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Rice detector: proposal of a novel objective response detection technique

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Abstract ***Introduction:*** *The detection of the somatosensory response (SR) is an important tool for the neurophysiological evaluation in the intra and post-operative period of some vascular and spine surgeries. Particularly, the SR identification with a maximum false positive ratio by means of Objective Response Detection (ORD) techniques could lead to a less subjective procedure. In this work a novel ORD, the Rice Detector (RD), is presented and its theoretical critical value is obtained.* ***Methods:*** *The probability of detection (PD) of RD is assessed for different numbers of eletroencephalographic (EEG) signal epochs (M = 30, 60, 120, 240) and signal-to-noise ratios (–20 to 10 dB, in steps of 1 dB) by means of simulation. The simulated PD curves (PDc) are compared with the theoretical ones and with the PDc of the Magnitude-Squared Coherence (MSC), a well-known ORD technique. The performance of RD and MSC are also compared for real EEG data. The comparison is based on the DP for estimates calculated with M = 30, 60, 120 and 240 epochs.* ***Results:*** *The results showed that the simulated PDc follow the theoretical ones and both the MSC and RD present similar performance, with slight advantage for this latter at low M-values. However, for real data, no statistical significant difference (proportion test with alpha=0.05) was found between MSC and RD.* ***Conclusion:*** *Both techniques presented mean detection rates varying from 70% to 90%, even for intermediate M-value (120 epochs), and can be useful for evoked response detection applications.*

Keywords *Objective response detection, Rice distribution, Rice detector, Magnitude-squared coherence, Evoked potential, Electroencephalography.*

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Introduction

The detection of the somatosensory response has been considered an important tool for the neurophysiological evaluation in the intra and post-operative period (Achouh *et al*., 2007). Such responses, usually elicited by current pulses, are estimated by averaging a number of electroencephalogram (EEG) epochs, synchronized with the stimulation, and present characteristic waves. The amplitude and latency (time of occurrence referenced to the stimulus) of these waves are, then, assessed by visual inspection, which is clearly a subjective method. Alternatively, Objective Response Detection (ORD) techniques have been investigated in order to overcome this limitation (see Melges *et al*. (2012a) for a review about frequency-domain ORD techniques). Such techniques are based on statistical tests and, therefore, allow inferring about the sensorial response occurrence with a previously established significance level (false positives). One of the univariate frequency-domain ORD techniques with most promising results in the somatosensory response detection is the Magnitude-Squared Coherence (MSC) (Melges *et al*., 2008; 2011). The univariate techniques are, in general, easier to apply for both clinical and surgical monitoring, since they allow a more compact acquisition apparatus, by recording only one EEG derivation.

In this work, a novel univariate frequency-domain ORD technique based on the Rice Distribution is proposed, its analytical critical values are calculated and the theoretical probability of detection (PD) curves are obtained, by simulation, for different M-number of EEG epochs (30, 60, 120 and 240) and different signal-to-noise ratios (–20 to 10 dB, with 1 dB steps). The simulated PD values are also obtained and compared with those simulated for MSC. Finally, the detection performances for both techniques are also compared for real EEG signals.

Rice Detector (RD)

Theoretical background: objective response detection technique

The ORD approach usually defines a statistical metrics (for instance let us name its independent variable θ) capable of distinguishing between the conditions of absence and presence of sensorial response. Based on the theoretical Probability Density Function (PDF) of the adopted metrics, it is possible to calculate θ_{crit}, the critical value for the no-stimulation condition. Once the critical value is known, H0 (the absence of response hypothesis) should be rejected if the estimated value of the metrics $\hat{\theta}$ exceeds θ_{crit}. The frequency-domain ORD techniques usually employ parameters from the Fourier Transformed EEG epochs.

Definition of a novel statistical metrics

Assuming the EEG signal in the no-stimulation condition, $x(t)$, to have a zero-mean Gaussian distribution with variance σ_t^2, then the real ($a = \mathrm{Re}(X(f))$) and imaginary ($b = \mathrm{Im}(X(f))$) parts of its Fourier transform $X(f)$ will also follow a zero-mean Gaussian distribution but with variance $\sigma^2 = L\sigma_t^2/2$, where L is the number of samples used in the $X(f)$ calculation. Hence, the square of both real and imaginary parts ($\mathrm{Re}^2(X(f))$ and $\mathrm{Im}^2(X(f))$) would be related to chi-squared distributions with 1 degree of freedom (χ_1^2) each. By summing these parcels $r^2 = a^2+b^2$ would, hence, lead to a chi-squared with 2 degrees of freedom (χ_2^2).

Hence, by calculating the square root of r^2, one can obtain the Euclidian distance between the ordinate pair ($\mathrm{Re}(X(f))$,$\mathrm{Im}(X(f))$) and the origin (0,0) of the Argand-Gauss plan. The PDF of a circular bivariate Gaussian random variable, such as $r = \sqrt{a^2+b^2}$, for any expected value of a and b, respectively, μ_a and μ_b (null for H0), can be given by the Rice Distribution (Rice, 1944; 1948):

$$p_r(y) = \frac{y}{\sigma^2}\exp\left(-\frac{y^2+v^2}{2\sigma^2}\right) I_0\left(\frac{yv}{\sigma^2}\right) \tag{1}$$

where y is the PDF parameter, $\nu = \sqrt{\mu_a^2+\mu_b^2}$, and I_0 corresponds to the Bessel modified function of the first type and with 0 order.

Null hypothesis and critical value

By assuming that there is no response to sensory stimulation, we can establish the null hypothesis H0 of “response absence”, that is $\nu = 0$ in (1). For this particular case that $I_0(0) = 1$, expression (1) could be simplified and the PDF is reduced to the Rayleigh distribution (White, 1975) as:

$$p_r(y)\big|_{H_0} = \frac{y}{\sigma^2}\exp\left(\frac{-y^2}{2\sigma^2}\right) \tag{2}$$

Figure 1, illustrates this PDF for different variance values.

For practical purposes, the response to sensorial stimuli, such as auditory or somatosensory presents amplitude many times lower than the spontaneous EEG; hence, it is usual to apply a series of stimuli and windowing the EEG signal synchronized with stimulation to obtain a detector. In this case, it would be more convenient to calculate the mean Euclidian distance:

$$r_m = \frac{1}{M}\sqrt{\left(\sum_{i=1}^{M}\mathrm{Re}(X_i(f))\right)^2 + \left(\sum_{i=1}^{M}\mathrm{Im}(X_i(f))\right)^2} \tag{3}$$

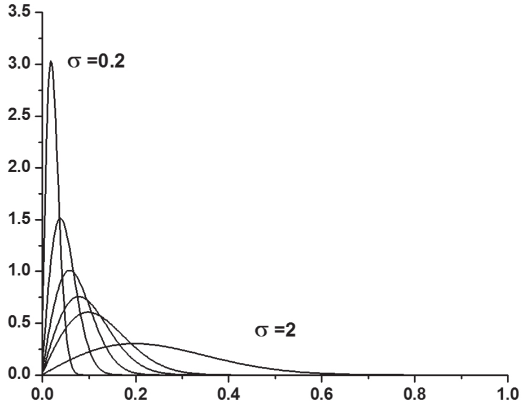

Figure 1. Probability Density Functions for the Rayleigh distribution (Rice distribution for ν = 0) with σ = 0.2, 0.4, 0.6, 0.8, 1.0 and 2.0.

where $X_i(f)$ is the Fourier Transform of the i-th EEG epoch (window). By using the known results for mean, variance and cumulative density function (CDF) of the Rayleigh distribution, one can calculate these parameters for the new metrics (White, 1975) as:

$$\mu_{r_m} = \sqrt{\frac{\pi\sigma^2}{2M}} \tag{4}$$

$$\sigma^2_{r_m} = \frac{4-\pi}{2M}\sigma^2 \tag{5}$$

$$cdf(r_m) = 1 - \exp\left(-M\frac{r_m^2}{2\sigma^2}\right) \tag{6}$$

It is possible to calculate the critical values for r_m by equaling expression (6) to the detection probability (1-α):

$$1-\alpha = 1 - \exp\left(-M\frac{r^2_{m_{crit}}}{2\sigma^2}\right) \tag{7}$$

where α is the significance level of the statistical test (maximum false positive rate), and M is the number of EEG epochs.

Rearranging expression (7) leads to:

$$r_{m_{crit}} = \sqrt{\frac{2\sigma^2 \ln(1/\alpha)}{M}} \tag{8}$$

Expression (8) highlights that the critical value for r_m depends on the signal variance. For this reason, in order to obtain a detector that is independent of this value, which is, a *priori* unknown, the statistics metrics can be redefined as:

$$\hat{\zeta}(f) = r_m/\sigma \tag{9}$$

and it will be named Rice Detector (RD) in this work. Thus, its critical value is a function only of the significance level (α) and number of epochs (M):

$$\hat{\zeta}_{crit} = \frac{r_{m_{crit}}}{\sigma} = \sqrt{\frac{2\ln(1/\alpha)}{M}} \tag{10}$$

The detection is, therefore, assumed based on the null hypothesis (H0) rejection, when the estimate values exceed the critical value $\left(\hat{\zeta}(f) > \hat{\zeta}_{crit}\right)$.

Power of the Statistical Test (PST)

When the response to the sensorial stimulation is assumed, that is, ν ≠ 0 in expression (1), one can calculate the detection probability (or the power of the statistical test – PST) for pre-defined values of significance (α), number of epochs (M) and signal-to-noise ratio (SNR) values. The PST, which consists of the probability of rejecting H0 when there is response to stimulation, can be calculated from the Rice CDF (Rice, 1944):

$$cdf(r) = 1 - Q\left(\frac{\nu}{\sigma}, \frac{y}{\sigma}, 1\right) \tag{11}$$

where Q corresponds to the Marcum Q function (Shnidman, 1989). Denoting $Q\left(\frac{\nu}{\sigma}, \frac{y}{\sigma}, 1\right) = Q_1\left(\frac{\nu}{\sigma}, \frac{y}{\sigma}\right)$, and using M epochs, the power of the test (P) can be expressed by:

$$P = 1 - cdf(r) = Q_1\left(\frac{\nu}{\sigma/\sqrt{M}}, \frac{r_{m_{crit}}}{\sigma/\sqrt{M}}\right)$$

$$P = Q_1\left(\frac{\nu}{\sigma/\sqrt{M}}, \frac{\sigma\sqrt{\frac{2\ln(1/\alpha)}{M}}}{\sigma/\sqrt{M}}\right) \tag{12}$$

$$P = Q_1\left(\frac{\nu\sqrt{M}}{\sigma}, \sqrt{2\ln(1/\alpha)}\right)$$

Since the signal-to-noise ratio is given by the ratio between the square of $\nu = \sqrt{\mu_a^2 + \mu_b^2}$ and the noise variance $\left(SNR = \nu^2/\sigma^2\right)$, the expression (12) can be simplified to:

$$P = Q_1\left(\sqrt{M\ SNR}, \sqrt{2\ln(1/\alpha)}\right) \tag{13}$$

From expression (13), it is possible to calculate the detection probability for different values of M, significance α and SNR.

The Magnitude-Squared Coherence (MSC)

The MSC is a frequency-domain ORD technique that can be interpreted as the power of the collected EEG that is due to the stimulation. For the case in which the stimulation is periodic and the EEG signal is discrete-time, finite-duration and windowed, the MSC can be estimated as (Melges *et al.*, 2008):

$$\hat{k}^2(f) = \frac{\left|\sum_{i=1}^{M} X_i(f)\right|^2}{M\sum_{i=1}^{M}\left|X_i(f)\right|^2} \tag{14}$$

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where "^" denotes estimation, $X_i(f)$ is the Fourier Transform of the i-th EEG epoch and M is the number of epochs employed in the calculation.

The analytical critical values for the MSC can be obtained as described in (Dobie and Wilson, 1993; Miranda de Sá and Infantosi, 2007):

$$\hat{\kappa}^2_{crit} = 1 - \alpha^{\frac{1}{M-1}} \tag{15}$$

where α is the significance level.

Similarly, the detection is reached with the null hypothesis rejection, for estimate values exceeding the critical ones $\left(\hat{\kappa}^2(f) > \hat{\kappa}^2_{crit}\right)$.

Methods

Simulated signals

The theoretical critical values for the Rice Detector (RD) were calculated according to the expression (10), by using different values of epochs M (30, 60, 120 e 240) and significance α (from 0.01 to 0.05, with steps of 0.01). Such critical values constitute the detection thresholds for response identification.

Then, the signals were simulated for the same values of M epochs (30, 60, 120 e 240) and 31 signal-to-noise ratio values (SNR varying from –20 to 10 dB, with steps of 1 dB). For each combination M-SNR, 10000 complex numbers were generated (mimicking the Fourier Transformed EEG epochs $X_i(f)$, as described in the Theoretical Background). These numbers were generated with the real and imaginary parts following zero-mean Gaussian distributions with unit variance. The mean Euclidian distance (r_m) of each one of 10000 values for each pair M-SNR was calculated (estimate of ν) and divided by the standard deviation estimates.

The estimated values of RD were compared with the critical values previously calculated for $\alpha = 0.05$ and 0.01. The detection is defined when the estimated value exceeds the respective critical value and the detection probability is obtained by the number of detections divided by the number of simulated signals (10000).

The detection rate values for the simulated signals were then compared with the theoretical detection probability values obtained with the expression (13).

Finally, the performance of the proposed technique was compared with the MSC by means of simulated curves of detection probability. The same set of signals was used for the evaluation of both techniques.

EEG acquisition and stimulation

EEG signals were collected during somatosensory stimulation from forty-five adult volunteers without history of neurological diseases. The signals were recorded according to the 10-20 International System at 600 Hz (16 bits resolution) using the BNT-36 EEG amplifier (EMSA, Brazil, www.emsamed.com.br). All derivations were reference to the earlobe average. The stimulation was applied to the right posterior tibial nerve at the frequency of 5 Hz and at the motor threshold intensity level. Further details about the experimental protocol and signal acquisition were described in (Melges *et al*., 2011). The local ethics committee (CEP-HUCFF/UFRJ) approved this research and all volunteers gave written informed consent to participate.

Pre-processing

The EEG signals were filtered (band-pass from 0.5 Hz to 100 Hz) and segmented into windows with one inter-stimulus duration, 207 ms, leading to a spectral resolution of 4.8 Hz. The samples corresponding to the first 5 ms post-stimulus were zeroed in order to avoid the stimulus artifact, which produces distortions in the frequency domain and interfere in the ORD analysis. Similar procedure was applied to the last 5 ms for window symmetry maintenance. Moreover, a Tukey window with rising (and falling) time of 7 ms was employed to minimize spectral leakage. Epochs with high amplitude artifacts were discarded by an algorithm described in Infantosi *et al*. (2006).

Comparison between ORD techniques

$\hat{\zeta}(f)$, $\hat{\zeta}_{crit}$, $\hat{\kappa}^2(f)$, $\hat{\kappa}^2_{crit}$ crit were calculated using expressions (9), (10), (14) and (15) respectively, with M = 240, 120, 60 and 30 epochs and α = 5%. Following, the number of volunteers for which the stimulation response was detected (detection percentage - DP) was determined for each frequency from 5 to 100 Hz and each technique. Moreover, the mean detection rate (MDR) within the low gamma band (30-55 Hz) was calculated for both techniques and 7 disjoint sets of M = 100 epochs in order to assess the detector behavior along the experiment. The DP and MDR were compared by means of the Proportion Test to verify whether there was significant difference between the MSC and RD performances.

Results

The critical values for the Rice Detector (RD) calculated with expression (10) are showed in Figure 2. As it can be seen, the values decrease with the increase of the number of epochs (M). The same is verified for the increase of the significance level (α).

Figure 3a shows that the theoretical and simulated detection probability values for RD for the significance level $\alpha = 0.05$, are almost overlapped. The same is verified for $\alpha = 0.01$ (Figure 3b).

By comparing the detection probabilities between RD and MSC for $\alpha = 0.05$ (Figure 4a) and $\alpha = 0.01$ (Figure 4b), a slight advantage for DR is noticed only for $M = 30$. The performance is similar for higher M-values.

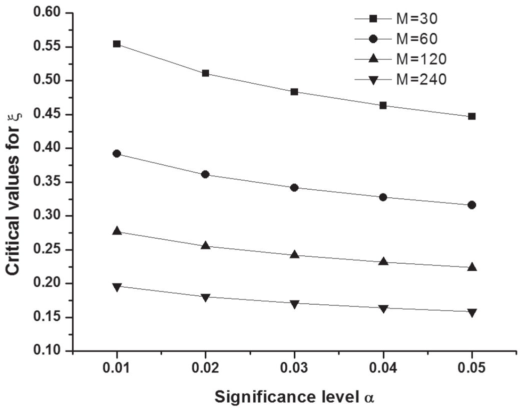

Figure 2. Critical values for ς for different values of M epochs and significance level a.

The detection percentages (DP) for both techniques calculated for the 5% significance level and $M = 240$ epochs are showed in Figure 5a. As it can be seen, both MSC and RD presented detection higher than 80% in the frequency band from 30 to 40 Hz. Hence, the gamma band is highlighted as the best for the somatosensory response identification. However, no significant difference was observed for any frequency.

When M is reduced to 120 epochs (Figure 5b), the detection rates vary from 69 to 87% for the same frequency band. Significant statistical differences are only noted at 10, 15 and 95 Hz, which present very low rates for both techniques. For the estimates calculated with $M = 60$ (Figure 5c), significant statistical differences are only found for 15, 25 and 30 Hz, frequencies for which the detection is also poor, confirming the performance similarity between the techniques. Using $M = 30$ epochs (Figure 5d), only RD for 50 Hz achieved percentage higher than 55%.

Figure 5a illustrates the time evolution of the mean detection rate (5 disjoint sets of $M = 120$ epochs)

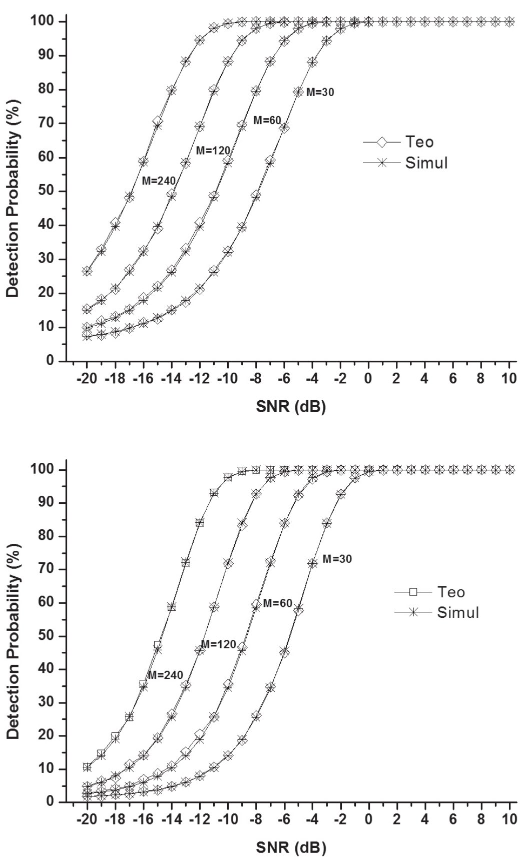

Figure 3. Comparison between the theoretical and simulated detection probability values obtained for a) $\alpha = 0.05$ and b) $\alpha = 0.01$.

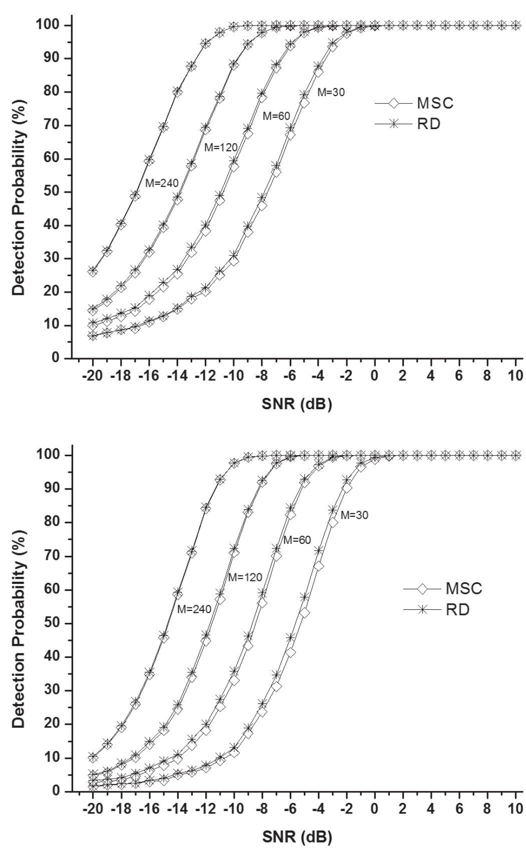

Figure 4. Comparison between the detection probabilities obtained by simulation for RD and MSC with significance level a) $\alpha = 0.05$ and b) $\alpha = 0.01$.

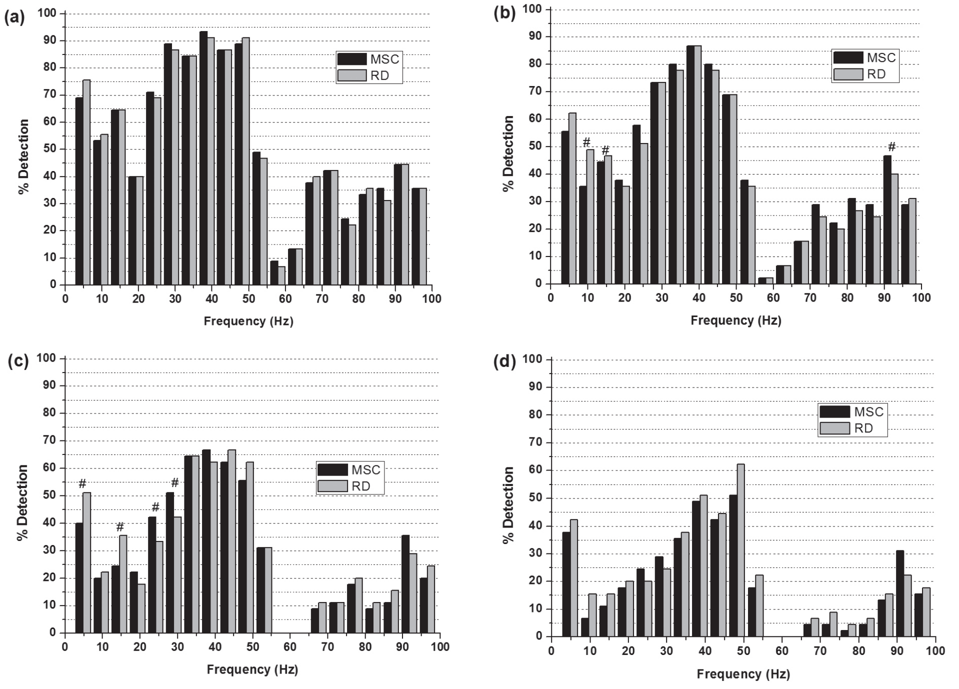

Figure 5. Detection Percentage for MSC and RD calculated with α = 5%. a) M = 240; b) M = 120; c) M = 60; d) 30 epochs. Statistical difference between the techniques indicated by (#).

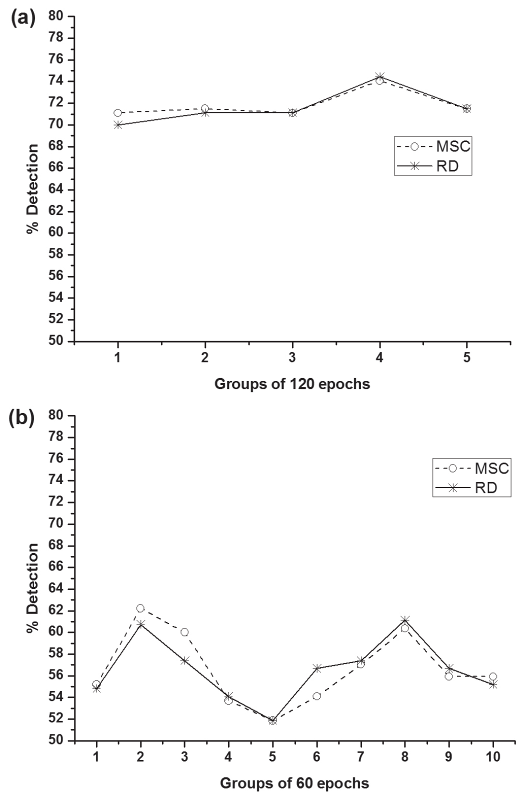

Figure 6. Time evolution of the mean detection rate (in the frequencies from 30 to 55 Hz) for MSC and RD. a) 5 disjoint sets of M = 120; b) 10 disjoint sets of M = 60. No statistical difference was found between the detection percentages.

in the frequency band from 30 to 55 Hz (28.99 to 53.14 Hz), where a similarity between the MSC and RD performances (varying from 70 to 74%) is noted. For 10 disjoint groups of 50 epochs (Figure 5b), the equivalence between the techniques is also evidenced. The proportion test did not show any statistically significant difference between the techniques.

Discussion

This work proposes a novel frequency-domain technique for objective response detection. This method is based on the Rice distribution, from which the theoretical critical values were derived. These values constitute detection thresholds for sensorial stimulus response identification.

Moreover, the detection probability curves (DPc – power of the test) were obtained analytically (theoretical values) and by simulation (simulated values) for the Rice Detector (RD) The comparison between the theoretical and simulated values showed overlapped probability of detection curves (PDc). The simulated PDc highlighted the performance equivalence for RD and MSC, with slight advantage of the proposed method for low values of M epochs.

Usually, the identification of response to the stimulation in the shortest time as possible, and, therefore, for the lowest M-value as possible, is desired. This requirement is particularly critical for intra-operative monitoring of spine or vascular surgeries in order to avoid early and late neurological damages caused by mechanical stress, hypotension or ischemia.

The application of the investigated techniques supposes that the stimulation response is identical for all stimuli and that it is uncorrelated with the spontaneous EEG. These conditions are not always achieved, leading to different detection rates for the same number of epochs, as observed for the mean detection rates along the experiment.

The results obtained for real signals agree with the simulated ones, in which a slight advantage for RD over the MSC was observed, particularly for low *M*-values. However, no statistical significant difference was found between the RD and MSC performances, indicating that these techniques are equivalent and both techniques could be employed for sensory response detection, with potential clinical and intra-operative application, during spine (Smith *et al*., 2007) and vascular (Astarci *et al*., 2007) surgeries. On the other hand, the simulated results showed that RD could perform better for higher SNR values, especially for low *M*-values. Thus, would be interesting to investigate and compare these techniques for other evoked responses such as visual evoked potential that usually presents higher SNR.

Moreover, the investigation of multivariate extensions of these techniques would allow obtaining detectors with higher performances for identification of visual (Felix *et al*., 2007) or somatosensory (Melges *et al*., 2012b) stimulation response. Finally, the frequencies with best detection rates were the same reported by Infantosi *et al*. (2006) and Melges *et al*. (2012b), which used uni and multivariate ORD.

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