Sebastián Durango
sebastiandi@autonoma.edu.co
Universidad Autónoma de Manizales
Grupo de Investigación en Diseño Mecánico
Antigua Est. Ferrocarril, Manizales, Colombia

Gabriel Calle
gcalle@utp.edu.co
Universidad Tecnológica de Pereira
Vereda la Julita, Pereira, Colombia

Oscar Ruiz
oruiz@eafit.edu.co
EAFIT University
Laboratory of CAD CAM CAE
Carrera 49 N° 7 Sur – 50, Medellín, Colombia

Analytical Method for the Kinetostatic Analysis of the Second-Class RRR Assur Group Allowing for Friction in the Kinematic Pairs

The calculation of forces in the kinematic pairs of mechanisms by inverse dynamics is usually performed without friction considerations. In practice, when examination of articulated mechanisms takes into account friction, the solution of the inverse dynamics results in a complex procedure. If a modular approach for the inverse dynamics is used, then exact solutions are available, but not necessarily are practical. For example, the analytical solution for a second-class first-type Assur group is a 16th degree equation. Previous researches proposed an approximated but practical (graphical) method to calculate the forces on the kinematic pairs taking into account the friction forces. In this article, an analytical interpretation of the Artobolevski approximated method is developed for the second-class Assur group with three rotational pairs. The final results for the reactions calculated with the implemented method present a good approximation with respect to the graphical solution. Future work should consider friction forces not only in second-class groups with rotational joints, but also in second-class groups with prismatic joints and high-class Assur groups.
Keywords*: modular approach, friction, kinetostatics, Assur group*

Introduction

In the context of the dynamic analysis of the articulated mechanisms it is usual to neglect friction effects in the kinematic pairs. However, when calculation of performance, accurate dynamics or power consumption is required, friction in the kinematic pairs needs to be considered.

The accurate calculation of friction forces and moments (by inverse dynamics) in the kinematic pairs of articulated mechanisms results in a complex procedure (Baranov, 1979). The problem complexity is due to the non-linear character of the required models. A graphical iterative solution to the problem of inverse dynamics (kinetostatics) of planar mechanism taking into account friction was proposed by Artobolevski (1988). This solution is limited by its graphical nature.

This paper presents a practical alternative to the kinetostatic analysis of second-class mechanisms with rotational pairs allowing for friction in the kinematic pairs. The developed approach is based on the modular concept developed by Assur (1916) for the analysis of planar mechanisms by its structure. The approach corresponds to a chain-based general purpose program in terms of computer aided analysis of mechanical systems (Hansen, 1996). Major advantages of chain-based general purpose programs are flexibility and computer efficiency, being possible to analyze a great variety of mechanisms (Hansen, 1996). To develop a modular approach it is necessary to codify independent analytical solutions (modules) for a number of kinematic chains with a special condition (Assur groups). The analysis of a particular mechanism is performed assembling the modules corresponding to the driving mechanisms and Assur groups forming the mechanism.

A methodology to obtain a kinetostatic model allowing for kinematic pair friction for any structural group is proposed. The proposed methodology is applied to the kinetostatic modeling of the second-class first-type Assur group. A four-bar mechanism formed by a driving rotational mechanism and a second-class first-type Assur group is used as a case study of the modular kinetostatic analysis. Finally, conclusions are developed and future work is proposed.

Nomenclature

f'	= *non-dimensional friction coefficient of a rotational pair*
F	= *force magnitude, N*
J	= *inertial moment of mass, kg m*^{2}
l	= *link length, m*
m	= *mass of a lin, kg*
M	= *moment of a force, force pair, Nm*
P	= *power, W*
r	= *shaft radius of a rotational pair, m*
R	= *rotational kinematic pair*
$\mathbf{R}_{AB}$	= *relative position vector of point A with respect to B*
$sgn()$	= *sign function*
T_c	= *drive input torque of a driving mechanism, Nm*

Greek Symbols

γ , δ	= *angular parameters, rad*
η	= *mechanical efficiency of a mechanism, dimensionless*
ω	= *angular velocity of a link, rad/s*

Subscripts

$_f$	relative to friction component of a force or moment.
$_{h,i,j,k}$	relative to links h, i, j, k

Superscripts

^{n}	relative to the force component normal to a link
^{t}	relative to the force component parallel to a link

Paper accepted February, 2010. Technical Editor: Domingos Alves Rade

Literature Review

Computer Aided Analysis Methods for Mechanical Systems

Computer aided analysis methods for mechanical systems are divided in two categories by Nikravesh (1988):

1. Special Purpose (SP), and
2. General Purpose (GP)

General-purpose methods are codified as libraries without any specific mechanism, but including the necessary elements to virtually assembling a mechanism. GP libraries could be developed over the kinematic joint concept or over the kinematic unit concept. For planar mechanisms a kinematic unit corresponds to a structural group or Assur group.

For joint based GP programs the library contains a number of kinematic joints and bodies. Multi-body system (MBS) dynamics became the basis of joint based programs for dynamic analysis of planar and spatial mechanisms. A modern tendency in MBS dynamics is to develop modular methods and hierarchical simulations (Schiehlen, 1997; Eberhard and Schiehlen, 1998; Kubler and Schiehlen, 2000). For planar linkages (Pennestri, Valenti and Vita, 2007) present a MBS dynamic simulator that allows (Dahl) friction in the kinematic pairs. Dynamics of a planar rigid-link mechanism with rotating slider joint and simplified friction calculations are presented in Stoenescu and Marghitu (2003, 2004). In Larochelle and McCarthy (1992), a specific case of static analysis of spherical closed chains allowing for joint friction calculations was developed. The estimation of friction is developed using a successive approximation method; however, with respect to the approach presented here the methodology requires to formulate a new set of equations for each new mechanism structure to be analyzed.

GP Programs Based on Kinematic Units

Libraries developed using the kinematic unit GP approach contain a number of kinematic chains with a special characteristic such that they can be assembled in a way to form a mechanism. In the case of planar chains, such a characteristic corresponds with the definition of Assur group. The most important feature of this kind of programs is that it comprises the advantages of both SP programs and joint based GP programs; Hansen (1996) lists them as: computational efficiency and flexibility.

Kinematics of mechanisms by a kinematic unit GP approach is reported by: Cavic, Kostic and Zlokolica (2007), who present an iterative method for the forward kinematics of high-class Assur groups by decomposing it into dyadic (second-class) forms; Zhang, Zou and Guo (2006), who develop a virtual searching method for the position analysis of higher-class Assur groups; Buśkiewicz (2006), who presents a general and optimal numerical method to calculate the kinematics of a mechanism when its structural classification is given; Mitsi, Bouzakis and Mansour (2004), Mitsi et al. (2003) and Mitsi (1999), who develop a position analysis in polynomial form of Assur groups of class 4 and 3 including some prismatic joints; Calle, Quintero and Díaz (2001), who research a modular approach for the kinematic analysis of mechanisms; and Han, Liao, and Liang (2000), who analyze the position of a eight-link Assur group using a vectorial technique.

GP programs based on kinematic units for kinetostatic analysis (inverse dynamics) of mechanisms were developed by Durango (2007), Marghitu and Crocker (2001), and Molian (1984) for second-class mechanisms, and Bràt and Lederer (1973) for second and third-class mechanisms. Friction in the kinematic pairs is neglected in those works.

Dynamics of mechanisms by a kinematic unit GP approach is reported by: Wang, Lin, and Lai (2008), who present a method for the dynamic analysis of planar mechanisms using Assur groups and a state space formulation of the dynamics; Hansen (1996), who develops a general method for the dynamic analysis of mechanisms using a modular approach based on the concept of neutral (Assur) and expansion modules. There are no kinematic pair friction considerations in these programs.

The present article proposes an analytical, but practical method to the inverse dynamics of Assur groups allowing for friction in the kinematic pairs. The analytical and modular form of the method allows it to develop a modular approach for the kinetostatic analysis of planar mechanisms.

Methodology

Methods of kinematic, inverse and forward dynamic analysis based on structural groups are general and modular. A structural group can be identified as a kinematic chain without degrees of freedom with respect to the links which forms pairs with it, and such that it is not possible to divide it in simpler chains with the same characteristic (Calle, Quintero and Díaz, 2001). The analysis of a structural group can be established independently and then codified as a computer function or module of a library. The variety of mechanisms that could be analyzed within a library depends on the number of modules that the library contains. However, most of the industrial mechanisms are formed by combinations of first-class (driving) mechanisms (Fig. 1) and second-class structural groups (Fig. 2).

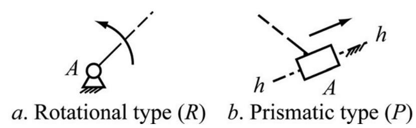

Figure 1. First-class (driving) mechanisms.

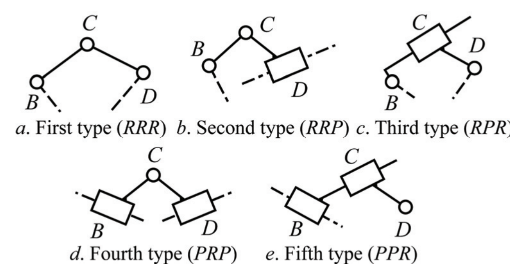

Figure 2. Second-class structural groups.

In inverse dynamics we assume that the laws of movement of the driving mechanisms are given. Therefore it is possible to stream the kinematics from group to group until the entire mechanism is solved. The solution is developed in the direction of the structural sequence of the mechanism using the required modules (Fig. 3). To complete the force analysis the last group in the structural sequence is isolated. The pair reactions are solved using the correspondent kinetostatics module. The solution comes from group to group in the opposite way of the structural sequence until the driving force or torque is solved (Fig. 3).

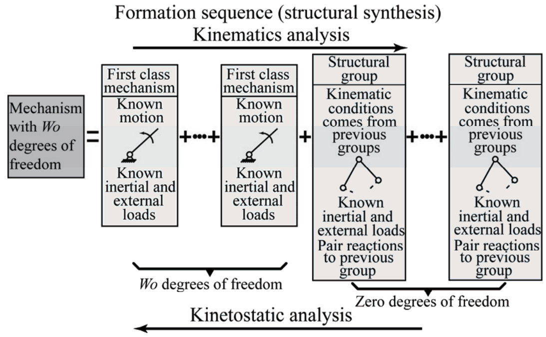

Figure 3. Kinematics and kinetostatic analysis of mechanisms with a modular approach.

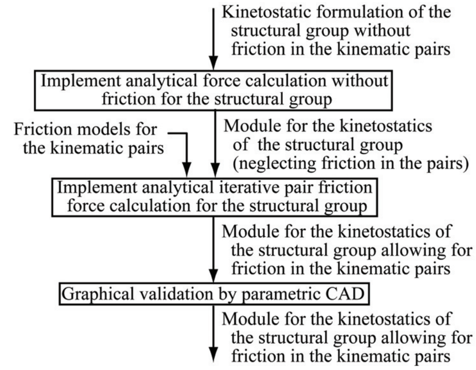

Figure 4. Kinetostatic modeling of structural groups allowing for friction in the kinematic pairs.

The analytical solution of the kinetostatics of structural groups without friction in the kinematic pairs results in a linear problem. However, when friction in the kinematic pairs is allowed the problem becomes non-linear and complex. In particular for a second-class first-type structural group, an exact analytical solution allowing for friction results in a 16-degree equation (Baranov, 1979). Therefore, a simpler estimation method is needed.

Artobolevski (1988) proposed a graphical approximated method for the inverse dynamics of structural groups allowing for friction. This method could be implemented with a parametric CAD program, but it is not practical for the analysis of complex mechanisms nor for multiple configurations of a mechanism. Baranov (1979) recommends not applying this method when the analyzed mechanism is near to self-locking. In such configurations, the influence of friction forces is major and the approximation could not be valid. In mechanisms self-locking is defined when the work of the driving forces is not enough to overcome the resistive forces different from the workloads, *e.g.* the friction forces actuating on the kinematic pairs. When a mechanism is near to self-locking its mechanical efficiency comes to zero, Eq. (1).

$$\eta \approx 0 \tag{1}$$

where the mechanical efficiency (η) is defined as the ratio of the work of the useful resistive loads (workloads) to the work of the driving forces. To analyze complex mechanisms and for multiple configuration analysis a computer aided analytical solution is desirable.

To obtain an iterative analytical solution of the kinetostatics of a structural group allowing for friction in the kinematic pairs that is suitable to be implemented as computer code, the methodology presented in Fig. 4 is proposed:

1. To obtain a kinetostatic (D' Alembert) formulation without friction forces and moments for the structural group. Analytical solutions of the pair reactions are formulated.
2. To establish a friction model for the kinematical pairs in the structural group. Several options are allowable for the pair friction models, from a simple Coulomb model to a Dahl model (Pennestri, Valenti and Vita, 2007).
3. To include the friction forces and moments in the kinetostatic formulation (as external loads).
4. To codify an independent computer function with successive approximation process for the pair force calculation of the structural group.
5. To develop a parametric computer aided drawing (CAD) validation for pair forces in the structural group. Calle, Quintero and Díaz (2002) proposed a parametric CAD method for the kinematics of planar mechanisms that is expandable to the kinetostatics.

As an example of the proposed methodology, the kinetostatic model allowing for friction of the second-class first-type structural group is developed in the next section. This methodology is expandable to other second-class or high-class Assur groups.

Kinetostatics of the Second-Class First-Type Assur Group

For the kinetostatic analysis of the second-class first-type Assur group neglecting for friction in the kinematic pairs are known parameters:

1. The geometry of the links,
2. The kinematics of the structural group, and
3. The inertial parameters of the links.

Figure 5 presents a dynamic equilibrium diagram of the second-class first-type structural group neglecting for friction in the kinematic pairs. Solving the dynamic equilibrium over the entire group by the x and y components:

$$F_{hi,x} + F_{i,x} + F_{j,x} + F_{kj,x} = 0 \tag{2}$$

$$F_{hi,y} + F_{i,y} + F_{j,y} + F_{kj,y} = 0 \tag{3}$$

where F_{pq} corresponds to the force of link p over link q, $pq = hi, kj$. F_i, F_j are the sum of the external and inertial forces over links i and j correspondently.

Taking force moments around C, for links i and j:

$$R_{BC,x} F_{hi,y} - R_{BC,y} F_{hi,x} + M_i + M_{Fi,C} = 0 \tag{4}$$

$$R_{DC,x} F_{kj,y} - R_{DC,y} F_{kj,x} + M_j + M_{Fj,C} = 0 \tag{5}$$

where M_p is the sum of the external and inertial moments over link p, and $M_{Fp,C}$ is the sum of the force moments of external and inertial forces actuating on link p with respect to C, $p = i, j$.

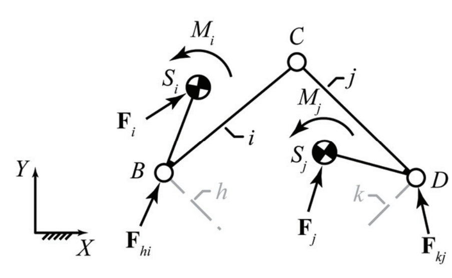

Figure 5. Second-class first-type Assur group, dynamic equilibrium diagram.

Equations (2)-(5) form a linearly independent system for the reactions $\mathbf{F}_{hi}$ and $\mathbf{F}_{kj}$ neglecting for friction in the kinematic pairs. To calculate the reaction on the *C* pair the sum of forces over link *i* or *j* can be solved. Taking the sum of forces over link *i* by the *x* and *y* components:

$$F_{hi,x} + F_{i,x} + F_{ji,x} = 0 \tag{6}$$

$$F_{hi,y} + F_{i,y} + F_{ji,y} = 0 \tag{7}$$

To estimate the kinematic pair reactions with friction effects it is necessary to calculate the friction moments in the kinematic pairs $M_{hi,f}$, $M_{ij,f}$, $M_{ji,f}$ and $M_{kj,f}$, Fig. 6. A model for friction in the kinematic pairs is required. A simplified Coulomb model is assumed, Eq. (8); however, a sophisticated model can be used, *e.g.* a smooth (Dahl) friction model is proposed by Pennestri, Valenti and Vita (2007). The assumed model for friction in rotational kinematic pairs is

$$M_f = Frf' \tag{8}$$

where r is the radius of cylindrical element on the pair, f' is the friction coefficient on the rotational pair and F is the load over the shaft. The friction moment is opposite with respect to the relative movement of the links forming the pair. The friction coefficient for a rotational pair is greater than the nominal friction coefficient between the materials forming the pair. An accepted estimation is to calculate it as 4/3 of the friction coefficient between the materials (Baranov, 1979). However, a more sophisticated estimation of the coefficient can include mechanical characteristics of the joint, *e.g.* parameters of the bearing forming the pair. With the assumed model (Eq. (8)) the initial estimation of the friction moments in the kinematic pairs of the second-class first-type structural group is as follows:

$$M_{hi,f} = -sgn(\omega_i - \omega_h) F_{hi} f'_B r_B \tag{9}$$

$$M_{ji,f} = -M_{ij,f} = -sgn(\omega_i - \omega_j) F_{ji} f'_C r_C \tag{10}$$

$$M_{kj,f} = -sgn(\omega_j - \omega_k) F_{kj} f'_D r_D \tag{11}$$

where ω_p is the angular velocity of the link p , f'_Q is the friction coefficient in the rotational pair Q, sgn() is for the sign function and r_Q is the radius of the cylindrical element forming the pair, $p = h,i,j,k$, $Q = B,C,D$. Including the friction moments in the kinetostatics formulation, Eqs. (4)-(5):

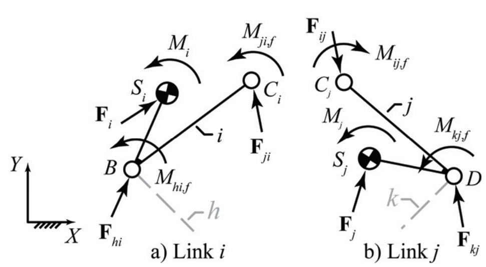

Figure 6. Second-class first-type Assur group, dynamic equilibrium diagrams with friction moments.

$$R_{BC,x} F_{hi,y} - R_{BC,y} F_{hi,x} + M_i + M_{Fi,C} + M_{hi,f} + M_{ji,f} = 0 \tag{12}$$

$$R_{DC,x} F_{kj,y} - R_{DC,y} F_{kj,x} + M_j + M_{Fj,C} + M_{ij,f} + M_{kj,f} = 0 \tag{13}$$

In Eqs. (12) and (13), the friction moments are functions of the magnitude of the radial forces on each rotational pair. Therefore, it is not possible to determine linearly the reactions $\mathbf{F}_{hi}$, $\mathbf{F}_{ji}$, and $\mathbf{F}_{kj}$ by means of Eqs. (2), (3), (6), (7), (12) and (13). A practical form to solve this problem is to use a method of successive approximations:

1. For the first approximation the moments of the friction forces are equal to zero, $M_{hi,f} = 0$, $M_{ji,f} = -M_{ij,f} = 0$ and $M_{kj,f} = 0$. The problem is reduced to the solution of the kinetostatics neglecting friction by the linear set of Eqs. (2)-(7).
2. Assuming a friction simplified model, Eq. (8), the initial estimation of the friction moments is calculated by Eqs. (9)-(11).
3. Using the estimated values of the friction moments calculated in 2, a new set of reactions $\mathbf{F}_{hi}$, $\mathbf{F}_{ji}$, $\mathbf{F}_{kj}$ are determined by the set of Eqs. (2), (3), (6), (7), (12) and (13).
4. To iterate the calculation process the new values of F_{hi} , F_{ji} , F_{kj} are used in step 2 for friction moments calculation. A new set of kinematic pair reactions is determinated in step 3.
5. The iteration process finishes when the difference of the magnitude of the reactions on consecutive calculations is small enough. The maximum difference can be defined depending on the units and magnitudes of the reactions calculated in the analysis. In this sense, the difference depends on the analyzed mechanism. As an example, the four-bar mechanism analyzed in forward section contains a second-class first-type Assur group. The magnitude of the reactions in the Assur group is of order $1 \cdot 10^3$ N. The reference value for the difference of the reactions between calculations was defined as $1 \cdot 10^{-3}$ N.

Figure 7 presents a diagram for the proposed model. The obtained model for the second-class first-type Assur group can be codified as an independent module or computer function. Such a function could be included in a GP chain based library for the analysis of planar mechanisms.

The method of modeling presented here for the kinetostatic analysis allowing for friction in the kinematic pairs of the second-class first-type Assur group is expandable to any other second-class or high-class Assur group. Next section presents the modeling of the rotational driving mechanism allowing for friction in the rotational pair.

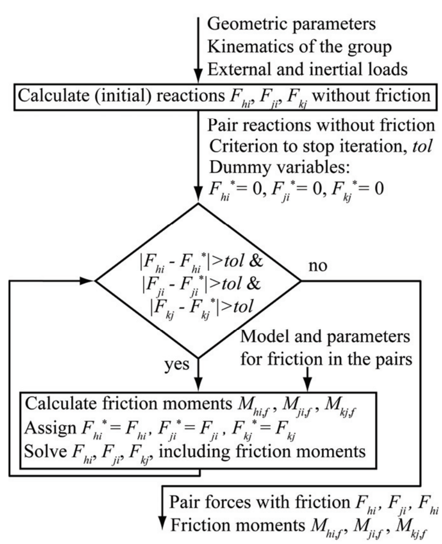

Figure 7. Kinetostatic model of a second-class first-type Assur group allowing for friction in the kinematic pairs.

Kinetostatics of the Rotational Primary Mechanism

Figure 8 presents the dynamic equilibrium diagram for a rotational driving mechanism with a torque input and friction moment in the rotational pair. The sum of forces for the dynamic equilibrium is:

$$\mathbf{F}_{0h} + \mathbf{F}_h = 0 \tag{14}$$

where $\mathbf{F}_h$ is the sum of the external and inertial forces over link *h* and $\mathbf{F}_{0h}$ is the reaction in the rotational pair. The effect of other links forming pairs with link *h* is included as a external force and a external moment.

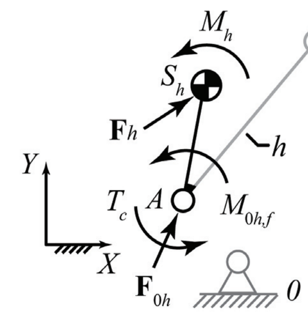

Figure 8. Rotational primary mechanism: dynamic equilibrium diagram allowing for friction in the rotational pair.

It is possible to solve the reaction $\mathbf{F}_{0h}$ using directly Eq. (14). Once the reaction is solved it is possible to determine the required input torque for the dynamic equilibrium. Calculating the sum of moments with respect to A (Fig. 8):

$$T_c + M_{0h,f} + M_1 + M_{Fh,A} = 0 \tag{15}$$

where T_c is the required input torque for the dynamic equilibrium, M_h is the sum of the external and inertial moments and $M_{Fh,A}$ is the sum of the force moments of the inertial and external forces with respect to *A*.

The friction moment $M_{0h,f}$ can be determined assuming a model for the friction in the rotational pair. Assuming a simplified friction model, Eq. (8), and including the sign function to determine the direction of the friction moment:

$$M_{0h,f} = -sgn(\omega_h - \omega_0) F_{0h} f'_A r_A \tag{16}$$

where ω_p is the angular velocity of link p, f'_A is the friction coefficient on the pair, r_A is the radius of the cylindrical element on the pair , and $p = 0, h$. The input torque (T_c) can be solved directly from Eq. (15).

Equations (14)-(16) can be codified as an independent module or computer function. Such a function could be included in a chain-based GP library for the analysis of planar mechanisms.

As an application and validation of the exposed method the inverse dynamics of a four-bar mechanism allowing for friction in the rotational kinematic pairs is developed in the next section.

Results

In this paper, a kinetostatic model of the second-class first-type Assur group is presented. The model allows friction in the kinematic pairs. The pair reactions are calculated using an iterative method avoiding non-linearities and complicated solutions. Although the obtained model is for the second-class first-type structural group, it is possible to extend the method of modeling (Fig. 4) to any second-class or high-class structural group.

A simplified friction model is assumed to estimate the friction moments in the rotational pairs (Eq. 8), however, other models are allowed, *e.g.* a Dahl model for friction in rotational pairs (Pennestri, Valenti and Vita, 2007).

Analytical models for the kinematics and kinetostatics of first-class mechanisms and second-class structural groups were codified as modules of a computer library. If a graphic user interface (GUI) is used to develop the modules, then each structural group or driving mechanism is represented by a block. The structural sequence of the mechanism can be established wiring properly the blocks, Fig. 9.

A computer aided graphical validation of the presented modules was developed using a parametric CAD software (SolidWorks). SolidWorks is a product of Dassault Systèmes SolidWorks Corp. A second-class mechanism including a first-class rotational mechanism and a second-class first-type structural group is used as a validation model.

Modular Kinetostatics of a Four-Bar Mechanism

Figure 10 presents a four-bar mechanism with parameters specified in Table 1. For the analysis, the mechanism is assumed to be in steady state. The input angular velocity is assumed to be constant $(\omega_1 = 10\text{rad/s})$. The structural classification of the four-bar mechanism is:

$$I^R_{0,1} \rightarrow II^{RRR}_{2,3} \tag{17}$$

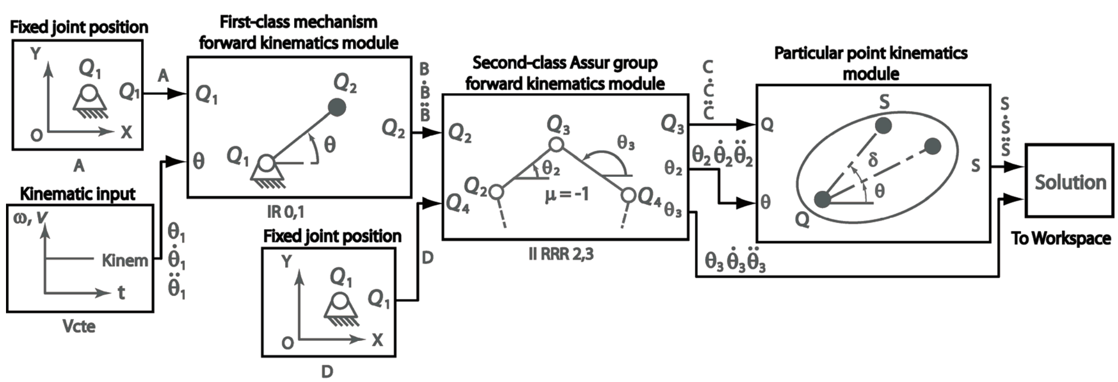

Figure 9. Kinematic modular analysis of a four-bar mechanism using a graphic user interface (GUI). Structural sequence: $I^R_{0,1} \rightarrow I^{RRR}_{2,3}$ **.**

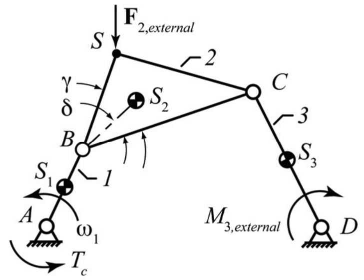

Figure 10. Four-bar mechanism.

Table 1. Four-bar mechanism: geometric and inertial parameters.

Parameter	Magnitude	Parameter	Magnitude	Parameter	Magnitude
l_{AB}	2.00 m	l_{DS3}	1.50 m	$F_{2,external}$	1,0 kN ↓
l_{BC}	6.00 m	m_1	4.8 kg	$M_{3,external}$	−250 Nm
l_{CD}	3.00 m	m_2	30.0 kg	J_1	1.70 kg m^2
l_{AD}	5.50 m	m_3	7.2 kg	J_2	48.80 kg m^2
l_{BS2}	2.80 m	δ	0.152 rad	J_3	5.50 kg m^2
l_{BS}	2.50 m	γ	$\pi/6$ rad	$f'^{(a)}$	0.40
l_{AS1}	1.00 m			$r^{(a)}$	7.50 mm

(a) Same radius and friction coefficient for all joints.

where I^R is for a driving mechanism with a rotational pair and II^{RRR} is for a second-class first-type Assur group.

We assume that the kinematics of the four-bar mechanism is given. The kinetostatic analysis is developed for the range of movement of the input link with evaluations each rad. The modular kinetostatic analysis allowing for friction in the kinematic pairs of the four-bar mechanism is as follows:

1. To solve the kinetostatics of the second-class structural group formed by links 2 and 3. The module for the kinetostatics of the second-class structural group corresponds with the codification of the methodology described in previous section.

2. To stream the solutions in step 1 to the first-class mechanism and solve the input drive torque and reactions. The module for the kinetostatics of the first-class mechanism corresponds with the codification of the solution of the set of Eqs. (14)-(16).

Figure 11 shows the kinetostatic analysis process of the four-bar mechanism.

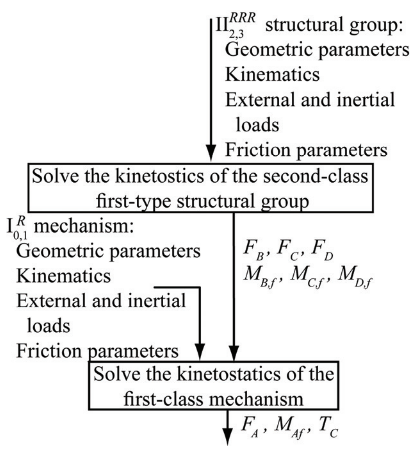

Figure 11. Kinetostatic analysis of a four-bar mechanism with a modular approach.

The results of the inverse dynamic analysis of the four-bar mechanism are presented in Figs. 12, 13 and 14. Three solutions are calculated:

1. Calculating pair friction effects using a modular approach and the successive approximation method proposed here.

2. Calculating pair friction effects using a joint-based MBS dynamic program (rough calculation of pair friction forces without iteration in SimMechanics). SimMechanics is a product of MathWorks Inc. Most of commercial MBS simulation softwares allow an approximate calculation of friction forces without iteration, e.g. classical (stiction) friction.

3. Neglecting friction in the kinematic pairs.

The following analysis outputs are presented: the required driving torque (Fig. 12), the required power to overcome the friction in the kinematic pairs (Fig. 13), and the reaction at pair *D* (Fig. 14). The criterion for breaking the iteration process was established as $1 \cdot 10^{(-3)}$ N for the difference on consecutive calculations of the magnitude of the pair reactions in the II^{RRR} structural group. The results solving the inverse dynamics problem by the method of successive approximation developed here and by using a MBS dynamic simulator are basically coincident. However, the solution presented here is straightforward with respect to the MBS dynamic simulation, because it does not require the solution of any differential equation of movement.

Computer Aided Graphical Validation

A computer aided graphical solution using a parametric CAD software (SolidWorks) was developed as a validation of the second-class first-type structural group module. Such a validation uses the successive approximations method by Artobolevski (1988) with three iterations. The validation was performed using the four-bar mechanism proposed in previous section. Ten equally-spaced configurations of the driving mechanism were evaluated for the reactions at pairs *B*, *C* and *D*. The maximum difference between the calculation of the pair reactions using the analytical method of successive approximations proposed here and the graphical method with 3 iterations was lower than 0.1%.

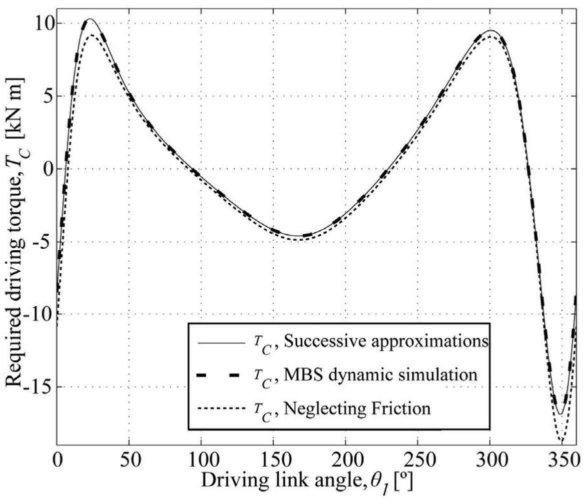

Figure 12. Kinetostatic analysis of a four-bar mechanism. Required driving torque.

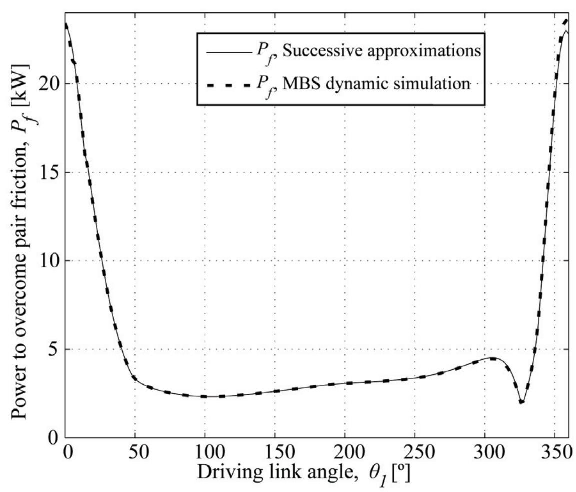

Figure 13. Kinetostatic analysis of a four-bar mechanism. Required power to overcome pair friction.

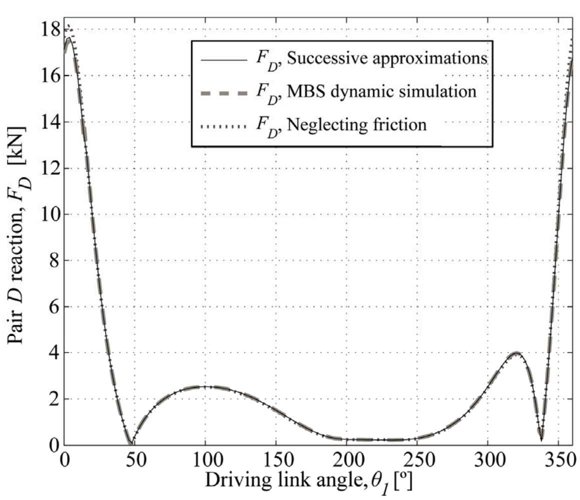

Figure 14. Kinetostatic analysis of a four-bar mechanism. Pair D reaction.

Known Limitations

The analytical method for the kinetostatic analysis of Assur groups allowing for friction in the Kinematic pairs presented here is not adequate in cases in which the mechanism is near to self-locking. Self-locking occurs when the work of the driving forces is not enough to overcome the resistive forces different from the workloads. In such configurations, the influence of friction forces is important and an approximated estimation comes to big calculation errors (Baranov, 1979).

The proposed analysis method does not consider singular configurations of the mechanism. The definition of singular configurations adopted here corresponds to the definition of (Gosselin and Angeles 1990). For serial singular configurations the analysis is not adequate if the mechanism is near to self-locking. For parallel and architectural singular configurations the proposed kinetostatic analysis method must be avoided.

Conclusion and Prospective

The performance of a mechanism during its periodic movement depends of the work needed to overcome the productive and non-productive resistances. Usually non productive resistances appears principally because of friction. In mechanisms the calculation of reactions with friction usually comes into a non-linear problem. Therefore, approximated methods of calculation as the ones developed here are practical and useful.

The analytical solutions for the kinetostatics of structural groups allowing for friction in the kinematic pairs presented in this paper correspond to the so called general purpose programs based on chains (modular approach). The modular approach allows to codify computer functions with emphasis placed on the users finding the input data structure easy to understand. Therefore, very complex planar linkages may be described using a simple and familiar form of the input data.

A methodology is proposed for the development of the kinetostatic analytical models of structural groups allowing friction in the kinematic pairs (Fig. 4). Such a methodology is based on an iterative, but practical solution. With respect to commercial software the proposed modular approach for the kinetostatic analysis of mechanisms has the following advantages:

1. The modules for mechanism analysis developed by this method preserve the characteristics of the GP programs based on kinematic chains: computational efficiency and flexibility. As consequence, with respect to commercial MBS simulation software the flexibility is comparable. In the case of MBS simulation software, the complexity of the mechanisms that can be analyzed depends on the number of joints that are coded. In the case of the modular approach, the complexity depends on the structural groups that are coded. However, most of industrial mechanisms are formed only by driving mechanisms and second-class structural groups.

2. With respect to commercial MBS simulation software the proposed methodology is straight-forward because it does not require the solution of any differential equation of movement.

Future work is proposed developing modules for the analysis of high-class structural groups, structural groups with prismatic joints, and inclusion of more elaborated friction models of the kinematic pairs.

Acknowledgements

The authors wish to acknowledge the financial support for this research by the Colombian Administrative Department of Science, Technology and Innovation (COLCIENCIAS), and the Colombian National Service of Learning (SENA), grant 1216-479-22001.

References

Artobolevski, I.I., 1988, "Theory of Mechanisms and Machines", Nauka, Moscu, 640 p. (In Russian).

Assur, L.V., 1916, "Investigation of plane hinged mechanisms with lower pairs from the point of view of their structure and classification (in Russian): Part I, II", Bull. Petrograd Polytech. Inst.

Baranov, G.G., 1979, "Curso de la Teoría de Mecanismos y Máquinas", 2 ed., MIR, 524 p.

Bràt, V. and Lederer, P., 1973, "KIDYAN: Computer-aided kinematic and dynamic analysis of planar mechanisms", *Mech. Mach. Theory*, Vol. 8, pp. 457-467.

Buśkiewicz, J., 2006, "A method of optimization of solving a kinematic problem with the use of structural analysis algorithm (SAM)", *Mech. Mach. Theory*, Vol. 41, pp. 823-837.

Calle, G., Quintero, H.F. and Díaz, A., 2001, "Análisis cinemático de mecanismos planos a partir del análisis estructural según Assur". V CIBIM Congreso Iberoamericano de Ingeniería Mecánica, Mérida, Venezuela.

Calle, G., Quintero, H.F. and Díaz, A., 2002, "Propuesta de enseñanza de la cinemática de mecanismos planos con la ayuda de programas CAD paramétricos", CREA'02 Innovación docente, La Habana, Cuba.

Cavic, M., Kostic, M. and Zlokolica, M., 2007, "Position analysis of the high class kinematic group mechanisms", 12th IFToMM World Congress, Besaçon, France.

Durango, S., 2007, "Dynamics of second-class planar mechanisms by means of a computer aided modular approach", ISPE 23rd International Conference on CAD/CAM Robotics and Factories of the Future, Bogota, pp. 421-428.

Eberhard, P. and Schiehlen, W., 1998, "Hierarchical modeling in multibody dynamics", *Archive of Applied Mechanics*, Vol. 68, pp. 237- 246.

Gosselin, C. and Angeles, J., 1990, "Singularity analysis of closed-loop kinematic chains", *IEEE Transactions on Robotics and Automation*, Vol. 6, No. 3, pp. 281-290.

Hansen, M.R., 1996, "A general method for analysis of planar mechanisms using a modular approach", *Mech. Mach. Theory*, Vol. 31, pp. 1155-1166.

Kubler, R. and Schiehlen, W., 2000, "Modular simulation in multibody system dynamics", *Multibody System Dynamics*, Vol. 4, pp. 107-127.

Han, L., Liao, Q. and Liang, C., 2000, "Closed-form displacement analysis for a nine-link Barranov truss or a eight-link Assur group", *Mech. Mach. Theory*, Vol. 35, pp. 379-390.

Larochelle, P. and McCarthy, J.M., 1992, "Static analysis of spherical nR chains with joint friction", ASME Flexible Mechanisms, Dynamics and Analysis, pp. 173-177.

Marghitu, D.B. and Crocker, M.J., 2001, "Analytical Elements of Mechanisms", Cambridge University Press.

Mitsi, S., 1999, "Position analysis in polynomial form of planar mechanisms with a closed chain of the Assur group of class 4", *Mech. Mach. Theory*, Vol. 34, pp. 1195-1209.

Molian, S., 1984, "Software for mechanism design", Proceedings of Mechanisms Conference, Granfield.

Nikravesh, P.E., 1988, "Computer Aided Analysis of Mechanical Systems", Prentice Hall, New Jersey.

Pennestri, E., Valenti, P.P. and Vita, L., 2007, "Multibody dynamics simulation of planar linkages with Dahl friction", *Multibody System Dynamics*, Vol. 17, pp. 321-347.

Mitsi, S., Bouzakis, K.-D. and Mansour G., 2004, "Position analysis in polynomial form of planar mechanism with an Assur group of class 4 including one prismatic joint", *Mech. Mach. Theory*, Vol. 39, pp. 237-245.

Mitsi, S., Bouzakis, K.-D., Mansour G. and Popescu, I., 2003, "Position analysis in polynomial form of planar mechanisms with Assur groups of class 3 including revolute and prismatic joints", *Mech. Mach. Theory*, Vol. 38, pp. 1325-1344.

Schiehlen, W., 1997, "Multibody system dynamics: roots and perspectives", *Multibody System Dynamics*, Vol. 1, pp. 149-188.

Stoenescu, E.D. and Marghitu, D.B., 2003, "Dynamic analysis of a planar rigid-link mechanism with rotating slider joint and clearance", *Journal of Sound and Vibration*, Vol. 266, No. 2, pp. 394-404.

Stoenescu, E.D. and Marghitu, D.B., 2004, "Effect of prismatic joint inertia on dynamics of kinematic chains". *Mech. Mach. Theory*, Vol. 39, No. 4, pp. 431-443.

Wang, H., Lin, Z. and Lai, X., 2008, "Composite modeling method in dynamics of planar mechanical system", *Science in China – Series E: Technological Sciences*, Vol. 51, No. 45, pp. 576-590.

Zhang, Q., Zou, H.J. and Guo, W.Z., 2006, "Position analysis of higher-class Assur groups by virtual variable searching and its application in a multifunction domestic sewing machine", *Int. J. Adv. Manuf. Technol*., Vol. 28, pp. 602-609.