension in the lower surface of the low waves) was about $+0.0034$ MPa. Strength values obtained according to Figure 18^[3] were $\sigma_3 = -0.0025$ MPa and $\sigma_1 = +0.0033$ MPa, where $\sigma_3 = \sigma_{\min}$ and $\sigma_1 = \sigma_{\max}$, respectively.

4. Conclusions

The span of sheet has a significant influence on the longitudinal stresses, i.e., for the same applied load, the stresses in the sheets are larger for larger spans. For the simulation of the bending tests with vertical load of 2500N/m the most intense longitudinal stresses of tension occur in the low waves, in the lower surface, in the central area of the sheet, while the most intense longitudinal stresses of compression occur in the high waves, in the upper surface. The flanks are subjected to low levels of stress. The stresses σ_{\max} (in the low waves) and σ_{\min} (in the high waves) for the applied load of 2500 N/m act in the longitudinal direction of the sheet, decreasing in intensity from the center of sheet to the points of supports. The stresses in the sheets are more pronounced in the first low wave (number 3) when compared with the other stresses in the low waves. This behavior is due the configuration of the support, which forms a balance, and load used in this area of the sheet that favors the increase of the stresses. The tension of stress (σ_{\max}) in the lower surface of the sheet is the critical stress and is responsible for the failure of the material. The conventional fiber cements without asbestos

present plastic behavior when submitted to the stresses above of the stress in the linear limit that correspond about 6 MPa. The asbestos cement has fragile behavior and linear elastic up to failure that occurs with approximately 13.2 MPa. The limit numeric value for the minimum load in corrugated sheets without asbestos considering the tension strength of the fiber cement is equal to 6 MPa and span of the sheet of 1100 mm is approximately 1547 N/m. The limit numeric value estimated for the minimum load in corrugated sheets of asbestos cement considering the tension strength equal to 13.2 MPa and span of 1100 mm is approximately 3402 N/m. The strains of the sheets in transverse and longitudinal directions present different configurations for the vertical applied load of 2500 N/m in the center of the span. The results of the analyzed models indicated that the stiffness of the sheet in the transverse direction is lower than longitudinal direction.

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