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# Effect of Hydrodynamic Interaction on the Motion of a Rotating Body

*The hydrodynamic interaction between two bodies with rotational motion through an inviscid and incompressible fluid is investigated theoretically. The dynamical behavior of an elliptic cylinder moving around a fixed circular cylinder is described first based on the dynamical equations of motion in the plane of motion. In a relative coordinate system moving with the stream, the kinetic energy of the fluid is expressed as a function of fifteen generalized added masses due to the planar motion of the two cylinders. By means of the generalized added masses, the planar motion of an elliptic cylinder around a fixed circular cylinder can be computed without considering the flow field. The trajectories of an elliptic cylinder around a fixed circular cylinder in planar motion are obtained and the effects of non-circularity, initial position and initial velocity on the interaction between two cylinders are discussed. Similarly, the planar motion of a prolate spheroid around a sphere is investigated. The numerical results show explicitly that the dynamical behaviors of the moving bodies with rotational motion appear nonlinear. Their moving properties exhibit significant difference from those in the particle dynamics.*

**Keywords:** Interaction hydrodynamics; two-body system; rotating body; potential flow; Lagrange equations of motion; added masses

## Introduction

Hydrodynamic interactions between two floating bodies, or between a floating body and a fixed body, have a variety of applications in the offshore and polar engineering. For example, the presence of an offshore structure affects the motion of surrounding ice floes. On the other hand, the motion of an ice floe induces a hydrodynamic interaction force on the offshore structure before an actual impact takes place. In many practical cases, a common feature of the flow is that the Reynolds number based on the characteristic size of the offshore structure is usually large, i.e., the inertia effect becomes predominant in comparison with the viscous effect. Potential theory may then be employed to calculate the interaction forces and to predict the trajectories of the moving body. As the ice floes move on the sea surface, the motion may be regarded as planar.

The relative motion between two circular cylinders moving in any manner in an unbounded fluid was first investigated by Hicks (1879). He studied the velocity potential due to the distribution of unit sources spread over each surface, which was first assumed to be stationary, and then found the velocity potential due to the relative motion of two circles by making the magnitude of each source proportional to the normal motion of the surfaces. However, his method may be too complicated for the study of two-dimensional motion involving a pair of arbitrarily shaped bodies. Müller (1929) studied the uniform flow past two stationary circular cylinders and developed an infinite series solution for the velocity potential by using the method of successive images. Dalton and Helfinstine (1971) considered the forces on more than two cylinders by using the method of successive images to express the complex potential in terms of an infinite series of doublets. Huang and Yong (1995) presented an approximate approach that transforms an analytical solution for uniform flow past two circular cylinders into a solution for uniform flow past two cylinders of arbitrary shape. This approximate approach is a combination of the analytical solution, which is obtained in bipolar cylindrical coordinates, and the numerical pseudo-conformal transformation.

Hydrodynamic interaction between a pair of three-dimensional bodies moving in a uniform flow has been investigated by a number

of researchers. Hicks (1880) and Herman (1887) first analyzed the kinetic energy of the fluid, due to the motion of two spheres along the line joining their centers, and obtained analytical solutions of added masses in terms of doublets interior to each body. Their expressions about the strengths and positions of the doublets were alternatively reduced to a set of recurrence formulas, which were suitable for computation, as shown by Lamb (1932) and Landweber in the book edited by Rouse (1976). However, no recurrence formulas for the derivatives of the added masses with respect to the separation distance between the centers were given by these investigators. Mitra (1944) and Shail (1962) also applied the method of successive potentials to the Dirichlet problem for two spheres in electrostatics and obtained a set of unknown coefficients involved in the series expansion of velocity potential by applying the Neumann-Liouville iteration process. This analysis was extended by Shail (1962) to the hydrodynamic-interaction problems for two spheres moving along the line of the centers. The motion of a solid body, influenced by hydrodynamic interaction, was investigated by Lamb (1932), who applied Lagrange's equations of motion in the generalized coordinates and related the fluid inertia to the equations of motion by means of the kinetic energy of the fluid. Landweber and Shahshahan (1992) studied the centroidal motion of two cylinders and two equal spheres. Their paper was focused on how to increase the accuracy of numerical solutions when two bodies are close to each other.

Kazi et al. (1998) conducted a series of experiments to investigate the hydrodynamic interaction between a fixed circular cylinder and several floating cylinders of different sizes and shapes approaching centrally toward it. Six cases were studied under the same flow conditions with one fixed cylinder and six floating cylinders of three geometric shapes and six sizes. Their results showed the presence of a repulsive interaction force between the floating and the fixed cylinders. However, the force on the fixed cylinder was somewhat below the predicted values. Reasons for the over-prediction of the theoretical model were attributed to the viscous effect. Wang and Wahab (1971) presented a solution of the two-dimensional potential problem associated with a semi-submerged twin-cylinder performing small vertical oscillations in a free surface. Results of the added-mass and damping coefficients were presented as functions of the oscillation frequency.

General oblique motion between two bodies is more complicated. Hicks (1880) used the method of successive images again and represented the added masses in terms of distributed and isolated dipoles. However, he was able to calculate only a few

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images owing to the complexity of the calculation. Herman (1887) and Basset (1887) investigated independently the same problem by the method of successive potentials. They took two sets of spherical polar coordinates at the centers of each sphere and obtained series expressions for the velocity potential due to the transverse motion of two spheres. The added masses were determined by Basset (1887) up to the twelfth inverse power of the distance between the centers of two spheres, although his iteration procedure can be continued to any desired power. Herman (1887) gave expressions of added masses up to the fifteenth inverse power of the center distance. However, he omitted a factor 2 in two of the three added-mass expressions. Although Herman (1887) stated that "by this method, it is possible, with no more recondite work than simple differentiation, to approximate as closely as we please to the value of the kinetic energy," the algebra involved in obtaining higher order terms than the fifteenth inverse power of the center distance is very tedious. Herman's paper is by no means an easy one to read. However, the expressions given by Basset and Herman for the added masses are not accurate enough when two spheres are very close to each other. Moreover, no analytical expressions for the derivatives of added masses with respect to the center distance due to the transverse motion of two spheres were given by Basset (1887) or Herman (1887), which are necessary in determining the moving trajectories of two spheres as well as the hydrodynamic-interaction forces acting on them.

Yamamoto (1976) derived an analytical expression for the flow around and the hydrodynamic forces on an arbitrary number of cylinders in arbitrary motion based on the potential flow theory. As a numerical example, he considered the relative motion of two circles and represented the complex potential field in terms of an infinite series of doublets by applying Milne-Thomson's (1968) circle theorem. He also derived a formula for forces on each circle based on the Blasius theorem. Thus, his solutions cannot be extended to three dimensions. Isaacson and Cheung (1988) gave a two-dimensional formulation for the flow field around an ice mass drifting in a current near a large offshore structure, using the potential flow theory. The motions of the ice mass and current interactions were represented by five unit potentials and the hydrodynamic forces were obtained by the Bernoulli equation. Isaacson and Cheung (1988) assumed that the unit potentials were invariant with time. Thus, the spatial derivatives of added masses were missing in their expressions for the forces.

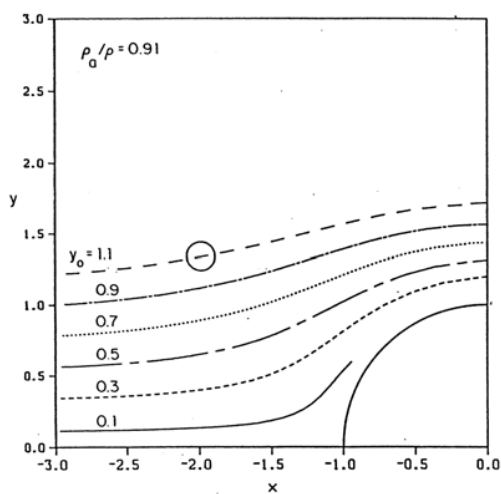


Figure 1. Trajectories of a moving cylinder around a fixed cylinder (Guo and Chwang, 1991).

Landweber et al. (1991) and Guo and Chwang (1991) studied the oblique impact of two cylinders in a uniform flow. Typical numerical results of Guo and Chwang (1991) are shown in Fig. 1, in which a circular cylinder (body 1) of radius  $a = 0.1$  and density  $\rho_a = 0.91$  moves around a fixed cylinder (body 2) of radius  $b = 1.0$ . The initial position of body 1 is at  $(x_0, y_0)$ , where  $x_0 = -20$ . At  $y_0 = 0.1, 0.3$ , etc., the trajectories of body 1 are plotted in Fig. 1 which corresponds to the case of an ice particle moving in fresh water. We note from this figure that for  $y_0 = 0.1$ , body 1 will eventually impact body 2.

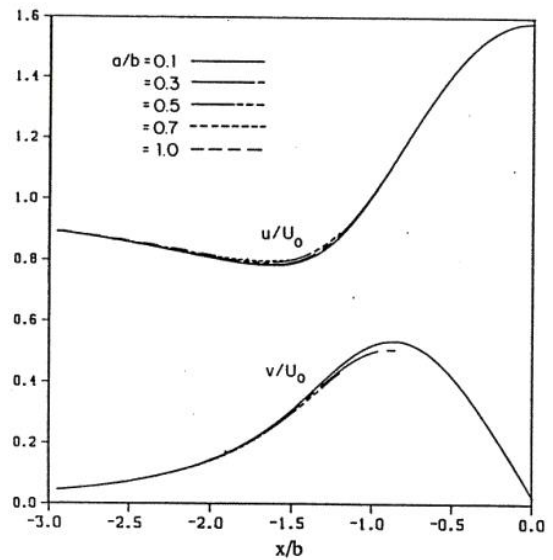


Figure 2. Velocity components  $u$  and  $v$  of a moving cylinder (Guo and Chwang, 1991).

For a variety of radius ratios  $a/b$  from 0.1 to 1.0 and a fixed density ratio 0.89, the velocity components  $u$  and  $v$  of the moving cylinder are shown in Fig. 2. The initial condition used in the calculation is  $x_0 = -20$ ,  $y_0 = 0.5$ ,  $u_0 = 1.0$  and  $v_0 = 0.0$ . This plot is consistent with the physical interpretation. The velocity  $u$  is reduced as the moving body approaches the fixed body, and after a certain position, it increases and reaches a maximum value around the top of the stationary cylinder. On the other hand, the velocity  $v$  increases from zero to a maximum value in approaching and decreases to zero after the cylinder passes over the fixed one. We also note from Fig. 2 that for  $x/b < -2.5$ , velocity components for cylinders of different radii are of the same value practically. It indicates that for two bodies apart from each other by a large distance, the hydrodynamic interaction does not have any effects on their motion.

In three dimensions, the hydrodynamic interaction between two spheres can be handled in the same manner (Guo and Chwang, 1992), although the numerical computation becomes quite complicated. Numerical results on the trajectories and velocity components of a moving sphere in the vicinity of another sphere are very similar to Fig. 1 and Fig. 2, respectively, for two cylinders. The hydrodynamic interaction between a three-dimensional body and an infinitely long cylinder was investigated by Guo and Chwang (1993). The hydrodynamic interaction of two vessels moving at the same speed in nearfield was considered by Yeung and Hwang (1977) applying the slender-body theory in potential flows. Theoretical predictions on sway force and yaw moment were generally high in comparison with available experimental

measurements. Fang and Kim (1986) studied the hydrodynamically coupled motions of two ships advancing in oblique waves. More references on challenging ship-ship interaction problems can be found in the ship hydrodynamics literature.

**Interaction with Rotation**

For the oblique motion of non-circular bodies in two dimensions or non-spherical bodies in three dimensions, the moment acting on each body is no longer zero. Therefore, the effect of rotation becomes important and the translational motion is coupled with the rotational one. Thus, the translational energy of a moving body can be transformed into the rotational energy and vice versa. Due to this coupling, the moving properties of these bodies have large differences from those in the particle dynamics. Sun and Chwang (1999) investigated analytically the general planar motion of an elliptic cylinder through an inviscid and incompressible fluid in the vicinity of a fixed circular cylinder, which is much larger than the elliptic cylinder. The velocity potential was derived by Sun and Chwang (1999) using the successive image method and the perturbation method. For an arbitrarily sized elliptic cylinder, Sun and Chwang (2001b) introduced a set of transformations of harmonics to derive the complete complex potentials for the elliptic and circular cylinder system by using the successive potential procedure, which is an extension of Milne-Thomson's (1968) circle theorem in two dimensions. In a relative coordinate system moving with the uniform stream, the kinetic energy of the fluid was expressed as a function of 15 generalized added masses due to the planar motion of the two cylinders. By means of the generalized added masses, the hydrodynamic interaction between an elliptic cylinder and a circular one in an ideal flow was studied.

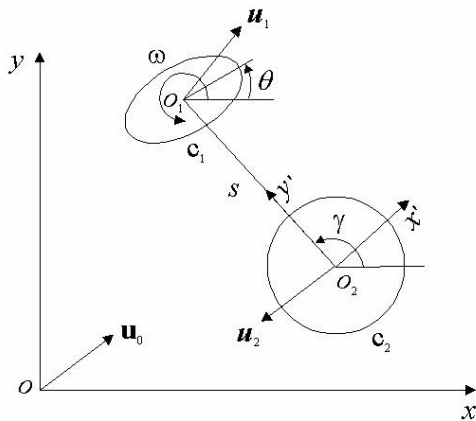


Figure 3. A moving elliptic cylinder in a uniform flow around a fixed circular cylinder (Sun and Chwang, 1999).

As shown in Fig. 3, relative to a moving frame of reference  $(x, y)$  in which the fluid is at rest at infinity, the elliptic cylinder with center  $O_1$  located at  $(x_1, y_1)$  moves with a translational velocity  $\mathbf{u}_1$ , whose components are  $u_{11}$  and  $u_{21}$  in the  $x$  and  $y$  directions respectively, and an angular velocity  $\omega = u_{31}$ , while the circular cylinder with center  $O_2$  located at  $(x_2, y_2)$  moves with velocity  $\mathbf{u}_2$  ( $\mathbf{u}_2 = -\mathbf{u}_0$ ) whose components are  $u_{12}$  and  $u_{22}$  in the  $x$  and  $y$  directions respectively, where  $\mathbf{u}_0$  is the uniform stream velocity in the absolute frame of reference fixed in space. The elliptic cylinder, whose semi-major and semi-minor

axes are  $a$  and  $b$  respectively, is inclined at an angle  $\theta$  with respect to the  $x$  axis in the counter-clockwise direction. The separation distance between the cylinder centers  $O_1O_2$  is denoted by  $s$ ,

$$s^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2. \tag{1}$$

If there is no external force acting on the elliptic cylinder apart from the fluid pressure, the Lagrange equations of motion for the elliptic cylinder are

$$M_1 \frac{du_{11}}{dt} = -\frac{d}{dt} \frac{\partial T}{\partial u_{11}} + \frac{\partial T}{\partial x_1}, \tag{2}$$

$$M_1 \frac{du_{21}}{dt} = -\frac{d}{dt} \frac{\partial T}{\partial u_{21}} + \frac{\partial T}{\partial y_1}, \tag{3}$$

$$J_1 \frac{d\omega}{dt} = -\frac{d}{dt} \frac{\partial T}{\partial \omega} + \frac{\partial T}{\partial \theta}, \tag{4}$$

where  $M_1$  is the mass of body 1 (the elliptic cylinder),  $J_1$  the moment of inertia per unit length of body 1 and  $T$  the kinetic energy of the fluid given by

$$T = -\frac{\rho}{2} \int \phi \frac{\partial \phi}{\partial n_\beta} dS_\beta = \frac{1}{2} A_{i\alpha j\beta} u_{i\alpha} u_{j\beta}, \quad (i, j = 1, 2, 3; \alpha, \beta = 1, 2). \tag{5}$$

In equation (5),  $\rho$  denotes the fluid density,  $S_\beta$  represents the surface of cylinder  $\beta$  and

$$A_{i\alpha j\beta} = -\rho \int \phi_{i\alpha} \phi_{j\beta} dS_\beta \quad (\text{no sum on } \beta) \tag{6}$$

is the generalized added mass tensor representing 36 added masses. As shown by Landweber & Chwang (1989),

$$A_{i\alpha j\beta} = A_{j\beta i\alpha}. \tag{7}$$

Hence, the total number of independent added masses is reduced to 21. However, for the present problem in which  $u_{32} = 0$ , the total number of independent added masses is further reduced to 15. Detailed expressions of equations (2) to (4) in terms of added masses were given by Sun and Chwang (1999).

Numerical results were given by Sun and Chwang (2001b) for an elliptic cylinder moving around a fixed circular cylinder of radius  $r_0$  in a uniform stream with velocity  $\mathbf{u}_0 = (1, 0)$ . In the calculation, the origin of the absolute coordinate system is located at the center of the fixed circular cylinder  $O_2$ . The absolute velocity components of the elliptic cylinder are  $u$  and  $v$  in the  $x$  and  $y$  directions respectively,

$$u = u_{11} + 1, \quad v = u_{21}. \tag{8}$$

To obtain the trajectories of the moving elliptic cylinder, the computation was started at the initial position  $x_1 = -20r_0$ ,  $y_1 = y_{10}$  and  $\theta = \theta_0$ , and the initial velocity  $u = u_0$ ,  $v = 0$  and  $\omega = \omega_0$ .

### A Purely Drifting Elliptic Cylinder

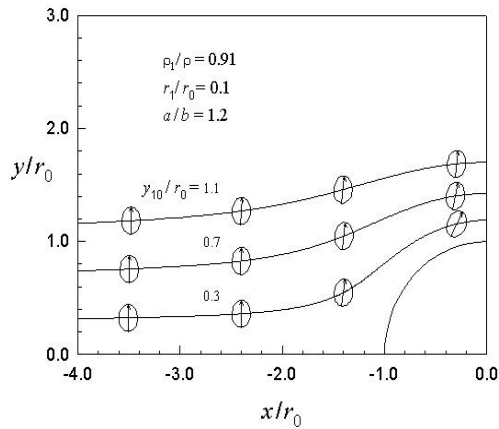


Figure 4. Planar motion of a purely drifting elliptic cylinder around a fixed circular cylinder with  $\theta_0 = 90^\circ$  (Sun and Chwang, 2001b).

For a purely drifting elliptic cylinder,  $u_0 = 1$  and  $\omega_0 = 0$ . Fig. 4 shows the trajectories and orientations of a purely drifting elliptic cylinder for a given aspect ratio and a density ratio with  $\theta_0 = 90^\circ$ . We note from Fig. 4 that rotation of the elliptic cylinder is induced by the presence of the fixed circular cylinder due to the relative translation and the non-uniformity of the flow field. As expected, when the elliptic cylinder approaches the circular cylinder, its orientation angle changes rapidly. Fig. 5 shows the variation of the orientation angle of an elliptic cylinder in pure drifting for three different aspect ratios  $a/b = 1.2, 2.0,$  and  $4.0$ . We note from this figure that the aspect ratio of a body affects strongly its rotation during drifting, the larger the aspect ratio, the greater the variation of the orientation angle. This is because a larger torque acts on the elliptic cylinder of larger aspect ratio. Sun and Chwang (2001b) also found that the trajectories of the center of a purely drifting elliptic cylinder are insensitive to the aspect ratio. The reason is that there is no obvious relative motion of the elliptic cylinder to the flow in pure drifting. Moreover, it is found that there is no difference between trajectories for  $x/r_0 < -2.0$ . It means that the hydrodynamic interaction becomes important only as the dimensionless separation distance  $s/r_0$  is less than 2.0.

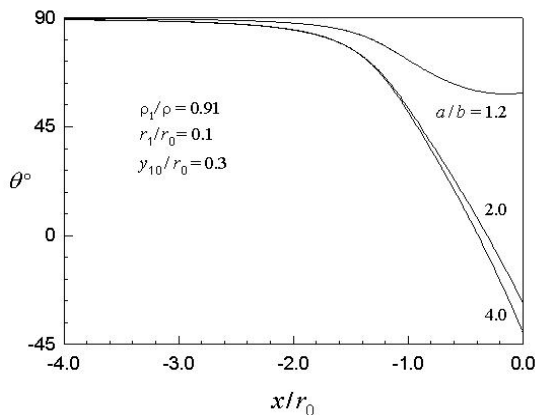


Figure 5. Orientation angle  $\theta$  of a purely drifting elliptic cylinder versus  $x/r_0$  with  $\theta_0 = 90^\circ$  (Sun and Chwang, 2001b).

Figures 6 and 7 show two rotational patterns of an elliptic cylinder around a circular cylinder with  $\theta_0 = 0^\circ$  and  $\theta_0 = 90^\circ$ , respectively. It is interesting to note from the figures that the rotation with  $\theta_0 = 0^\circ$  is in the counter-clockwise direction opposite to that with  $\theta_0 = 90^\circ$ . These mean that the initial orientation angle of an elliptic cylinder in pure drifting determines the development of its rotation. The figures also reveal that for  $x/r_0 < -2.0$ , the orientation angles change very little. It agrees with Guo and Chwang's (1991) result that when the two bodies are far apart, they have little effect on each other's motion.

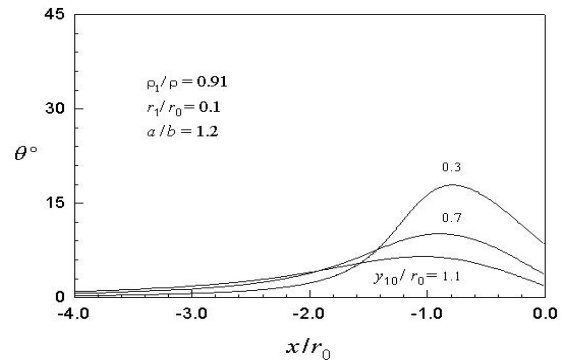


Figure 6. Orientation angle  $\theta$  of a drifting elliptic cylinder versus  $x/r_0$  with  $u_0 = 1, \omega_0 = 0$  and  $\theta_0 = 0^\circ$  (Sun and Chwang, 2001b).

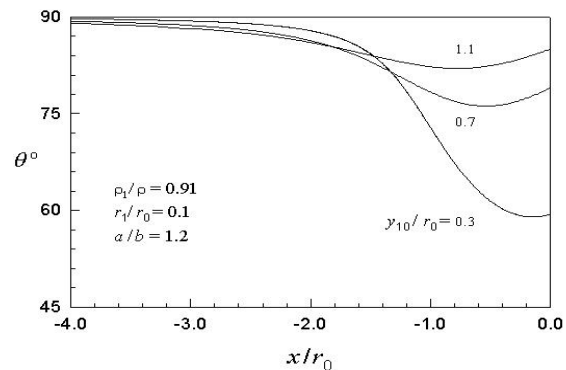


Figure 7. Orientation angle  $\theta$  of a drifting elliptic cylinder versus  $x/r_0$  with  $u_0 = 1, \omega_0 = 0$  and  $\theta_0 = 90^\circ$  (Sun and Chwang, 2001b).

### A Moving Elliptic Cylinder

As stated by Sun and Chwang (2001b), when the elliptic cylinder has an initial translational velocity and an initial rotational velocity, its subsequent motion demonstrates much more complicated features and some interesting phenomena when compared with those for pure drifting. Figures 8 and 9 show the trajectories of the center of an elliptic cylinder around a fixed circular cylinder and the orientation angle  $\theta$  versus the horizontal distance  $x/r_0$  with  $u_0 = 2, \omega_0 = -1$  and  $\theta_0 = 90^\circ$ . It is noted from these two figures that when the elliptic cylinder is far away from the fixed one, its motion exhibits an oscillatory behavior.

It implies that the translation and rotation of a moving elliptic cylinder are oscillatory in space. With its translational energy being transformed into rotational one and vice versa, its translational motion is coupled with the rotational one. It is also shown in these figures that with a larger aspect ratio, the period becomes shorter and the amplitude of the variation of the orientation angle  $\theta$  becomes smaller. The periodicity changes as the elliptic cylinder is close to the circular cylinder. It is understood that close to a second body, the moving pattern of the body will change. Furthermore, it is noted from Fig. 8 that the trajectory of an elliptic cylinder curves down when it approaches the  $x=0$  plane. This plot is consistent with the physical interpretation. When the elliptic cylinder moves over the circular cylinder, the interaction force due to this type of motion is attractive and draws the elliptic cylinder toward the circular one (Guo & Chwang, 1991).

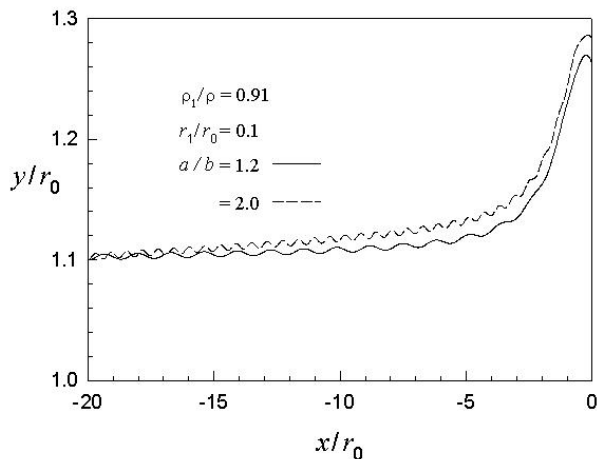


Figure 8. Trajectories of a moving elliptic cylinder around a fixed circular cylinder with  $u_0 = 2$ ,  $y_{10}/r_0 = 1.1$ ,  $\omega_0 = -1$  and  $\theta_0 = 90^\circ$  (Sun and Chwang, 2001b).

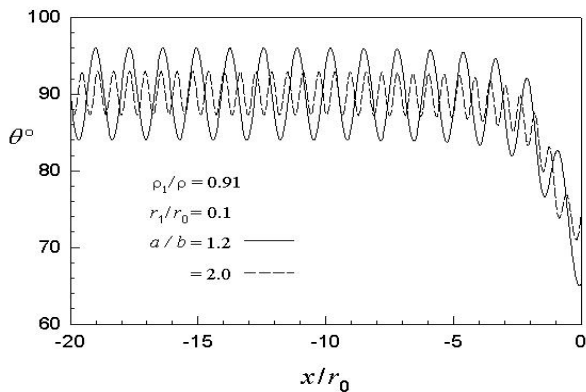


Figure 9. Orientation angle  $\theta$  of a moving elliptic cylinder versus  $x/r_0$  with  $u_0 = 2$ ,  $y_{10}/r_0 = 1.1$ ,  $\omega_0 = -1$  and  $\theta_0 = 90^\circ$  (Sun and Chwang, 2001b).

With the same initial conditions except for  $\omega_0 = 1$  and  $\theta_0 = 0^\circ$ , Figures 10 and 11 show another motion pattern. Note that the elliptic cylinder keeps rolling and oscillating around the circular cylinder. When the elliptic cylinder has a large aspect ratio, the amplitude of its oscillating motion becomes large; the centerline of its trajectory becomes curved off the fixed circular cylinder. As a result, it is easily running away from the fixed cylinder without

collision. On the other hand, it is observed from these two figures that the trajectories appear to be saw-pattern in shape. It verifies that the translation and rotation of a moving elliptic cylinder are periodic in space. This phenomenon occurs as a combined effect of three factors: the non-circular body shape, the inertia force and the hydrodynamic interaction between two bodies.

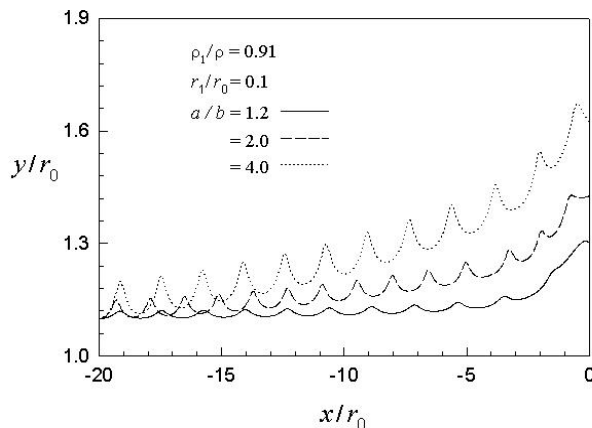


Figure 10. Trajectories of a moving elliptic cylinder around a fixed circular cylinder with  $u_0 = 2$ ,  $y_{10}/r_0 = 1.1$ ,  $\omega_0 = 1$  and  $\theta_0 = 0^\circ$  (Sun and Chwang, 2001b).

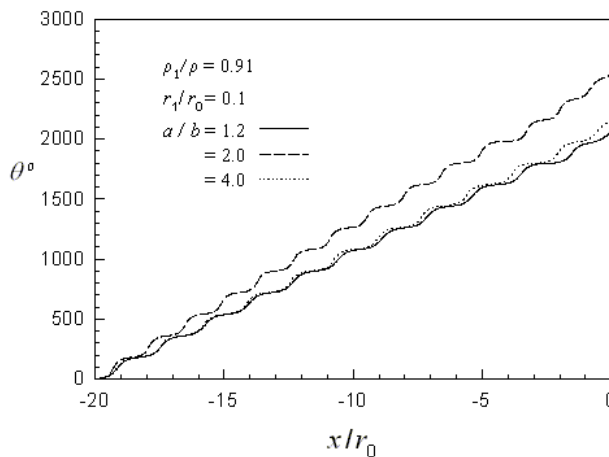


Figure 11. Orientation angle  $\theta$  of a moving elliptic cylinder versus  $x/r_0$  with  $u_0 = 2$ ,  $y_{10}/r_0 = 1.1$ ,  $\omega_0 = 1$  and  $\theta_0 = 0^\circ$  (Sun and Chwang, 2001b).

Hydrodynamic interaction between three-dimensional bodies with rotation is very complicated. Sun and Chwang (2001a) investigated the planar motion of a slightly distorted sphere around a fixed sphere in an unbounded fluid by a perturbation approach. An approximate velocity potential was derived in terms of sets of singularities by using the successive potential method. Approximate analytical solutions of added masses in series form were obtained and applied to determine the trajectories of the slightly distorted sphere around a fixed sphere. The hydrodynamic interaction between two bodies was computed based on the Lagrange equations of motion. It was found by Sun and Chwang (2001a) that the presence of a sphere generates an effect on the planar motion of the slightly distorted sphere and the initial configuration of the slightly distorted sphere has a decisive influence on the development of its subsequent rotational motion.

Yang and Luh (1998) used the Lagrange equations of motion to study the unsteady ground effect on the motion of a body. They conducted numerical simulations for a prolate spheroid moving in the fore-and-aft direction at constant speed past a flat ground with a protrusion. Recently, exact analytical expressions for the velocity potentials of a prolate spheroid translating and rotating around a fixed sphere were obtained by Sun (1999).

## Conclusions

The effect of hydrodynamic interaction on the motion of a rotating body has been briefly discussed in the present paper. The hydrodynamic interaction of two bodies without rotation in two dimensions or three dimensions was discussed first and followed by the general oblique motion between two bodies with rotation. Finally, the translational and rotational motion of an elliptic cylinder around a fixed circular cylinder and its three-dimensional counterpart were studied.

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