Sports Sciences

Efficiency and productivity evaluation of basketball players' performance

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Abstract - Aim: Basketball players' performances have been traditionally summarized in indices that rely on the game-related statistics (e.g. rebounds, field goals, etc). Indices are defined according to different methods (e.g. Efficiency Index - Ei, Plus-Minus, Wins Produced), seeking distinct analytic objectives. Ei is frequently used given its simplicity. However, it has questionable validity since it measures productivity instead of efficiency and uses biased calculations for scoring. This study aimed to define indices of efficiency (Basketball Efficiency Index (BEi)) and productivity (Basketball Productivity Index (BPi)) of a player's contribution to the team performance with greater validity than Ei. **Methods:** We gathered public NBA game-related statistics (2014/2015 – 2018/2019). We analyzed: Ei's and BEi's winning prediction accuracy; Ei's and BEi's point spread prediction accuracy; the correlation between GRS and Ei, BEi, BPi, and PER; players' rank correlation between indices. **Results:** In comparison to Ei, both BEi and BPi reduced the weight of points scored on the final value. Less reliance on points scored results in a more accurate comparison of the contribution of players independent of their tactical roles. **Conclusion:** These indices may improve coaches' understanding of the real contribution of each player to team performance.

Keywords: box-score, NBA, discrete data, game-related statistics, index.

Introduction

The performance of a basketball player is frequently summarized through a set of game-related statistics (GRS) - e.g. field-goal conversion percentage, defensive rebounds - displayed in a box score (i.e. a structured summary of the end-of-possession events of a game). The GRS provides information about the player's contribution to the team performance and provides some general information about the history of the game[1,2].

In most official tournaments, higher-resolution methods of data acquisition, for instance, automatic player tracking technologies or scouting services, are not available and GRS represents the only official data provided. Hence, GRS are frequently the main source of objective information about a player's performance both for the coach and for researchers in several related subjects such as the contribution of a player to winning[3], tactical versatility of a player[4,5] or the effects of situational variables in the player's performance[6,7].

GRS is used for within and between player comparisons through GRS-based indices[8,9]. Different performance indices apply different calculation methods, seeking an improvement of measurement validity[8]. One of these indices is the Efficiency Index (Ei), whose value is presented in several official tournaments. The Ei consists of an addition of all GRS with a positive outcome (e.g. made shots) and subtraction of those with a negative outcome (e.g. turnovers) in the box score. The final number stands as the player contribution to team [10]. It is simple to calculate and provides an intuitive result. Unfortunately, it leads to a biased comparison of players by not accounting for different players playing time, the game pace, and GRS weights. Players with more playing time and teams with a higher game pace may produce more GRS. Furthermore, GRS are weighted differently - a missed field goal subtracts one point from the index value while a converted field goal adds two or three points to the index, depending on if the shot was taken from inside or outside the three-point line.

Besides the aforementioned sources of bias, there is a conceptual flaw in the index since Ei is not an efficiency index but a productivity one. Efficiency can be defined as the ratio of positive outcomes over total inputs[11]. In basketball, it may be calculated, for instance, in terms of the number of made shots given the total possessions of the team[12]. Nevertheless, Ei's value is generated by the addition and subtraction of GRS, not considering the total number of opportunities the player had.

The National Basketball Association (NBA) also currently computes the Player Impact Estimate (PIE), which is related to the Ei. The PIE measures a player's overall GRS contribution and normalizes it by the total frequency of GRS in the game. It prevents the bias created by the game pace and enables intra-player comparisons between games. However, it does not fix Ei's issues related to weighting GRS differently (e.g. converted and missed field goals). Additionally, there is a controversial 0.5 weight for offensive rebounds and blocks.

Alternatively, the Player Efficiency Rating (PER)[13] measures the GRS performed per minute by the player, converts the GRS produced into points and corrects the game pace[8,913]. A made shot is worth two points minus the correction for assisted-made shots. PER assumes that the shot is affected by the competence of both the passing and the shooting players. The shooting player is, therefore, responsible for two-thirds of the shot result and the correction for the made shot is the two-thirds multiplication that is the shooter's responsibility. A missed shot is worth one point multiplied by the average value of possession and depends on the average defensive rebound efficiency. The missed shot is only considered negative when there is a defensive rebound afterward. The average defensive rebounding efficiency indicates the frequency at which a missed shot is negative for the player. The use of different corrections for made and missed shots affects the influence of points on the final value of the metric.

Ei, PIE, and PER have the common feature of assessing player performance focusing on the impact of her productivity on scoring. Some methodological issues have been progressively fixed, such as the corrections for game pace (PIE, PER) and minutes played (PER). Nonetheless, sources of bias remain related to the weighting of inefficient scoring since a high volume of shooting with a low scoring rate still performs well in this regard[8]. Improvements in performance indices should address issues related to the scoring variables[14], and enable differentiation of efficiency and productivity.

Despite the aforementioned flaws of methods such as Ei and PER, these indices reportedly perform well, for instance, at explaining players' salaries[8]. Other alternatives - e.g. Plus-Minus methods - focus on explaining wins[8] or aim at equilibrating an individual's performance and wins' prediction - e.g. Wins Produced[8]. In this sense, an index may be preferable depending on the analytic purpose. Specifically, for the Ei, the negative consequences of its biased results in the evaluation of players in several leagues indicate it is worth to be improved in order to support more accurate comparisons within and between players' performance.

The goal of this study was to attempt to summarize a player's performance in a game through indices that overcome: i) conceptual flaws related to efficiency and productivity; and ii) biased influences of playing time, game pace, GRS weights, and a player's participation in the game actions. We departed from the Ei's because of its well-known structure which may favor the understanding of the adjustments and reinforce the practical application of the proposed indices in the basketball community. We hope to reference the player's index value to an estimate of points they contribute to the team's total score and elucidate how this contribution occurred in terms of GRSs produced by the player. In the end, it may support the better judgment of a player's performance.

Methods

In this section, we take the Ei as a reference, describing its sources of bias and the respective adjustments we will perform in order to summarize a player's performance in terms of efficiency and productivity.

Limitations of the efficiency index

The Ei calculation is based on the set of GRS available in the box-score. GRS may be distinguished between those that favor the team (offensive: points, offensive rebounds, and assists; defensive: defensive rebounds, steals, and blocks) and those that favor the opponent (offensive: turnovers; defensive: personal fouls). The Ei calculation presents three main sources of bias: i) the player's playing time; ii) the team pace; iii) the GRS weights.

First, a player's playing time influences the number of GRS they may perform given the minutes played. Greater playing time may lead to more GRS performed, which increases the index value attributed to players with more minutes played. The current form of the index is actually a measure of productivity (i.e. the addition over time of the GRS the player generates) despite being called an efficiency index.

Second, a higher game pace of a team increases the number of GRS since there is a greater number of ball possessions. Thus, it is difficult to compare players in different games and on different teams.

Third, GRS are computed with different weights. Made shots may earn two or three points while all other GRS earns one point, including missed shots. Thus, if a player converts a two-point field-goal and misses the next two attempts or if a player converts a three-point field-goal and misses the next three attempts their Ei value would be 0. If a player has over a 33% of efficiency on two-point field-goals, or over a 25% on three-point field-goals, their performance will contribute positively to the index[8]. These field-goal rates are below average for most leagues; therefore, they are commonly achieved by the players.

Corrections of the efficiency index

In order to overcome some of the limitations of the current Ei we define a method for calculating a player's efficiency in a game. Our approach for calculating efficiency overcomes the bias derived from both the player's playing time and game pace - greater playing time and greater pace may lead to greater productivity, not necessarily efficiency. We also fix issues related to GRS weights. A player's efficiency was defined as follows.

First, we estimated the total opportunities to perform each GRS a player had in the game: i) compute the team's total opportunities of each GRS (e.g. total rebounds on the team defensive court); ii) assess the minutes played by the player in the game to infer the total opportunities for performing each GRS; iii) standardization of the product between the total GRS in the game and the minutes of the player in the game by the minutes in the game (see Equation (1)). The appendix displays the calculation for each specific GRS.

$$\text{GRS Opportunities} = \frac{\text{Team GRS opportunities} * \text{Minutes}}{\text{Minutes in the game}} \tag{1}$$

Second, we calculated the efficiency of the player in performing each particular GRS. The player's frequency of a specific GRS was divided by the estimated total possible number for the GRS, computed in Equation (1), leading to an efficiency value (see Equation (2)). This overcomes the bias from the game pace and allows for comparisons among players from different teams.

$$\text{GRS Efficiency} = \frac{\text{GRS}}{\text{GRS opportunities}} \tag{2}$$

Third, we estimated the frequency of each GRS for a player in the "league average game" (i.e. the average values of each GRS computed from all League games, in the seasons of interest). For instance, if there are 30 defensive rebounds and 10 offensive rebounds in the league average, the number of opportunities for rebounds (defensive or offensive) for a team in the league average is 40. Then, the average number of opportunities for each GRS is multiplied by the efficiency value, calculated in Equation (2). This estimates a player's performance for each GRS standardized by the league averages (see Equation (3)).

$$\text{GRS (av. game)} = \text{GRS efficiency} * \text{GRS opportunities (av. game)} \tag{3}$$

Fourth, we estimated the equivalent value of in-game points generated by each GRS performed by a player (League's GRS ppp). Rebounds, steals, and turnovers relate to gaining or losing a ball possession. Their equivalent values in points were considered as the League average points per possession (see the formulas in the Appendix). GRS data supports a more accurate calculation of points generated after shots, assists, blocks, or personal fouls as follows. The GRS points naturally express the value of points per possession. Nevertheless, it requires a correction for assisted points, since two-thirds of the value of an assisted point is attributed to the shot. Consequently, the points per possession of an assist are calculated as one-third of the ratio between points and field-goals made (Equation (4) illustrates the calculation with the specific case of the GRS "points"). Points per possession for a block estimate the points a block prevents in the League average. Thus, it is equivalent to the ratio between points and non-blocked field-goal attempts with a correction for the resulting rebound afterward. A defensive or offensive rebound implies the opponent's possibility of keeping possession, which influences the ultimate value of points per possession after a blocked shot. Personal fouls' points per possession are related to the frequency of generated free throws and their respective conversion rate. Besides the example below with "points", each calculation of the League's GRS ppp is explained in detail in the Appendix.

$$\text{League's PTS ppp} = 1 - \left(\frac{2}{3} * \frac{\text{AST}}{\text{FGM}}\right) \tag{4}$$

Fifth, we calculated the equivalence in points generated for each GRS performed by a player (GRS_{eq}). For this purpose, we multiplied the player's number of GRS (Equation (3)) by each League's GRS ppp, see Equation (5).

$$\text{GRSeq} = \text{GRS (av. game)} * \text{League's GRS ppp} \tag{5}$$

Finally, the results of each GRS_{eq} were summed to yield the Basketball Efficiency Index (BEi), see Equation (6).

$$\text{BEi} = \text{PTSeq} + \text{OREBeq} + \text{ASTeq} - \text{TOVeq} + \text{DREBeq} + \text{STLeq} + \text{BLKeq} - \text{PFeq} \tag{6}$$

In a game, players perform different numbers of GRS - e.g. the main shooter of a team might have up to 40% of the team's shots while on the court. Maintaining high efficiency with greater participation is more challenging than when participation time is lower. To establish a comparison among players that is sensitive to their distinct GRS contributions, we should add a game participation factor to the BEi.

We computed this factor as the ratio between the GRS performed by the player and the GRS of the team while the player is on the court. GRS performed by the player were chosen as the criterion since they represent a player's contribution better than criteria such as minutes played. Equivalent participation among each of the five players on the court would lead to a player performing 20% of each GRS in the game (i.e. one fifth of the GRS performed by the team players). Thus, we normalized the ratio between a player's GRS and a team's GRS by 0.2 to assess the deviation of the player's participation from a hypothetical equivalent contribution among all team players, see Equation (7).

$$\text{Participation Factor} = \frac{\frac{\text{GRS}}{\text{Team's GRS} * \frac{\text{Minutes}}{\text{Minutes in the game}}}}{0.2} \qquad (7)$$

Finally, to calculate the Basketball Productivity Index (BPi) of the player, the efficiency calculated in Equation (6) was multiplied by the participation factor (Equation (7)), as indicated in Equation (8).

$$\text{BPi} = \text{Participation Factor} * \text{BEi} \qquad (8)$$

The BPi is focused on the player level, to discriminate among players by taking into account their contribution in terms of the GRS. However, on the team level, the team's BPi will have the same value as the team's BEi, since the participation factor of a team is 100%.

Validity

To test the validity of BEi and BPi, these indices were compared with the currently used Efficiency Index (Ei) and Player Efficiency Rating (PER). We assessed the value of BEi and Ei in predicting the probability of a win and point spread. Also, we calculated the correlations between the indices (BPi, BEi, Ei, and PER) and each GRS in order to assess the influence of each GRS in the index value. For this analysis, we used individual player data of each GRS and each index, in each game. Finally, for the last season of the sample (2018-2019), also with individual players' data in each game, we performed Kendall's rank correlation to assess the differences in the players' ranking according to each index.

Sample

We used data from five seasons (2014/2015 – 2018/2019) of the National Basketball Association (NBA), publicly available at www.nba.com. It consisted of the GRS of every player, in every game, over all seasons of the sample (900 players; 12300 games – 30 teams, 82 games per team, 2460 games per season). Players who averaged less than 25% of the possible playing time per game (12 minutes) or played in fewer than 50% of the possible games played (41 games) in a season were excluded from the analysis. The data set was scraped with the nbastatR package for R software. The Brigham Young University institutional review board cleared this project relative to any issue involving human subjects.

Data analysis

GRS (seasons 2014–2015 – 2018–2019) were used to calculate the BEi, BPi, Ei, and PER for every player that met the inclusion criteria. Individual players' values for each of these indices were used to perform the following analysis. First, we performed a mixed-model logistic regression to appropriately account for the team-to-team variability and compare BEi and Ei in terms of their winning prediction accuracy. For this purpose, BEi's and Ei's values of each team player, in each game, were added. The team values of Ei and BEi were used for the prediction. The categorical binomial dependent variable game result (i.e. winning or losing) was predicted based on the value of the two independent variables Ei and BEi. Prediction results were compared with the Akaike Information Criterion (AIC) - a lower AIC indicates a better fit for the model. Second, we performed a mixed linear regression to appropriately account for both home team and visiting team variability to assess the accuracy of BEi and Ei in predicting the point spread in the final score of the games in the sample[15]. The final point spread was partitioned into three arbitrarily defined groups (ranges: ≥ 20 - unbalanced games, < 20 and > 5 - balanced games, ≤ 5 - close games). Again, BEi and Ei were the independent variables. Results were compared using AIC and residual standard errors. Third, we computed the Kendall's rank correlation coefficient among the four measures (BPi, BEi, Ei, and PER) across all players, all games, and all seasons. The Kendall's coefficient informed the strength of the relationship between pairs of indexes for the players' classification (i.e. the similarity of players' ordering when ranked by different indexes). Fourth, we presented the rank of the Top-20 players in the 2018-19 season for three measures (BPi, Ei, and PER). All analyses were performed using R software, version: 4.0.1[16], packages: nbastatR, dplyr, lme4, ggplot2.

Results

BEi and Ei were compared in terms of predicting a win. The following AICs were obtained: i) BEi: 12,696; ii) Ei: 11,489, thus the Ei model yielded a better fit. Figure 1 displays the output.

Table 1 displays the results for BEi and Ei in predicting the final point spread in three-game scoring ranges: i) ≥ 20 points; ii) < 20 and > 5; iii) ≤ 5 points. Ei better predicted large and intermediate point spreads, while BEi was better at modeling close games.

Table 2 displays the correlations among BPi, BEi, Ei, PER, and each GRS. The greatest correlations were between Ei and points (0.84), Ei and field-goals made (0.82), and PER and points (0.71).

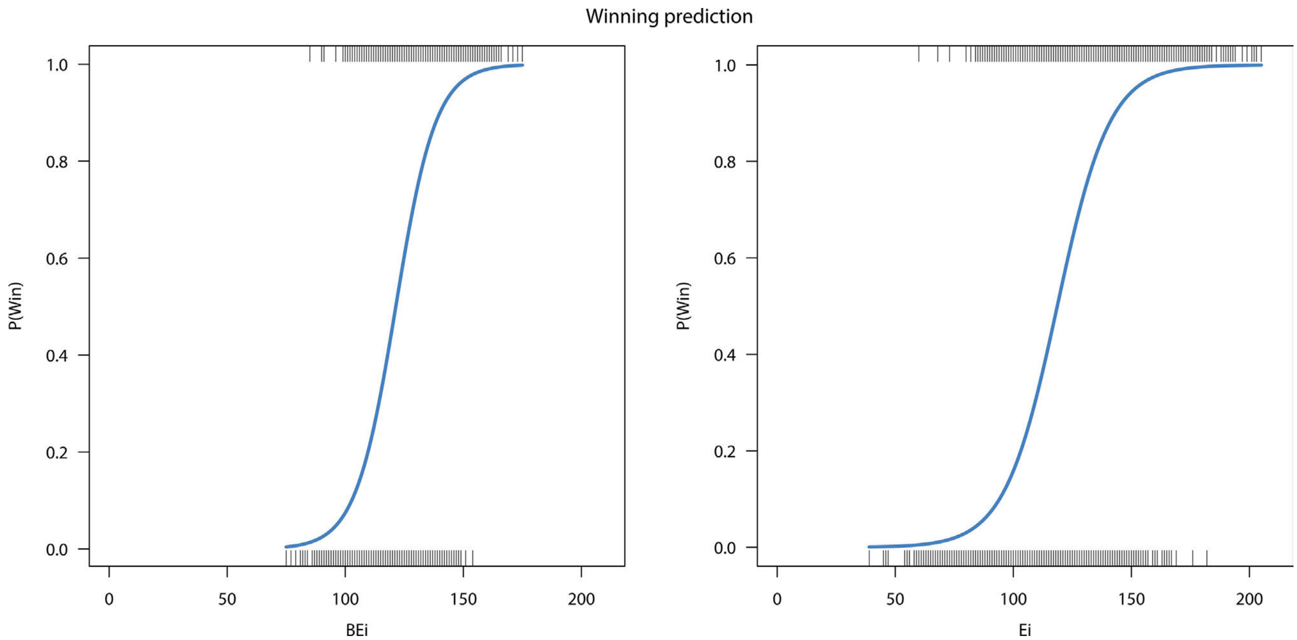

Figure 1 - Winning prediction based on BEi and Ei.

Table 1 - Point spread predictions for BEi and Ei, in three ranges of game scores. Where: Akaike information criterion (AIC), residual standard error (RSE), home team standard error (HTSE), visiting team standard error (VTSE).

	BEi				Ei			
	AIC	RSE	HTSE	VTSE	AIC	RSE	HTSE	VTSE
≥ 20	5,501.80	4.47	0.70	0.49	5,145.79	3.63	0.65	0.94
< 20 and > 5	18,590.36	3.31	0.41	0.31	17,550.44	2.83	0.74	0.62
≤ 5	5,774.80	1.32	0.09	0.16	5,857.11	1.34	0.16	0.23

Figures 2A, 2B and 2C show Kendall's rank correlation coefficient among indices (BPi - Ei: 0.75; BPi - PER: 0.72; Ei - PER: 0.80). Ei and PER had the highest correlation between indices (0.80).

Table 2 - Correlations between BPi, BEi, Ei, PER and each GRS (Minutes: Min; Points: Pts; Field-Goals Made: FGM; Field-Goals Attempted: FGAt; Assists: Ast; Defensive Rebounds: DReb; Offensive Rebounds: OReb; Steals: Stl; Turnovers: TO; Personal Fouls: PF).

	BPi	BEi	Ei	PER
Min	0.22	0.20	0.59	0.24
Pts	0.56	0.45	0.84	0.71
FGM	0.55	0.45	0.82	0.69
FGAt	0.39	0.17	0.60	0.38
Ast	0.39	0.27	0.43	0.26
DReb	0.61	0.49	0.57	0.27
OReb	0.43	0.32	0.33	0.27
Stl	0.27	0.22	0.28	0.24
TO	0.18	−0.05	0.20	−0.03
PF	0.05	−0.14	0.15	−0.02

Table 3 presents the ranking of players (season 2018-2019, a minimum average of 24 minutes played per game, 41 or more games' appearances in the season) for BPi, BEi, Ei, and PER. For the top 20 ranked players, we found the following playing positions' tendencies (Perimeter players (P) - point guards, shooting guards, small forwards. Inside players (I) - power forwards, centers) for each index. BPi (Perimeter players: 40%, Inside players: 60%); BEi (Perimeter players: 20%, Inside players: 80%); Ei (Perimeter players: 55%, Inside players: 45%); PER (Perimeter players: 45%, Inside players: 55%).

Discussion

The purpose of the present study was to define indices that could be used to summarize a basketball player's performance in terms of both efficiency and productivity. We also sought an index that would estimate the number of points the player would contribute to the team in a game. The algebraic solution improved the construct validity of a previous index used as a reference (Ei) since the player contribution is calculated in terms of the efficiency concept (BEi) and adjusted for player

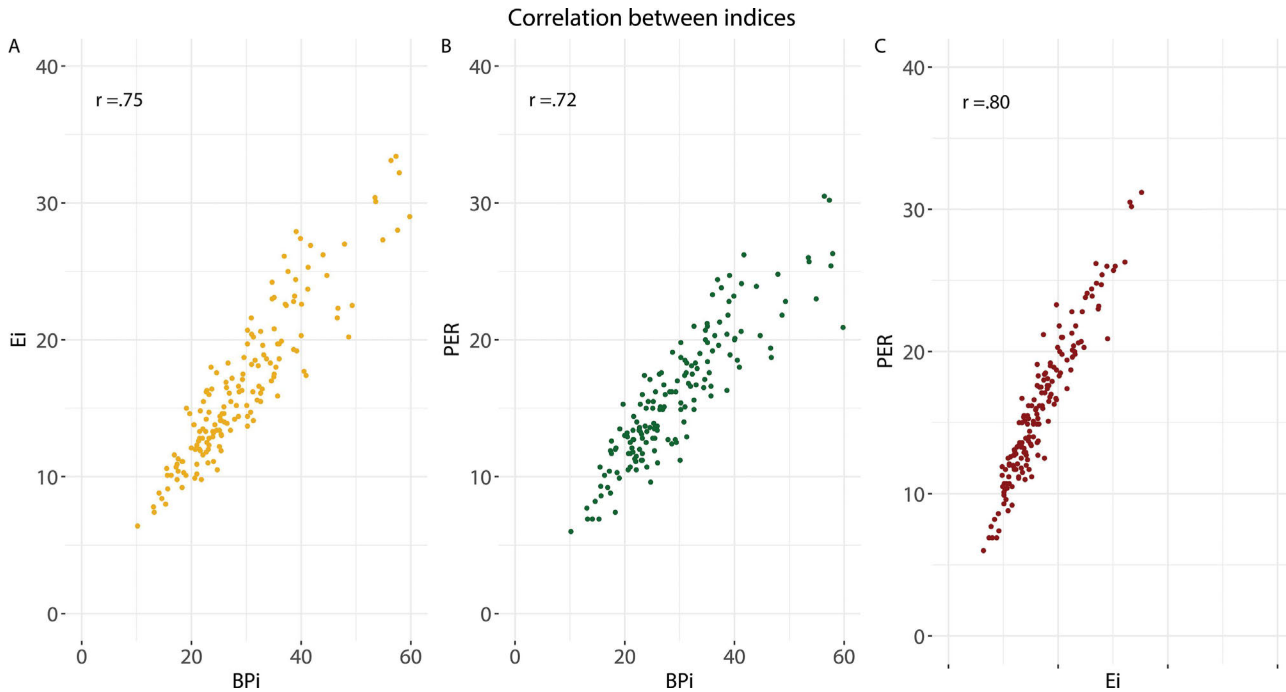

Figure 2 - Parts A, B, and C: Correlation between pairs of indices.

participation (BPi). BEi and BPi demonstrated lower variability in relation to Ei for the correlation between each GRS and the index value. Ei's value is highly influenced by points (r = 0.84) and for this reason, it predicted teams' winning better than BEi. However, BEi outperformed Ei in predicting the point spread in close games, suggesting the importance of all GRS for the team's success when teams play closely matched opponents. Lower correlation between Ei - BPi (0.75) and Ei - PER (0.80) reinforce the perspective that BPi improves the solution for comparing players' performance, with greater sensitivity to their distinct tactical roles in comparison to previous indices.

BEi and BPi improve the construct validity of Ei's calculation since they are consistent with efficiency and productivity definitions, respectively. Precise assessment of efficiency and productivity seems to be particularly important in basketball. Some of basketball's rules (e.g. offensive shot clock, defensive three-second violation) greatly constrain ball possession time, demanding high rates of positive outcomes over opportunities in order to be successful both on offense and on defense. On offense, the creation of opportunities happens at a high pace. However, the quality of the opportunities created, encompassing spacing and timing features, impact the probability of a positive outcome[12]. In comparison to sports that do not have a large number of offensive opportunities (e.g. association football)[12] or sports that have low efficiency (e.g. ice hockey)[12], success in basketball relies on efficient conversion of opportunities. Since the GRS used in the BEi and BPi calculation relate to end-of-ball-possession events, their scope naturally leads to an understanding of efficiency.

We took Ei as a reference to perform adjustments in the assessment of a player's efficiency. Ei's definition leads to an influence of playing time in the index value that is not coherent with the conception of efficiency[11] and may be evidenced by the correlation of 0.59 between Ei and minutes played. BEi conception is based on the opportunities to perform a GRS, which is consistent with the efficiency concept and resulted in a lowered correlation with playing time (r = 0.20, see Table 2).

The BEi's playing time adjustment improved Ei's bias related to the game pace. Previous indices, e.g. PER, applied a per-minute calculation, however, the procedure has limitations since teams can perform different numbers of GRS in a similar playing time interval[17,18]. For instance, the number of defensive rebounds is greatly affected by the game pace, even if there are few differences in field-goal efficiency[19]. The efficiency computation based on GRS opportunities reduces the influence of a team's pace as the player's efficiency will vary in accordance with the number of GRS performed given the total number of opportunities.

Since the BEi's calculation method is based on the ratio between opportunities to perform a GRS and those GRS effectively performed, we estimated the weights of the GRS contribution to points, considering the expected number of points generated by each GRS[20] (see Appendix for details). The value of a GRS to the game score is nei-

Table 3 - Players' ranking for BPi, BEi, Ei, and PER in the 2018-2019 season. The first two ranked players for BPi are highlighted to favor tracking among indices.

	BPi	BEi	Ei	PER
1	G. Antetokounmpo - P	D. Jordan - I	G. Antetokounmpo - P	G. Antetokounmpo - P
2	N. Jokic - I	R. Gobert - I	A. Davis - I	J. Harden - P
3	R. Westbrook - P	N. Jokic - I	J. Harden - P	A. Davis - I
4	J. Embiid - I	N. Vucevic - I	J. Embiid - I	J. Embiid - I
5	N. Vucevic - I	G. Antetokounmpo - P	KA. Towns - I	K. Leonard - P
6	A. Davis - I	C. Capela - I	L. James - P	N. Jokic - I
7	J. Harden - P	A. Davis - I	R. Westbrook - P	KA. Towns - I
8	A. Drummond - I	A. Drummond - I	N. Jokic - I	L. James - P
9	L. James - P	R. Westbrook - P	N. Vucevic - I	N. Vucevic - I
10	KA. Towns - I	L. James - P	K. Durant - P	R. Gobert - I
11	J. Nurkic - I	J. Embiid - I	P. George - P	K. Durant - P
12	D. Sabonis - I	D. Sabonis - I	A. Drummond - I	S. Curry - P
13	R. Gobert - I	B. Simmons - P	R. Gobert - I	K. Irving - P
14	D. Jordan - I	T. Thompson - I	K. Leonard - P	C. Capela - I
15	L. Doncic - P	KA. Towns - I	C. Capela - I	D. Lillard - P
16	B. Simmons - P	L. Nance Jr. - I	S. Curry - P	M. Harrell - I
17	C. Capela - I	D. Ayton - I	K. Irving - P	P. George - P
18	K. Leonard - P	J. Allen - I	D. Lillard - P	A. Drummond - I
19	K. Irving - P	J. Nurkic - I	B. Simmons - P	J. Nurkic - I
20	B. Griffin - I	A. Horford - I	L. Aldridge - I	L. Aldridge - I

ther influenced by its occurrence frequency during the game nor by its performance efficiency[21]. Consequently, it has to be accounted for in the BEi's computation apart from playing time and game pace corrections. GRS conversion into points helps us better understand the value of each GRS to the BEi value of a player.

BEi calculation weights all players equally relative to the performance of GRS. Nonetheless, game participation is unequal and an adjustment in this regard is crucial since the greater the number of minutes played the harder it is to perform efficiently [22]. Hence, BPi was created to acknowledge that players with greater participation in the game in terms of GRS performed usually have greater physical and technical demands[22], which is challenging in terms of sustaining efficiency. Considering the adjustment for player participation in terms of their GRS performed we added it into the BPi. BEi may be oriented towards the team-level assessment while BPi may be more useful for the player level.

On the team level, Ei (AIC: 11,489) predicted winning better than BEi (AIC:12,696), which may be explained by the greater reliance on point production (correlation 0.84) for Ei in comparison with BEi (correlation 0.45). Ei outperformed BEi when predicting games with larger point spreads. However, BEi (AIC: 5,774.80) predicted better than Ei (AIC: 5,857.11) when games were close ($\leq$ 5 points). This result may be because when games are close, other GRS have an increased importance[23,24]. For example, in close games, winning might be explained by rebounding efficiency or lack of turnovers[23]. The ability of BEi to predict better in close games reinforces the relevance of using a different scheme to weight each GRS when calculating the summary of a team's performance.

At the player level, the high correlation between Ei and points (r = 0.84), and PER and points (r = 0.71) may lead to the idea that scoring players tend to be more efficient or contribute more to the team. Moreover, the correlation of Ei with field-goal attempts (0.60) indicates that players with high shooting rates are benefited from the calculation method applied in the Ei despite their real efficiency[14].

While productivity indexes, such as Ei and PER overvalue scoring, BPi may provide a better balance between scoring efficiency and participating in other GRS. This is illustrated by the players' ranking according to BPi criteria. Among the top twenty players for BPi (see Table 3), only eight (40%) were perimeter players. Inside players generally have a higher shooting efficiency as their shots are taken closer to the basket, and their rebounding efficiency is much higher[25]. Alternatively, perimeter (P) and inside (I) players proportions in the top twenty ranking positions had smaller differences when considering Ei (P: 55%, I: 45%) and PER (P: 45%, I: 55%).

BPi is less influenced by a focus on point productions, suggested by the ranking differences between BPi and Ei/PER. For instance, in BPi, perimeter players that rely solely on shooting, independently of an extremely high conversion rate, are analyzed for their whole participation in-game actions. Therefore, BPi may be interpreted as more representative of the total player contribution in terms of GRS which is useful when comparing a player's impact on team performance.

A limitation of all box-score-based analyses is that a player's defensive actions are not accounted for at the same level as the offensive ones. Play-by-play data, often publicly available, may support some advances in this regard. Additionally, the use of play-by-play data may help with the assessment of both efficiency and productivity of more subtle in-game skills of players[26,27].

Conclusions

The present work presents two new indices for summarizing a basketball player's performance based on box-score data. BEi corrects biases related to playing time, game pace, and the weight of each GRS to the index value. BPi provides an additional adjustment for the time a player participates in the game. The calculation of BEi and BPi makes them less dependent on shooting-related GRS and increases the sensitivity to the overall contribution of the player.

In practice, the two indices are complementary - BEi and BPi may be more useful at the team and player levels, respectively. They may help coaches gain an understanding of a player's real impact in the game and, consequently, help motivate a player to make improvements in all game actions.

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Appendix

Abbreviations

Table - Appendix - Abbreviations used in the text's formulas.

Game-related statistics	Abbreviation	Game-related statistics	Abbreviation
Minutes Played	MP	Points per Possession	ppp
Opponent	Opp	Team	T
Offensive Rebound	OREB	Defensive Rebound	DREB
Assist	AST	Steal	STL
Turnover	TOV	Block	BLK
Personal Foul	PF	Points	PTS
Field-Goal Attempt	FGA	Free Throw Attempt	FTA
Field-Goal Made	FGM	Free Throw Made	FTM
Two-Point Field-Goal Made	2PM	Three-Point Field-Goal Made	3PM

Player's GRS opportunities

Calculation of each GRS opportunities (generalized in Formula (1) in the text):

$$\text{PTS opportunities} = \text{FGA} + (0.44 * \text{FTA})$$

*Points do not require the opportunity correction since the opportunities are already related to the individual player

$$\text{OREB opportunities} = \frac{(\text{OREBt} + \text{DREBopp}) * \text{MP}}{\frac{\text{MPt}}{5}}$$

$$\text{AST opportunities} = \frac{(\text{FGMt} + (0,44 * \text{FTMt}) - \text{FGM} - (0,44 * \text{FTM})) * \text{MP}}{\frac{\text{MPt}}{5}}$$

$$\text{TOV opportunities} = \frac{(\text{FGAt} + (0,44 * \text{FTAt}) + \text{TOVt}) * \text{MP}}{\frac{\text{MPt}}{5}}$$

$$\text{STL opportunities} = \frac{(\text{FGAopp} + (0,44 * \text{FTAopp}) + \text{TOVopp}) * \text{MP}}{\frac{\text{MPt}}{5}}$$

$$\text{BLK opportunities} = \frac{\text{FGAopp} * \text{MP}}{\frac{\text{MPt}}{5}}$$

$$\text{PF opportunities} = \frac{(\text{FGAopp} + (0,44 * \text{FTAopp}) + \text{TOVopp}) * \text{MP}}{\frac{\text{MPt}}{5}}$$

Player's GRS efficiency

Calculation of the particular GRS efficiency (generalized in Formula (2) in the text):

$$\text{PTS Efficiency} = \frac{\text{PTS}}{\text{PTS opportunities}}$$

$$\text{OREB Efficiency} = \frac{\text{OREB}}{\text{OREB opportunities}}$$

$$\text{AST Efficiency} = \frac{\text{AST}}{\text{AST opportunities}}$$

$$\text{TOV Efficiency} = \frac{\text{TOV}}{\text{TOV opportunities}}$$

$$\text{DREB Efficiency} = \frac{\text{DREB}}{\text{DREB opportunities}}$$

$$\text{STL Efficiency} = \frac{\text{STL}}{\text{STL opportunities}}$$

$$\text{BLK Efficiency} = \frac{\text{BLK}}{\text{BLK opportunities}}$$

$$\text{PF Efficiency} = \frac{\text{PF}}{\text{PF opportunities}}$$

GRS (av. game)

Calculation of the particular GRS on the average game (generalized in Formula (3) in the text). All GRS are related to the League's Average Game:

$$\text{PTS (av. game)} = \text{PTS Efficiency} * \frac{\text{FGA} + (0.44 * \text{FTA})}{5}$$

*In the formula, the fraction corresponds to the number of shots during the game divided by the number of players (5).

$$\text{OREB (av. game)} = \text{OREB Efficiency} * (\text{OREB} + \text{DREB})$$

$$\text{AST (av. game)} = \text{AST Efficiency} * \frac{\text{FGM} + (0.44 * \text{FTM})}{\frac{4}{5}}$$

*The amount of made shots during the game minus the amount by the player.

$$\text{TOV (av. game)} = \text{TOV Efficiency} * (\text{FGA} + (\text{FTA} * 0.44) + \text{TOV})$$

$$\text{DREB (av. game)} = \text{DREB Efficiency} * (\text{DREB} + \text{OREB})$$

$$\text{STL (av. game)} = \text{STL Efficiency} * (\text{FGA} + (\text{FTA} * 0.44) + \text{TOV})$$

$$\text{BLK (av. game)} = \text{BLK Efficiency} * \text{FGA}$$

$$\text{PF (av. game)} = \text{PF Efficiency} * (\text{FGA} + (\text{FTA} * 0.44) + \text{TOV})$$

League's GRS ppp

The result of each GRS needs to be considered in order to calculate its value. Considering the GRS for the league average game, the number of points per possession is calculated as:

Points: The point is almost the intended final value. The player that scores the point can be responsible for two-thirds of the point, since, passing, shooting and spacing can also be considered as responsible for scoring[9]. Then, the value of points is equal to two-thirds of the actual value, as follows:

$$\text{League's PTS ppp} = 1 - \left(\frac{2}{3} * \frac{\text{AST}}{\text{FGM}}\right)$$

Offensive rebounds: An offensive rebound generates a new possession for the team whose player grabbed it. Therefore, for each offensive rebound, the player is contributing one extra possession for the team. Then, the value of the offensive rebound is equal to the value of one possession, as follows:

$$\text{League's OREB ppp} = \frac{\text{PTS}}{\text{FGA} + (\text{FTA} * 0.44) + \text{TOV}}$$

Assists: An assist is related to points made. The assist occurs when there is a made field-goal. The assist can be considered one-third of the point[9]. Thus, the value of the assist is equal to one-third of the average number of points per field-goal made, as follows:

$$\text{League's AST ppp} = \left(\frac{\text{PTS} - \text{FTM}}{\text{FGM}}\right) * \frac{1}{3}$$

Turnovers: A turnover is the loss of possession by the team. Therefore, for each turnover, the player is costing one possession for the team. The value of turnover is equal to the value of one possession, as follows:

$$\text{League's TOV ppp} = \frac{\text{PTS}}{\text{FGA} + (\text{FTA} * 0.44) + \text{TOV}}$$

Defensive rebounds: A defensive rebound generates a new possession for the team whose player grabbed it. Therefore, for each defensive rebound, the player is preventing one extra possession for the opponent. The value of a defensive rebound is equal to the value of one possession, as follows:

$$\text{League's DREB ppp} = \frac{\text{PTS}}{\text{FGA} + (\text{FTA} * 0.44) + \text{TOV}}$$

Steals: A steal generates a new possession for the team whose player grabbed it. Therefore, for each steal, the player is removing a possession from the opponent. The value of a steal is equal to the value of one possession, as follows:

$$\text{League's STL ppp} = \frac{\text{PTS}}{\text{FGA} + (\text{FTA} * 0.44) + \text{TOV}}$$

Blocks: A block prevents the opportunity to make a shot for the opponent. The block might lead to a defensive or an offensive rebound. An offensive rebound after the block indicates a block with less value as the opponent still has another opportunity to score. Therefore, for each block followed by a defensive rebound, the player is denying the opportunity of the opponent to make a field-goal. For each block followed by an offensive rebound, the player is denying one opportunity of the opponent to make a field-goal but the opponent receives another opportunity to score. Then, the value of the block is equal to the denial of an opportunity score when there is a defensive rebound plus the denial of an opponent shot minus the value of the new possession when there is an offensive rebound, as follows:

$$\text{League's BLK ppp} = \left(\frac{\text{DREB}}{\text{OREB} + \text{DREB}} * \frac{(2\text{PM} * 2) + (3\text{PM} * 3)}{\text{FGA} - \text{BLK}}\right) + \left(\frac{\text{OREB}}{\text{OREB} + \text{DREB}} * \left(\frac{(2\text{PM} * 2) + (3\text{PM} * 3)}{\text{FGA} - \text{BLK}} - \frac{\text{PTS}}{\text{FGA} + (\text{FTA} * 0.44) + \text{TOV}}\right)\right)$$

Personal foul: A personal foul can generate free throw attempts for the opponent. When it does, the opponent has an efficiency value for those shots. Thus, the value of the personal foul is equal to the likelihood of generating a free throw attempt and the likelihood of the player converting it, as follows:

$$\text{League's PF ppp} = \frac{\text{FTM}}{\text{FTA}} * \frac{\text{FTA}}{\text{PF}}$$

GRS equivalence in points generated

Calculation of each GRS_{eq} (generalized in Equation (5) in the text):

$$\text{PTSeq} = \text{PTS (av. game)} * \text{League's PTS ppp}$$

$$\text{OREBeq} = \text{OREB (av. game)} * \text{League's OREB ppp}$$

$$\text{ASTeq} = \text{AST (av. game)} * \text{League's AST ppp}$$

$$\text{TOVeq} = \text{TOV (av. game)} * \text{League's TOV ppp}$$

$$\text{DREBeq} = \text{DREB (av. game)} * \text{League's DREB ppp}$$

$$\text{STLeq} = \text{STL (av. game)} * \text{League's STL ppp}$$

$$\text{BLKeq} = \text{BLK (av. game)} * \text{League's BLK ppp}$$

$$\text{PFeq} = \text{PF (av. game)} * \text{League's PF ppp}$$

Participation factor

Calculation of the Participation Factor (generalized in Equation (7) in the text):

$$\text{GRS} = \text{FGA} + (0.44 * \text{FTA}) + \text{OREB} + \text{AST} + \text{TOV} + \text{DREB} + \text{STL} + \text{BLK} + \text{PF}$$

$$\text{Team's GRS} = \text{FGAt} + (0.44 * \text{FTAt}) + \text{OREBt} + \text{ASTt} + \text{TOVt} + \text{DREBt} + \text{STLt} + \text{BLKt} + \text{PFt}$$

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