IBRACON Structures and Materials Journal
Revista IBRACON de Estruturas e Materiais
ISSN 1983-4195 ismj.org

ORIGINAL ARTICLE

Analysis of shear strength of complementary mechanisms trends in reinforced concrete beams

Análise das tendências nos mecanismos complementares de resistência ao cisalhamento de vigas de concreto armado

Igor José Santos Ribeiro[a]
José Renato de Castro Pessoa[b]
Túlio Nogueira Bittencourt[a]

[a]Universidade de São Paulo – USP, Programa de Pós-graduação em Engenharia Civil, São Paulo, SP, Brasil
[b]Universidade Estadual de Santa Cruz – UESC, Programa de Pós-graduação em Ciência, Inovação, e Modelagem de Materiais, Ilhéus, BA, Brasil

Received 31 July 2021
Accepted 16 August 2022

Abstract: The study of shear failure in concrete beams is one of the subjects growing in importance due to both the recent reformulations, and increasingly higher cross-section depths used. For instance, the recent updates in the ACI 318 (2019) shows the need to incorporate the size effect in the design of reinforced concrete elements. In this study, the same database adopted by the ACI-ASCE Committee DAfStb 445-D has been used to calculate shear strength, with and without the consideration of size effect, i.e., the design prescribed by ACI 318 (2014), ACI 318 (2019), Frosch et al. (2017), and the ABNT NBR 6118 (2014). Later these predictions are compared with test results. A dispersion analysis has outlined the trends regarding compressive strength, span to depth ratio, longitudinal reinforcement ratio and beam depth. Every variable was discussed per interval, delineating the causes related to the observable trends. Regarding the most prominent influences (effective depth and longitudinal reinforcement ratio), the approaches considering them directly through factors had provided results with no appreciable trends, with lower coefficients of variation (COV) and substantially more conservative for higher cross-section depths. As the Brazilian code does not consider both, a correction factors, determined by a two-step regression analysis on these parameters to adjust this design, are briefly introduced.

Keywords: fracture, size effect, concrete shear strength, reinforced concrete beams without stirrups, complementary mechanisms.

Resumo: O estudo da ruptura ao cisalhamento em vigas de concreto armado é uma dessas áreas que têm avultado em importância tanto devido às recentes reformulações efetuadas quanto as peças de seções cada vez maiores utilizadas. Por exemplo, as recentes atualizações no código ACI 318 (2019) apontam para a necessidade que tem sido demonstrada de incorporar o efeito escala no projeto de seções de concreto armado. Nesse estudo, o banco de dados adotado pelo Comitê ACI-ASCE DAfStb 445-D foi utilizado para cálculo da resistência ao cisalhamento, com e sem efeito escala, prescritas nas formulações ACI 318 (2014), ACI 318 (2019), Frosch et al. (2017), e ABNT NBR 6118 (2014). Em sequência, as predições foram comparadas com os resultados de ensaios. Uma análise da dispersão delineou as tendências concernentes à resistência à compressão, taxa vão-cisalhamento, taxa de reforço longitudinal e altura da viga. Cada uma das variáveis foi discutida por intervalo, discutindo as causas relacionadas às tendências observadas. No que tange as maiores influências (altura útil e taxa de reforço longitudinal), das abordagens que os consideram diretamente, resultaram em saídas sem tendências apreciáveis, com menores coeficientes de variação (COV) e satisfatoriamente mais conservadoras para seções com maiores alturas. Como o código brasileiro não considera esses fatores, fatores de correção obtidos mediante uma análise de regressão efetuada em duas etapas, são brevemente introduzidos.

Palavras-chave: fratura, efeito escala, resistência ao cisalhamento do concreto, vigas de concreto armado, mecanismos complementares.

Corresponding author: Igor José Santos Ribeiro. E-mail: ijoseribeiro@gmail.com
Financial support: Fundação de Amparo à Pesquisa do Estado da Bahia – FAPESB.
Conflict of interest: Nothing to declare.
Data Availability: The data that support the findings of this study are available from the corresponding author, Igor JS Ribeiro, upon reasonable request.

How to cite: I. J. S. Ribeiro, J. R. C. Pessoa, and T. N. Bittencourt, "Analysis of shear strength of complementary mechanisms trends in reinforced concrete beams," *Rev. IBRACON Estrut. Mater.*, vol. 16, no. 2, e16208, 2023, https://doi.org/10.1590/S1983-41952023000200008

1 INTRODUCTION

The progression of acquisition and monitoring systems, in juxtaposition with theoretical development and new approaches, provide different perspectives to safely achieve the goal of design structures more economically. The study of concrete shear strength of complementary mechanisms is one of those areas that have attracted researchers due to recent reformulations of the codes, together with the more demanding designs.

First, Ritter approached the cracked behavior of concrete due to loads by a truss, with parallel chords, with compressed struts that were inclined with a fixed angle of 45°, later expanded for torsion applications by Mörsch. Because this approach had provided conservative results compared with available tests in the period, it was disseminated in the literature [1].

Several approaches and models followed, regarding the change of the diagonal crack angle, load transfer mechanisms considered, and tensile strength after cracking, juxtaposed with its prominence in the final shear strength. In the attempt to better describe the shear behavior, these complementary mechanisms were included in the design codes. Nevertheless, its calculation is still done in a multitude of approaches delineating the lack of scientific consensus, which tends to reflect in empirical expressions in various codes [2].

Illustrating, the long standing ACI 318 [3] expression for the shear strength of the cross-section mechanisms (V_c), was updated to consider both the size effect and the longitudinal reinforcement ratio (ρ_L) in the ACI 318 [4]. One of the proposals to the north American committee, the Unified Approach by Frosch et al. [5] although was not adopted, presents an approach based on the depth of the compression zone. The authors state that this means to consider the average shear stress over the uncracked cross-section instead of over the effective depth.

Despites the better fitting of the newest version of north American code, the results obtained through ACI-DAfStb 445-D Database [6]–[8], shows a wide range in the influencing parameters, whose tendencies could be even more clear with additional analysis [6]–[8].

Simultaneously, in Brazil, the calculation of shear strength for concrete beams is governed by ABNT NBR 6118 [9] prescribing simple design formulation for the complementary mechanisms, based on concrete tensile strength for both models. Therefore, the current code may be improved regarding the contribution of the aforementioned mechanisms.

Hence, the comparison between the results predicted by the ACI 318 [3], ACI 318 [4], The Unified Approach [5] and ABNT NBR 6118 [9], regarding the scatter though the intervals in relation to main design parameters may allow a broader comprehension of localized or overall trends, as well as possible optimization of the current design by inclusion of correction factors related to these trends.

2 SHEAR STRENGTH OF COMPLEMENARY MECHANISMS OF REINFORCED CONCRETE BEAMS

The first approach to the shear strength of concrete beam, was originally proposed by Ritter, and later generalized by Mörsch. Next, aiming for the simplicity of application for designs, some codes have incorporated other load transfer mechanisms to this model, as well as formulations with variable compression strut angle. The generalized truss, considering the contribution of shear transfer cross-section mechanisms, is present in the Brazilian code ABNT NBR 6118 [9].

2.1 Shear Transfer Mechanisms

After cracking, different mechanisms allow the transfer of shear stresses. The principals have been summarized into cantilever action, shear transfer at the interfaces, dowel action, residual concrete tensile strength, and arch effect [1], [3], [4], [10]:

- **Cantilever effect:** Cracked concrete may transfer shear stresses between two flexural cracks, a region designated as "tooth", fixed in the compression zone [11]. The concept was brought up by Kani [12], who, considering bending cracks do not transmit shear stresses, states that the beam would resist this effort through a compressed region in the upper envelope of the cracks formed together with the bending of each of these regions.
- **Interface Friction:** The phenomenon of shear transfer through cracks is defined as interface shear transfer, or crack friction. However, this designation infers the dependence of the crack surface conditions, not being a property of the material. Sato et al. [13] affirmed that the relative slip between the interfaces and concomitantly this action, were higher for lower *a/d* rates. Fernández Ruiz et al. [14] corroborate this understanding and state that the load transfer capacity of this mechanism is limited by the surface roughness that is influenced by the aggregate size

(micro level), ripples and changes in the direction of the crack (meso level), and by relative displacement (macro level).

- **Dowel action:** This mechanism is defined by the capacity of the beam to transfer stresses through the concrete longitudinal reinforcement, which acts as a pin between the interfaces generated in the propagation of a diagonal crack.
- **Residual concrete tensile strength:** According to the ACI 445R [1], the basic explanation for this mechanism is because after cracking, a few portions of concrete bridge the cracks and continue to transmit stresses to small openings. In juxtaposition, concrete has a quasi-brittle fracture behavior, summarily characterized by the stress relaxation curve that occurs after the peak tensile load.
- **Arch effect:** The previously described mechanisms are modeled from the consideration of a constant lever arm between the compressed and tensioned fibers, which implies in the variation of the tensioned reinforcement stresses according to the loads for which the cross-section is submitted. These mechanisms are classified as shear transfer mechanisms. Alternatively, the forces on the stressed reinforcement may be fixed and the lever arm varies, which corresponds to a compression field of the plasticity theory with force transmission through a direct compressed strut. This mechanism develops through the failure of all the others and is associated with the longitudinal reinforcement loss of adhesion [14], [15].

2.2 Kani's Valley

The mechanisms have different relevance, varying with parameters such as the transverse reinforcement, height, shear span to effective depth ratio, or longitudinal reinforcement ratio. When studying how slenderness influenced the preponderance of shear transfer or the arch effect, Kani [12] exposed the Kani Valley, where four distinct regions may be seen on the response due to slenderness, as illustrated in Figure 1.

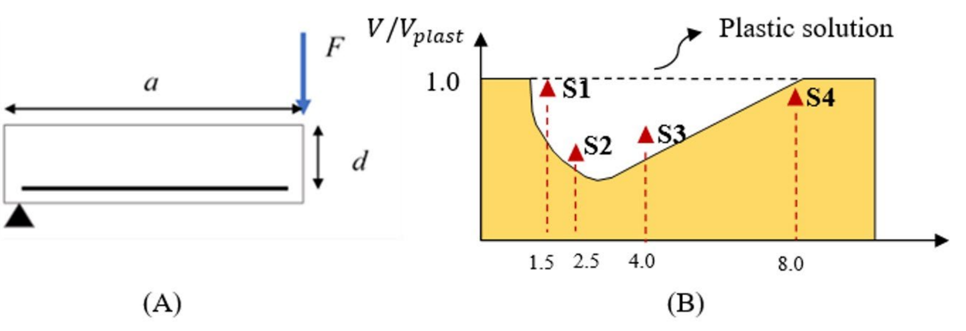

Figure 1 – (A) Shear span (*a*) to effective depth (*d*) to a point load (B) Kani's valley

Specimens S1, S2, S3, and S4 are beam tests with several slenderness available from Leonhardt and Walther [16]. The tests show for small *a/d* ratios, as S1, results closer to those predicted by the theory of elasticity, and the shear strength is governed by the ρ_L, calculated by Equation 1:

$$\rho_L = \frac{A_s}{b_w d} \quad (1)$$

where A_s is the area of the steel in the cross-section, b_w is the width of the beam and d is the effective depth, i.e., the distance from the centroid of tensile reinforcement to the most compressed fiber.

Then, even around the *a/d*=2.4 ratio, where S2 is located, the governing mechanism is the arch effect, in which bending cracks propagate in the stably compressed struts [15]. For slightly larger spans the cross-section transfer mechanisms predominate, where S3 is, until the longitudinal reinforcement begins to govern with the shear stress transfer mechanisms still developing.

3 SHEAR STRENGTH MECHANISMS EXPRESSIONS

Several concrete design codes had incorporated the concrete strength to adjust the results of an obtained dataset. The ACI 318 originally, came from the observation of several parameter in numerous tests where the longitudinal reinforcement rate (ρ_w), a/d ratio, and "concrete quality", which had per measure f_c' (MPa), were the most influent variables. After fitting two tendency lines to the dataset of that period, result in the long stand relation, which lasts until the new version, in S.I. units on Equation 2:

$$V_c = 0.166\lambda\sqrt{f_c'}b_w d \tag{2}$$

where b_w(mm) is the width of the cross-section, d(mm), λ is the aggregate factor, being $\lambda = 1.0$ for normal weight type. However, in the last version of the code [4], if the provided transversal reinforcement is less than the minimum, a new expression (3), most be used:

$$V_c = 0.644\lambda\lambda_s(\rho_L)^{1/3}\sqrt{f'_c}\, b_w d \tag{3}$$

where ρ_L (%) is the longitudinal reinforcement ratio and λ_s is a dimensionless size effect factor calculated by Equation 4:

$$\lambda_s = \sqrt{\frac{2}{1+\frac{d}{254}}} \tag{4}$$

Some other similar proposals were made by Frosch et al. [5]. The approach inferred that since reinforcement stiffness is a primary parameter in shear strength, an effective reinforcement ratio could be defined according to the Equation 5:

$$\rho_{eff} = \rho_L n \tag{5}$$

where n is calculated by Equation 6:

$$n = \frac{E_r}{E_c} \tag{6}$$

where E_r is the modulus of elasticity of the longitudinal reinforcement and E_c the modulus of elasticity of the concrete. Since the stiffness of the reinforcement also affects the location of the neutral axis, the author sought to develop a formulation considering the depth of the cracked cross-section of the concrete, through the Equation 7:

$$c = kd \tag{7}$$

where k is defined by Equation 8:

$$k = \sqrt{2\rho_L n + (\rho_L n)^2} - \rho_L n \tag{8}$$

The use of "c", instead of the usual approaches, allows the incorporation of other effects, as simplifying the design when multiple layers become necessary [5]. From these considerations, the authors fitted an expression to a dataset, with the Equation 9, in S.I. units:

$$V_c = \left(0.415\sqrt{f_c'}b_w c\right)\gamma_d \tag{9}$$

where γ_d is a size effect factor that must be taken as $\gamma_d = 1{,}00$ if the depth from the first layer of reinforcement (d_t) is less than 10 in (25.4 cm), or if d_t is simultaneously less than 100 in (254 cm), and the transverse reinforcement is higher than the minimum ratio. If these conditions are not fulfilled, the size effect should be calculated by Equation 10:

$$\gamma_d = \frac{1.4}{\sqrt{1+\frac{d_t}{d_0}}} \tag{10}$$

where $d_0 = 254$mm, if the transverse reinforcement is less than the minimum, or $d_0 = 2540mm.$ The proximity to the Type II size effect law (SEL), proposed by Bažant, is observed. The author had performed an analysis in energy terms through an asymptotic approach describing the transitional behavior between the plasticity theory and Linear Elastic Fracture mechanics [17]

Finally, the current Brazilian code, the ABNT: NBR 6118 [9] does not consider the size effect and is solely based on concrete resistance to compression. On the model I, a fixed angle truss model, the shear strength of the complementary mechanisms is calculated by the expression 11 in S.I. units:

$$V_c = 0{,}6 f_{ctd} b_w d \tag{11}$$

where f_{ctd} is the concrete tensile design resistance, calculated by Equation 12:

$$f_{ctd} = \frac{0{,}7 f_{ctm}}{\gamma_c} = \frac{0.7 f_{ctm}}{1.4} = 0.5 f_{ctm} \tag{12}$$

Where γ_c is the concrete compressive strength safety factor and f_{ctm} is the concrete tensile average resistance, calculated by (13) if the concrete has $f_{ck} < 55MPa$ or else (14):

$$f_{ctm} = 0{,}3(f_{ck})^{\frac{2}{3}} \tag{13}$$

$$f_{ctm} = 2{,}11\ ln(1 + 0{,}11 f_{ck}) \tag{14}$$

where f_{ck} is in MPa. As the safety factor is included in this analysis is not desirable that this prediction returns shear strength smaller than tests results.

4 MATERIALS AND EXPERIMENTAL PROGRAM

The survey carried out included 1356 beam tests from the ACI-DAfStb database, from the American and German committees. These data were obtained from several authors and initially filtered using the criteria set out in Reineck et al. [6], removing specimens with lack of information. The primary outputs were two sets: 1008 slender beams and 348 non-slender beams. The filters also ensure that the failure under analysis was due to shear, that the beams have the same type of anchorage in the longitudinal reinforcement and with a width greater than 5 cm.

Another additional filter was applied so that only concrete for structural purposes, with 20 MPa < f_{ck}< 100MPa, would be part of the set. Nonetheless, the North American code adopts the control of 10% chance of failure for this parameter, being f_c' the resistance meeting this criterion. The European and Brazilian codes, on the other hand, adopt 5%, and the f_{ck} is the strength fulfilling these criteria. Therefore, the value of f_c' was set to f_{ck} for both the filter and for the calculation of the Brazilian code, for this database. The conversion was the same performed by Reineck et al. [6] to comparisons with codes using similar control, calculated by Equation 15:

$$f_{ck} = f_c' - 1{,}6 \quad (15)$$

This equation is obtained considering a scatter of $\Delta f = 4\ MPa$ [6] to the same database. For instance, considering the cylinder compressive strength of 24th reference of Annex A, the Table 1 is obtained:

Table 1- Expression of Reineck et al. [6] applied to two specimens

Author	Specimen	$f_c'\ (MPa)$	$f_{ck}(MPa)$
Drangsholt, G.; Thorenfeldt, E. (1992)	B11	51,60	$f_{ck} = 51{,}6 - 1{,}6 = 50{,}00$
	B21	75,38	$f_{ck} = 75{,}38 - 1{,}6 = 73{,}78$

Moreover, the samples were restricted to data with *a/d* > 2.4 and beams with point loads. After this process, a database with 617 slender beams without stirrups with point loads used was obtained from 88 studies collected in the literature, according to Annex A.

4.1 Design Models

Having filtered data as input, the model codes in ACI 318 [3], ACI 318 [3], the Unified Approach [5], and ABNT: NBR 6118 [9] were used to calculate the shear strength of the complementary mechanism. Each of the models generates a prediction of the shear strength of the complementary mechanisms ($R = V_c$), by using data such as b_w, d, compressive strength, and ρ_L for each of the beams. Furthermore, each of these beams were tested until failure, and the database contains the ultimate strength of the test ($S = V_{test}$).

A satisfactory approach would have no tendencies, be as close to one as possible and have a small coefficient of variation. Additionally, when partial safety factors are considered, they should have an S/R>1 for most of the database, as shown in Reineck et al. [6], who used the 95% percentile for analysis of the ACI 318 [3]. This condition implies that the model predicted a lower resistance than measured in the test; therefore, the design would be reliable. Concomitantly S/R should not be much higher than 1, for project optimization. Henceforth, two upper limits (UL), establishing the percentiles of 5 and 10 of the results were set to analysis, and will be represented by a light blue dashed line (UL 5%) and a dark blue dashed line (UL 10%). This limit allows to identify optimum responses, i.e., closer to 1. The Figure 2 illustrates these concepts.

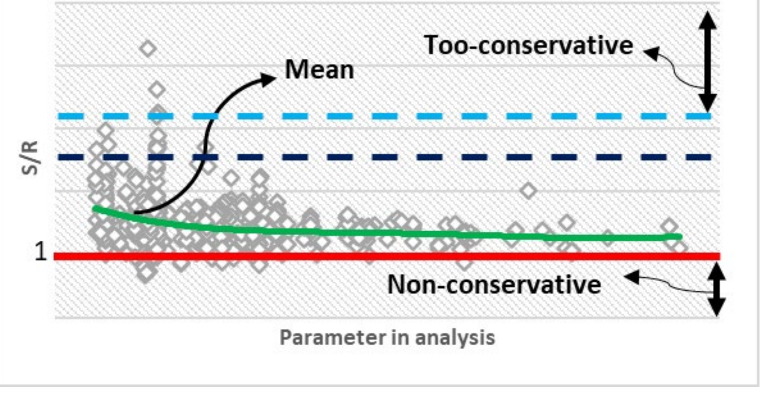

Figure 2- Dispersion points of analysis

5 RESULTS AND DISCUSSIONS

The distribution of the database through the analyzed parameter is in Figure 3, where the frequency per ranges of the parameters is exhibit:

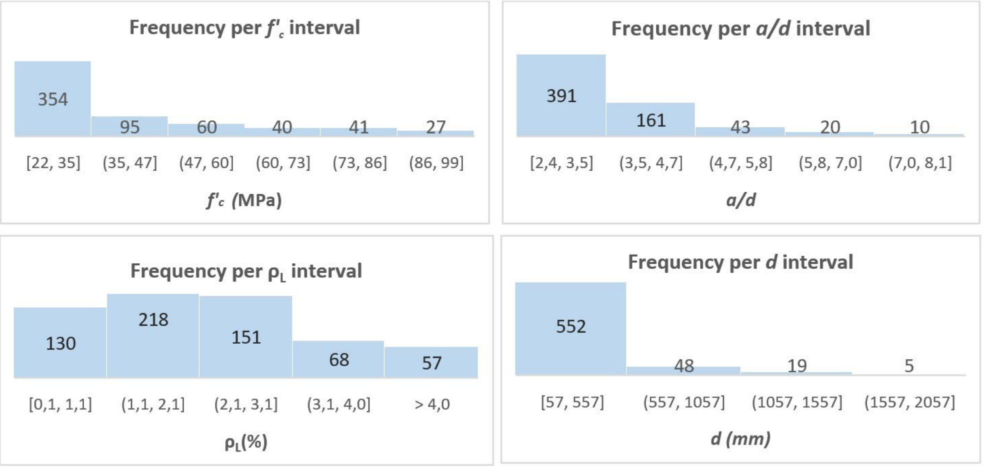

Figure 3- Distribution of the data

From the distribution, becomes clear the concentration of data in the first interval, except for ρ_L, associated with the technical difficulties with higher f_c', a/d, and d, hence, there is a need to expand the data to a better analysis. All the dispersion plots of S/R in relation to each of these parameters are in the following section. Additionally, in the Table 2 are shown the upper limits (UL) to both 5 and 10% limits, obtained by establishing limits to the dataset in such way that the UL fractile are reached.

Table 2- Upper Limits to 5 and 10%

Model	Upper Limit (5%)	Upper Limit (5%)
ACI 318-14	2,42	2,09
ACI 318-19	2,17	1,87
Frosch et al. [5]	2,37	1,97
NBR6118-14	2,61	2,20

The Unified Approach by Frosch et al. [5] and the Brazilian code present higher limits, which indicates more conservative results to the database. Furthermore, an expression related to mean and COV by the Equation 16 is proposed:

$$UL = \mu + kCOV \tag{16}$$

where μ is the mean and k is a factor related to the model sensitivity. The difference between UL and μ is the distance from the mean to the corresponding upper limit. Thus, k indicates a distance normalized by the COV. From Table 2 and Equation 16, considering the mean and COV obtained by the design codes model applied to the dataset (to be presented in sections 5.1 to 5.3), Table 3 is obtained:

Table 3- Upper Limits to 5 and 10%

Model	Upper Limit (5%)	Upper Limit (10%)
ACI 318-14	$UL = \bar{\mu} + 2.33COV$	$UL = \bar{\mu} + 1.50COV$
ACI 318-19	$UL = \bar{\mu} + 2.70COV$	$UL = \bar{\mu} + 1.50COV$
Frosch et al. [5]	$UL = \bar{\mu} + 3.14COV$	$UL = \bar{\mu} + 1.92COV$
NBR6118-14	$UL = \bar{\mu} + 2.51COV$	$UL = \bar{\mu} + 1.51COV$

The Table 3 shows the Frosh et al. [5] approach as the more conservative, followed by the ACI 318 [4], instead of ABNT NBR 6118 [9].

5.1 ACI 318

Initially the two models of the North American Code are considered. The mean of S/R of the ACI 318 [3] for the filtered database was 1.49 with COV=0.40. Also, 6.96% of the values that have S/R<0.75. In juxtaposition, the new code (ACI 318 [4]) had a mean of 1.48 with COV= 0.25 with 0.48% of the values, for which S/R<0.75. The values agree with Kuchma et al. [8] although he has applied different filters to the same set. Additionally, the Upper Limits to both 5 and 10 percentiles are smaller in the newest version; therefore, the model presents an optimized design. Accounting the multitude of studies accumulating data in each portion, the dispersion could be better evaluated per intervals.

5.1.1 Compressive Strength

Figure 4, exhibit ACI 318 [3] (yellow) and ACI 318 [4] (blue) regarding compressive strength. As shown in Figure 3, the interval from 20 to 40 MPa, comprehends most of the test results. Hence, is expected that standard deviation increases. However, is possible to see a strong trend of decreasing in S/R mean as compressive strength increases that is not related to this. The ranges concentrating most of the values against safety are 60-100 MPa, with higher COV and lower means. The standard under analysis stipulates, for the calculation of a maximum value for V_c of $f_c' = 82.76$MPa, which may help to reduce the variation associated with this parameter. Moreover UL_5 and UL_{10} delineates a more conservative model between 20 and 40 MPa. Although the newest code version has a smaller band of dispersion there are more results above UL_{10}.

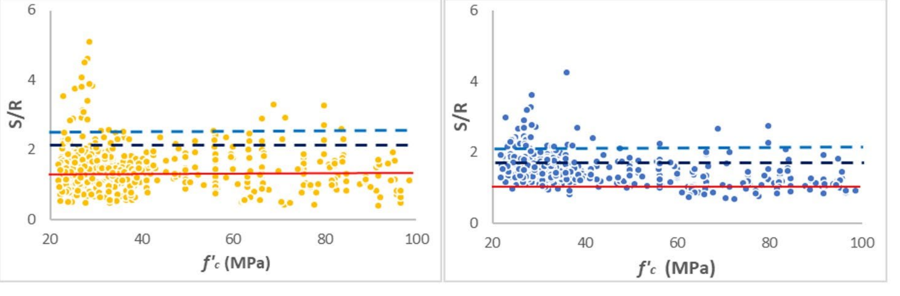

Figure 4- S/R in relation to f_c' to ACI 318 [3] (yellow) and ACI 318 [3] (blue)

The Table 4 expose in detail the tendencies of this dataset.

Table 4- ACI 318 [3] and ACI 318 [4] results

N	f_c' (MPa)	ACI 318 [3]			ACI 318 [4]		
		S/R<1 (%)	Mean	COV	*S/R*<1 (%)	Mean	COV
423	20-40	13.71	1.52	0.40	1.18	1.56	0.24
86	40-60	10.47	1.48	0.29	1.16	1.44	0.19
60	60-80	33.33	1.40	0.49	30,00	1.24	0.30
48	80-100	20.83	1.29	0.35	18.75	1.2	0.23

In the newest code version, a reduction in the S/R<1 values, is noted, with 33 tests for this formulation and only three, considering the factor φ=0.75 indicating both model guarantees the safety, and it can still be optimized. Lower COV's in all intervals, with an observed tendency to decrease with the increasing in compressive strength, is presented, even more accentuated than the previous formulation. Similar patterns are also observed in relation to the results with an S/R<1 in the intervals. Bažant et al. [18], observed the same trend, attesting the proportionality with $\sqrt{f_c}'$ as satisfactory, which Kuchma et al. [8] ratify, when analyzing this parameter in the new standard. Since both curves use this proportionality, the result reiterates this understanding.

5.1.2 Shear Span to Effective Depth Ratio

The Kani's Valley stats that when $a/d \geq 2.40$ the contributions of the shear transfer cross-section mechanisms are preponderant, linearly approaching itself to the prediction of the theory of plasticity as the ratio reaches values ranging from 6 to 8 [10], [12], [15], [16]. Hence, it is useful to group in intervals that may allow a better analysis among them. Figure 5 show the model error (S/R) in relation to span to depth ratio (*a/d*). This filtered dataset only has only beams with $a/d \geq 2.40$. There are no trends concerning the data in the ACI 318 [3]. In turn, the ACI 318 [4], presents a slight decrease trend. Finally, the UL_5 and UL_{10} demonstrates most of the highly conservative values located between $2.4 < a/d < 3$, where the shear transfer cross-sections mechanisms are preponderant, as stated in section 2.2. Notably, ACI 318 [4] is more conservative (more values above UL_{10}).

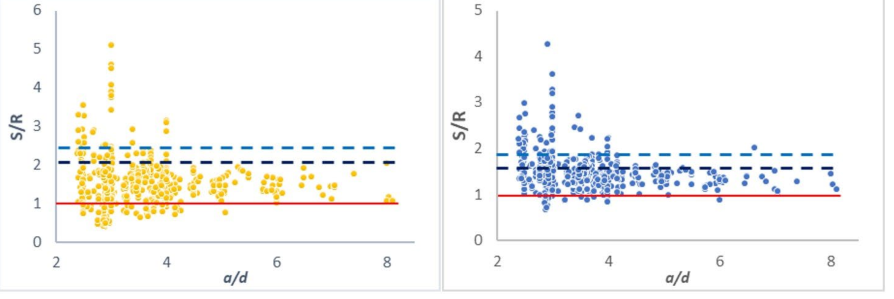

Figure 5 - S/R in relation to a/d to ACI 318 [3] (yellow) and ACI 318 [3] (blue)

The Table 5 shows the results obtained per interval in detail.

Table 5 – S/R x *a/d* results

N	a/d (-)	ACI 318 [3]			ACI 318 [4]		
		S/R<1 (%)	Mean	COV	*S/R*<1 (%)	Mean	COV
390	2.40 – 3.55	19.74	1.48	0.45	5.90	1.54	0.46
154	3.55 – 4.70	11.04	1.54	0.30	4.54	1.43	0.19
45	4.70 – 5.85	2.22	1.47	0.16	2.22	1.36	0.10
28	5.85 – 8.15	0.00	1.41	0.19	3.57	1.29	0.16

Two main trends appear from the analysis, i.e., the COV's are smaller for the ratios above 4.70 and most of results where S/R<1 is in the first intervals. Both have feasible explanations in the Kani's study. For higher values of *a/d*, the approximation of the current version leads to satisfactory results. Nonetheless, the first intervals, where the cross-section mechanisms govern the shear transfer, have the most significant fraction with S/R<1.

The simplified version approximates the *a/d* ratio through the Vd/M term, through the 0.166λ coefficient without significant changes. Additionally, the decreasing values of the mean in the new formulation, may be related to the consideration of longitudinal reinforcement by the term ρ_L, to be discussed in next section.

5.1.3 Longitudinal Reinforcement Ratio (ρ_L)

Figure 6 exhibit a clear tendency to increase in the S/R mean in the ACI 318 [3] that does not explicitly include the influence of this parameter on strength. To light reinforced beams parameters this design has most of the non-conservative values. Similarly, most of excessively conservative results lay on the beams with greater reinforcement ratios.

The ACI 318 [4] presents no notable trends regarding this parameter, indicating the introduction of ρ_L as an efficient way to correct this design. Furthermore, the upper limit to oldest version shows interval between 2 to 3%, holding most of the highest conservative values. However, there is a strong tendency which affect this analysis. The newest code is more conservative with no appreciable localized changes regarding UL_5 and UL_{10}. To analyze the changes in the database, it is useful to group through intervals, once more, obtaining the Table 6.

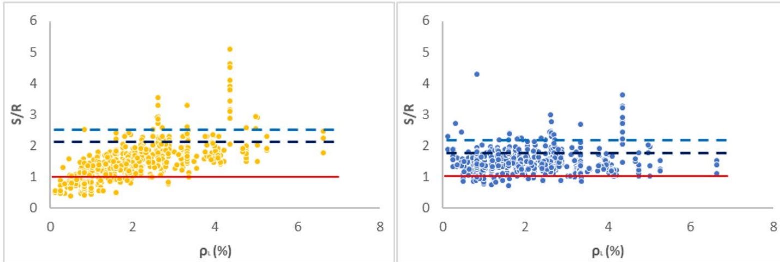

Figure 6- S/R in relation to ρ_L to ACI 318 [3] (yellow) and ACI 318 [3] (blue)

Table 6- S/R x ρ_L results

N	ρ_L (%)	ACI 318 [3]			ACI 318 [4]		
		S/R<1 (%)	Mean	COV	*S/R*<1 (%)	Mean	COV
168	0-1.30	50.00	1.00	0.30	10.71	1.41	0.20
232	1.30 – 2.60	4.74	1.47	0.23	4.74	1.47	0.21
154	2.60 – 3.90	1.32	1.75	0.26	2.63	1.50	0.24
68	3.90 – 6.70	0.00	2.29	0.39	0.00	1.62	0.39

The most of results against safety are found in lightly reinforced beams whereas higher COV's and predictions with higher means are observed for the experiments with higher ρ_w rates. When considering longitudinal reinforcement rate with the proportion $\sqrt[3]{\rho_L}$ directly, trends were not recognized. Lower COV's were obtained compared with the previous approach, although there is still a tendency for the S/R ratio to increase with the reinforcement ratio. In addition, lightly reinforced beams retain, proportionally, most of the designs against safety, and for higher reinforcement ratio, oversizing occurs, with higher COV’s.

This effect was studied by El-Ariss [19] who, when adjusting a numerical model, specifically for the contribution of the pin action of the longitudinal reinforcement, observed its contribution was essential to lightly reinforced beams, for a correct prediction, pointing the need to investigate how other parameters as compressive strength and bar diameter affected the contribution of this mechanism.

5.1.4 Effective Depth

The Figure 7 show a remarkable trend of mean decreasing as the effective depth increases to the ACI 318 [3]. In the turn, the new design code, significantly corrects the model error in first intervals, leading to smaller S/R, and increasing its value in the rest of dataset.

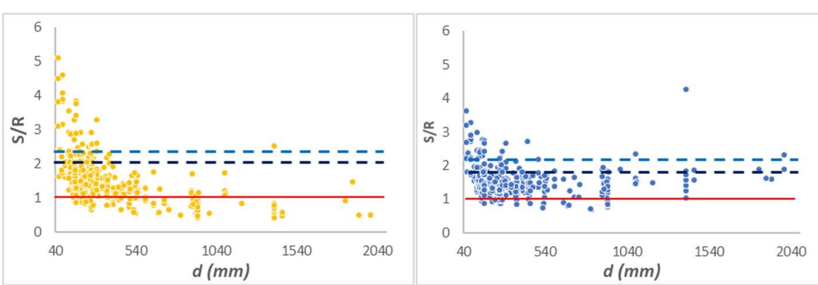

Figure 7- S/R in relation to d to ACI 318 [3] (yellow) and ACI 318 [3] (blue)

Table 7 exhibit in detail the trends regarding the beam depth. There is an increase in predictions against safety (S/R<1) with the increase in the height of the beams under analysis, which together with the COV’s in the settled intervals, attest the reliability of the trend. Regardless, in the ACI 318 [4] there is a significant reduction in data with inadequate design, with a lower mean of 1.31 in the intervals taken, with lower coefficients of variation, indicating a good fit through the adoption of the factor for the size effect. The proposed upper limits delineate the most conservative values in the smaller beam depth to both design code. Even though the ACI 318 [4] is more conservative there were small changes regarding the values above UL_5 and UL_{10}, but with less tendencies.

Table 7- S/R x d results to ACI 318

N	d (mm)	ACI 318 [3]			ACI 318 [4]		
		S/R<1 (%)	Mean	COV	S/R<1 (%)	Mean	COV
225	0-250	1.78	1.86	0.35	2.22	1.58	0.28
298	250-500	10.40	1.40	0.27	4.36	1.42	0.19
37	500-750	48.65	1.09	0.34	18.92	1.39	0.25
33	750-1000	81.82	0.78	0.37	27.27	1.31	0.28
24	1000-2000	75.00	0.78	0.39	4.17	1.61	0.18

Bažant's approach, which was adopted in the new version of the code, performs an asymptotic analysis of concrete, as a quasi-brittle material, between the constant resistance prescribed by the theory of plasticity, and the Linear Elastic Fracture Mechanics (LEFM), which claims the inelastic process zone as negligible compared to cross-section dimensions.

Since even for data with dimensions of the order of 2 m in height, good fits were obtained by applying the factor, the FPZ did not become negligible for the range adopted, with a transitional formulation between the LEFM and the plasticity theory being adequate.

5.2 Unified Approach

Using the proposed expression of Frosch et al. [5], the mean to the database was 1.59 with COV=0.27. Only eight results had S/R$\leq$ 1 and one bellow 0.75. This proposal was calibrated to this database on Frosch et al. [5]. The model has the highest upper limit in the analysis with the more conservative design.

5.2.1 Compressive Strength (f'_c)

The Figure 8 show S/R in relation to f'_c to this approach. First, no trends are noted, and the approach is the more conservative. This is also corroborated by the upper limits, which shows more values above them across the intervals. However, the same interval (20-40 MPa), still has the most values above the proposed upper limits. A more detailed analysis is possible by filtering the data similarly to the last section, as show in Table 8.

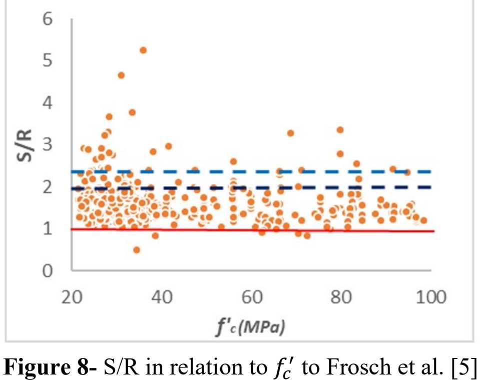

Figure 8- S/R in relation to f'_c to Frosch et al. [5]

Table 8- S/R x f'_c to the Unified Approach

N	f'_c (MPa)	S/R<1 (%)	Mean	COV
423	20-40	0.44	1.61	0.28
86	40-60	0.00	1.59	0.20
60	60-80	10.00	1.50	0.35
48	80-100	0.00	1.52	0.21

The COV's have more uniformity among the groups and the same decreasing trend whereas f_c' increases. The formulation remains satisfactorily in favor of safety along the compressive strengths. Since f_c' is considered with the same proportion of the previous code, considering the depth of the uncracked compressed zone and the scale effect may be one of the generators of the distinction among the analyzed data.

5.2.2 Shear Span to Effective Depth Ratio

Figure 9 shows S/R in relation to *a/d*. Once more, no strong trends are present. This is expected because this parameter is not directly considered. However, the dispersion band and S/R through the intervals are different. Thus, the Table 9 show the dataset information in detail. Once more the upper limits allow to demonstrate that the highest values are between 2.0 to 3.0 as explained in section 2.2.

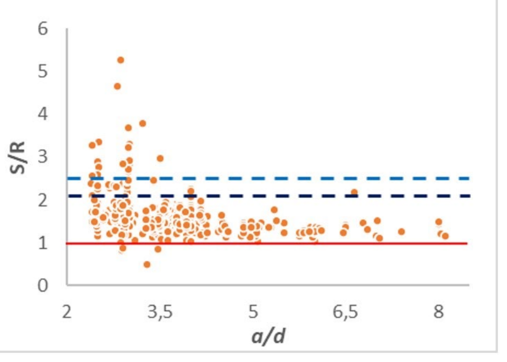

Figure 9- S/R in relation to a/d to Frosch et al. [5]

Table 9- S/R x *a/d* to the Unified Approach

N	*a/d* (-)	S/R<1 (%)	Mean	COV
392	2.40 – 3.55	0.26	1.84	0.25
154	3.55 – 4.70	0.00	1.51	0.16
47	4.70 – 5.85	0.00	1.43	0.12
25	5.85 – 8.15	0.00	1.45	0.20

In general, there is a reduction in COV's while *a/d* ratio approaches itself to the right level of the Kani valley. Muttoni and Ruiz Fernandez [10] obtained good adjustments for the ranges 2.47 to 3.0, extended in Ruiz Fernandez et al. [2] to 4.5, considering the contribution of the shear transfer mechanisms of the cross-sections.

In juxtaposition, the formulation by Frosch et al. [5] is close to these authors, considering the depth of the cracked compression zone, calculated to contemplate the higher rigidity of the reinforcement and consequent alteration of the compressed zone and obtained designs with satisfactory performance in all ranges.

5.2.3 Longitudinal Reinforcement Ratio (ρ_L)

Figure 10 shows the S/R in relation to ρ_L to this approach. Notably, no trends are present, and the results are dispersed in a more uniform manner in a smaller band, over all the dataset. The same interval (2 to 3%) has most of the values above the upper limits.

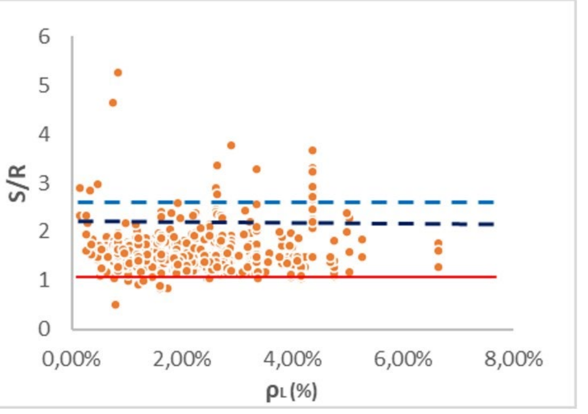

Figure 10- S/R in relation to ρ_L to Frosch et al. [5]

Dividing in intervals by the same criteria, Table 10 is obtained:

Table 10 – S/R x ρ_L to the Unified Approach

N	ρ_L (%)	S/R <1 (%)	Mean	COV
168	0-1.30	2.98	1.61	0.30
232	1.30 – 2.60	1.29	1.54	0.19
152	2.60 – 3.90	0.00	1.57	0.28
65	3.90 – 6.70	0.00	1.72	0.37

In the intervals taken there are no observable trends; therefore, there is a correct consideration of the term, although higher COVs are still observed for higher rates of longitudinal reinforcement. The dispersion band is the smallest among the considered design to this dataset, nevertheless, this approach also has the more conservative approach.

5.2.4 Effective Depth

Figure 11 shows S/R in relation to d to this approach. No trends until d is over 1000mm where an increasing trend occurs. This dispersion is like ACI 318 [4] but is more conservative, mainly to the smaller beam depths. Even after the size affect factor, the smallest beams depth concentrates most of the values above UL_5 and U_{10}. This may be related to correlation between longitudinal reinforcement and size effect, which was not considered on this approach. Utilizing the same intervals to this parameter to the previous code the Table 11, is obtained to analyze in detail.

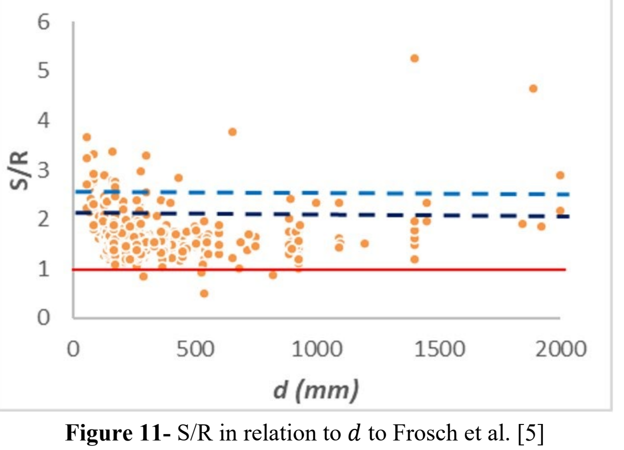

Figure 11- S/R in relation to d to Frosch et al. [5]

Table 11 – S/R x d to the Unified Approach

N	d (mm)	S/R <1 (%)	Mean	COV
225	0-250	0.00	1.70	0.25
298	250-500	0.34	1.48	0.20
37	500-750	10.81	1.49	0.32
33	750-1000	9.09	1.50	0.24
24	1000-2000	0.00	2.06	0.47

Good adequacy is denoted by the smaller dispersion band, by the mean of S/R demonstrating results closer to those tested and with less variation for the distinct ranges, with adequate values for practical purposes.

The size effect is also calculated with $\alpha \times \ d^{1/2}$, but there is a difference. The transitional dimension d_0 is a function of the ZPF, and it is sensitive to the inhomogeneities of the material [20]. In this model, even when a transversal reinforcement greater than the minimum is provided, a size effect could be applied if $d \geq 2{,}54m$, i.e., a suppression to size effect may occur, but it will not become negligible as the height increases. The adjusted value for the database under analysis provides more conservative results for the lower range, and the with adequate adjustment for higher beams.

5.3 ABNT: NBR 6118 [9]

The application of the Model I formulations of the Brazilian standard provides a mean of 1.58 with COV= 0.42 and 14.10% with S/R<1. Moreover, the upper limit to the fractile of 5 ($UL_5 = 2{,}61$), is the most conservative among all the analyzed approaches.

5.3.1 Compressive Strength (f_{ck})

It is important to comprehend that f_{ck} is different from f_c' (which is related to the 10%) control of acceptable results. The same trends concerning the highest conservative models are obtained to this model, i.e., on the range 20 to 40 MPa. The results obtained after the calculations are exposed in the Figure 12. The S/R in relation to f_c' have a similar trend to ACI 318 [3], i.e., there is a decrease trend over the dataset, slightly more prominent. The Table 12 shows the dataset results in detail.

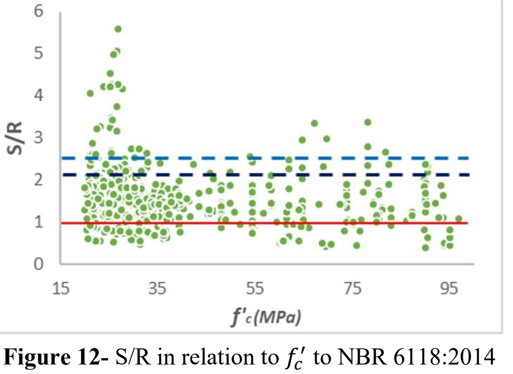

Figure 12- S/R in relation to f_c' to NBR 6118:2014

Table 12 – S/R x f_{ck} to NBR 6118: 2014

N	f_{ck} *(MPa)*	S/R <1 (%)	Mean S/R	COV
423	20-40	11.35%	1.65	0.41
86	40-60	11.63%	1.48	0.29
60	60-80	31.67%	1.43	0.32
48	80-100	22.92%	1.29	0.37

Eighty-seven of S/R results are below 1. As the NBR factor of 1.4 was considered, the ϕ=0.75 usual to American codes are not considered. In addition, there is a high dispersion, which is expected, based on what was previously discussed for the American code ACI 318 [3], i.e., the contribution of concrete calculated by Model I of the code does not consider the size effect, nor the change of the rate of longitudinal reinforcement. The proportionality to V_c with the compressive strength is calculated by $\sqrt[3]{f_{ck}^2}$ for values below 55 MPa and by a function of the natural logarithm for higher values, distinct from the previous codes.

In the initial ranges there are fewer predictions against safety, compared with the ACI code 318 [3], with non-negligible coefficients of variation. The trend with increasing resistance is also towards a reduction in the S/R factor, as shown by the increase in the S/R<1 column and decrease in the mean. When compared with both ACI 318 [4] and the Unified Approach, the trends regarding this parameter are similar.

5.3.2 Shear Span to Effective Depth Ratio (*a/d*)

The Figure 13 shows S/R in relation to *a*/d. First, no trends are presented in the dispersion, being the differences observed in the previous approaches in the COV`s observed, as well as the dispersion band of the dataset.

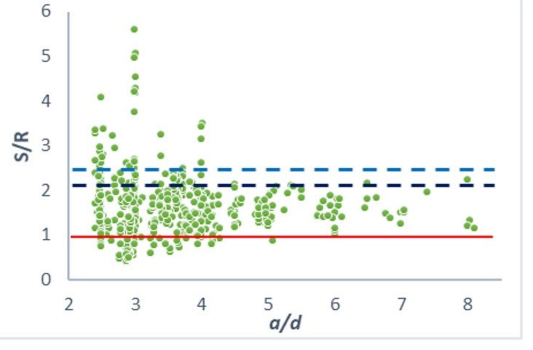

Figure 13- S/R in relation to a/d to NBR 6118:2014

Considering the explained intervals, the Table 13 is obtained. Where the Brazilian code shows the same behavior presented in the ACI 318 [3], i.e., no trends concerning the mean, most of the results with S/R<1 are located next to inflection point of Kani's valley and the lesser COV's are also in the higher *a/d* values. Hence, the influence of not considering *a/d* appears increasing the variance, and consequently the fraction to which S/R<1 near to a $a/d = 2.4$.

Table 13 - S/R x *a/d* to NBR 6118: 2014

N	$\boldsymbol{a/d}$ (-)	S/R <1 (%)	Mean	COV
392	2.40 – 3.55	17.44	1.58	0.48
154	3.55 – 4.70	10.39	1.61	0.31
47	4.70 – 5.85	2.22	1.61	0.17
25	5.85 – 8.15	0.00	1.55	0.20

5.3.3 Longitudinal Reinforcement Ratio (ρ_L)

The results of S/R in relation to ρ_L, to Brazilian code, are in the Figure 14. The current Brazilian design code has most of the values to which S/R<1 to light reinforced beams, with a strong increase trend. The same pattern was observed and analyzed in the ACI 318 [3], pointing some similar correction to this trend may be effective. Simultaneously, the range of 2 to 3% has most of the values above the proposed upper limits. Alternatively, the unified approach proposal [5], could be use. Nevertheless, this change implies in considering the depth of the compressed zone, a substantial transition from our current design.

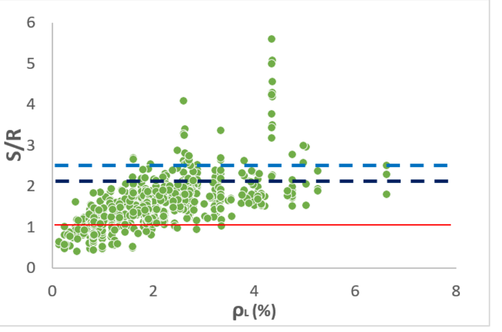

Figure 14- S/R in relation to ρ_L to NBR 6118:2014

The details of this dispersion are shown in the Table 14. Once more, the trends of this code are like ACI 318 [3], with the most results to which model error are below 1 regarding longitudinal reinforcement occurring for the light reinforced beams and the excessively safe outcomes in the higher ρ_L.

Table 14 - S/R x ρ_L to NBR 6118: 2014

N	ρ_L (%)	S/R <1 (%)	Mean	COV
168	0-1.30	44.05	1.05	0.32
232	1.30 – 2.60	5.17	1.54	0.25
152	2.60 – 3.90	1.32	1.87	0.27
65	3.90 – 6.70	0.00	2.42	0.42

Samora et al. [21] state that, from tests like those contained in this database, within the same range of compressive strength, for the lowest rates of longitudinal reinforcement, there was a greater contribution of the other complementary mechanisms, increasing with the increase in strength in compression and decreasing with the diameter of the bar.

An explanation is based on the study of Krefeld and Thurston [22], which is also used by Ruiz Fernandez et al. [2] in the model incorporated in the Swiss standard, Critical Shear Crack Theory (CSCT), which considers other parameters such as spacing between bars of reinforcement, diameter of bars, concrete tensile strength, and deformations in the reinforcement, obtaining adequate fits. Hence, the formulations under analysis presenting a direct proportionality only with the rate of reinforcement may underestimate the contribution of this mechanism, which would induce high *S/R* values.

5.3.4 Effective Depth

The Figure 15 exhibit S/R in relation to d for the Brazilian code. A strong decrease trends, like ACI 318 [3] occurs, i.e., excessively conservative design for shallow beams, decreasing until non conservative results to higher beam depth. The excessively conservative design occurs to smallest beams depth. Once more, as the trends are near the old north American code, the incorporation of a size effect factor could result in a more reliable design.

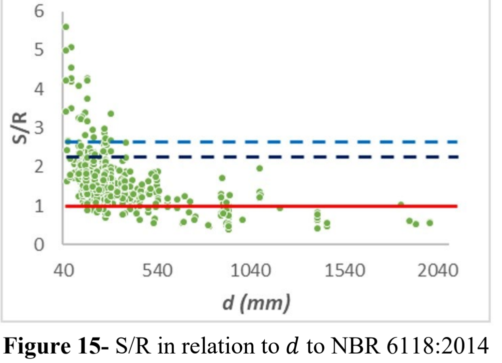

Figure 15- S/R in relation to d to NBR 6118:2014

Finally, disposing the results to analysis in the same intervals, Table 15 is obtained. There is a clear trend towards a reduction in the S/R mean with increasing height, with COV`s in the same range as those found in the ACI 318 [3]

There is a gradual increase in predictions against safety with the effective depth, juxtaposed with lower means, even below 1, for the last range. This parameter, together with the longitudinal reinforcement rate, are the strongest influences, which could provide optimized dimensions if adjusted to the Brazilian standard.

Table 15 - S/R x d to NBR 6118 [9]

N	d (mm)	S/R <1 (%)	Mean	COV
225	0-250	2.22	1.96	0.38
298	250-500	9.40	1.49	0.28
37	500-750	37.84	1.14	0.38
33	750-1000	75.76	0.80	0.39
24	1000-2000	70.83	0.85	0.42

Considering that increasing higher beams depths are being used in engineering practice there is a need to correct this trends that may lead to unsafe design conditions to higher beam depths. A correction may be realized in two steps:

1- A minimum square regression using a power law (like ACI 318 [4]) to a ρ_L factor;
2- A linear regression to dataset after (1), after applying a transformation in d using size effect law.

These steps result in:

$$V_c = 5.2\gamma_e f_{ctd} b_w d \rho_L^{0.44} \tag{17}$$

where γ_e is:

$$\gamma_e = \sqrt{\frac{1,53}{1+\frac{d}{254}}} \tag{18}$$

Leading to Figure 16, where S/R is represented as Model Error ***(ME)***. No notable trends are present, pointing to a better approach, which still must calibrate its partial safety factors. The purple line represents UL_{10} to this model in $UL_{10} = 1.60$. The fitting curve and the analysis of this model is still in study.

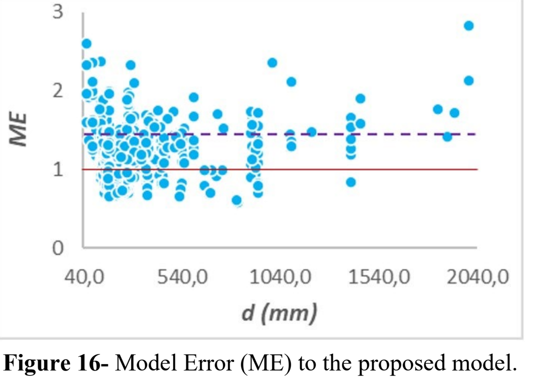

Figure 16- Model Error (ME) to the proposed model.

6 CONCLUSIONS

The analysis of the selected parameters has allowed to outline trends, as well as to analyze sources of dispersion of predictions not only in a global manner, but in localized phenomena, e.g., the a/d influence over the model error, and the tendencies regarding longitudinal reinforcement ratio. In Annex B there is a summary of the main conclusions obtained through this study

When comparing the ACI 318 [3] and ACI 318 [4], the non-consideration of the size effect and the longitudinal reinforcement ratio directly generates a larger dispersion band and non-conservative results for higher depths and lightly reinforced concrete beams, respectively. Therefore, the benefit of incorporating these parameters is notorious. The improved observed behavior with the proper consideration of these effects emerges even clearer when the Unified Approach by Frosh et al. [5] is introduced. In this model, the calculation considering the concrete and reinforcement stiffnesses, also the reinforcement ratio through the depth of compression zone, leads to the best results. However, it was also the more conservative model.

Finally, the ABNT: NBR 6118 has provided results like the North American previous code (ACI 318 [3]), which suggests the benefits of incorporating the size effect and the urgency to adapt to the Brazilian standard, considering the increasing dimensions of the structural elements used currently. The joint exposure of American codes (previous and current version) to the filtered database provides an overview of the changes generated by applying ρ_L and λ_s factor, which provide a basis for the analysis of the current Brazilian code. As a suggestion, two steps adjust is suggested considering the obtained results. The results briefly introduced exhibits the proposed model reduced tendencies and may be calibrated to the targeted safety, which is still in study.

REFERENCES

[1] American Concrete Institute, *Recent Approaches to the Shear Design of Structural Concrete*, ACI-ASCE Committee 445R, 1999, 55 p. (reapproved 2015).

[2] M. Fernández Ruiz, A. Muttoni, and J. Sagaseta, "Shear strength of concrete members without transverse reinforcement: a mechanical approach to consistently account for size and strain effects," *Eng. Struct.*, vol. 99, pp. 360–372, 2015.

[3] American Concrete Institute, *Building Code Requirements for Structural Concrete*, ACI Committee 318, 2014, 519 p.

[4] American Concrete Institute, *Building Code Requirements for Structural Concrete*, ACI Committee 318, 2019, 629 p.

[5] R. J. Frosch et al., "A unified approach to design," *Concr. Int.*, no. September, pp. 147–158, 2017.

[6] K. H. Reineck et al., "ACI-DAfStb database of shear tests on slender reinforced concrete beams without stirrups," *ACI Struct. J.*, vol. 110, no. 5, pp. 867–876, 2013.

[7] K. H. Reineck, E. Bentz, B. Fitik, D. A. Kuchma, and O. Bayrak, "ACI-DAfStb database of shear tests on slender reinforced concrete beams with stirrups," *ACI Struct. J.*, vol. 110, no. 5, pp. 1, 2014.

[8] D. A. Kuchma, S. Wei, D. H. Sanders, A. Belarbi, and L. C. Novak, "Development of the one-way shear design provisions of ACI 318-19 for reinforced concrete," *ACI Struct. J.*, vol. 116, no. 4, pp. 285–296, 2019.

[9] Associação Brasileira de Normas Técnicas, *Design of Concrete Structures: Procedures*, NBR 6118, 2014.

[10] A. Muttoni and M. Ruiz Fernandez, "Shear strength of members without transverse reinforcement as function of critical shear crack width," *ACI Struct. J.*, vol. 105, no. 2, pp. 163–172, 2008.

[11] F. Cavagnis, J. T. Simões, M. F. Ruiz, and A. Muttoni, "Shear strength of members without transverse reinforcement based on development of critical shear crack," *ACI Struct. J.*, vol. 117, no. 1, pp. 103–118, 2020.

[12] G. N. Kani, "The riddle of shear failure and its solution," *ACI J. Proc.*, vol. 61, no. 4, pp. 441–468, 1964.

[13] Y. Sato, T. Tadokoro, and T. Ueda, "Diagonal tensile failure mechanism of reinforced concrete beams," *J. Adv. Concr. Technol.*, vol. 2, no. 3, pp. 327–341, 2004.

[14] M. Fernández Ruiz, A. Muttoni, and J. Sagaseta, "Shear strength of concrete members without transverse reinforcement: a mechanical approach to consistently account for size and strain effects," *Eng. Struct.*, vol. 99, pp. 360–372, 2015.

[15] F. Cavagnis, "Shear in reinforced concrete without transverse reinforcement: from refined experimental measurements to mechanical models (8216)," M.S. thesis, Ec. Polytech. Fed. Lausanne, Suisse, 2017, 223 p.

[16] F. Leonhardt and R. Walther, "The Stuttgart shear tests" *Beton Stahlbetonbau*, vol. 56, no. 12, 1961.

[17] Q. Yu, J.-L. Le, M. H. Hubler, R. Wendner, G. Cusatis, and Z. P. Bažant, "Comparison of main models for size effect on shear strength of reinforced and prestressed concrete beams," *Struct. Concr.*, vol. 17, no. 5, pp. 778–789, 2016.

[18] Z. P. Bažant et al., "Justification of ACI 446 proposal for updating ACI code provisions for shear design of reinforced concrete beams," *ACI Struct. J.*, vol. 105, no. 4, pp. 512–515, 2008.

[19] B. El-Ariss, "Behavior of beams with dowel action," *Eng. Struct.*, vol. 29, no. 6, pp. 899–903, 2007.

[20] C. G. Hoover and Z. P. Bažant, "Cohesive crack, size effect, crack band and work of fracture compared to comprehensive concrete fracture tests," *Int. J. Fract. Mech.*, vol. 187, no. 1, pp. 133–143, 2014.

[21] M. S. Samora, A. C. D. Santos, L. M. Trautwein, and M. G. Marques, "Análise experimental da contribuição do concreto na resistência ao cisalhamento em vigas sem armadura transversal," *Rev. IBRACON Estrut. Mater.*, vol. 10, no. 1, pp. 160–172, Feb 2017.

[22] W. Krefeld and C. W. Thurston, "Contribution of longitudinal steel to shear resistance of reinforced concrete beams," *ACI Struct. J.*, no. 63, pp. 325–344, 1966.

Author contributions: JRCP: conceptualization, supervision, writing, methodology. TNB: Conceptualization, data curation, formal analysis, writing, methodology. IJSR: Conceptualization, data curation, formal analysis, writing, methodology

Editors: Edgar Bacarji, Guilherme Aris Parsekian.

ANNEX A

Data for Concrete Beams Without Stirrups

1. Adebar, P.E. (1989): Shear Design of Concrete Offshore Structures. A Thesis submitted in conformity with the requirements for the Degree of Doctor of Philosophy, University of Toronto 1989

2. Adebar, P.; Collins, M.P. (1994): Shear design of concrete offshore structures. ACI Structural Journ., V. 91, No. 3, May-June 1994, 324-335

3. Adebar, P.; Collins, M.P. (1996): Shear Strength of members without transverse reinforcement. Canadian Journal of Civil Engineering 23 (1996), No. 1, 30-41

4. Ahmad, S.H.; Khaloo, A.R.; Poveda, A. (1986): Shear Capacity of Reinforced High-Strength Concrete Beams. ACI-Journ. V.83 (1986), März/April, 297-305

5. Ahmad, S.H.; Park, F.; El-Dash, K. (1995): Web reinforcement effects on shear capacity of reinforced high-strength concrete beams. Magazine of Concrete Research 47 (Sep.1995), No. 172, 227-233

6. Al-Alusi, A.F. (1957): Diagonal tension strength of reinforced concrete T-beams with varying shear span, ACI Journal, May 1957, S. 1067-1077

7. Angelakos, D.; Bentz, E.C.; Collins, M.P. (2001): Effect of Concrete Strength and Minimum Stirrups on Shear Strength of Large Members. ACI Structural Journ. V.98 (2001), No.3, May/June, 290-300

8. Aster, H.; Koch, R. (1974): Schubtragfähigkeit dicker Stahlbetonplatten. BuStb 69 (1974), H.11, 266-270 Baldwin, J.W.; Viest, I.M. (1958): Effect of Axial Compression on Shear Strength of Reinforced Concrete Frame Members. ACI-Journ. V.30 (1958), Nov., 635-654

9. Bazant, Z.P., Kazemi, M.T. (1991): Size effect on diagonal shear failure of beams without stirrups.ACI-Struct. Journ. V.88 (1991), May-June, 268-276

10. Bentz, E. C.; Buckley, S. (2005): Repeating a classic set of experiments on size effect in shear of members without stirrups. ACI Structural Journal 102 (2005), Nov.-Dec., 832-838

11. Bernander, K. (1957): An investigation of the shear strength of concrete beams without stirrups or diagonal bars. RILEM-Symp., Stockholm, Vol.1, 1957

12. Bernhardt, C.J.; Fynboe, C.C. (1986): High strength concrete beams. Nordic Concrete Research, Norske Betongforening, Oslo 1986

13. Bhal, N.S. (1968): Über den Einfluß der Balkenhöhe auf die Schubtragfähigkeit von einfeldrigen Stahlbetonbalken mit und ohne Schubbewehrung. Otto-Graf-Institut, H.35, Stuttgart, 1968

14. Birrcher, D.; Tuchscherer, R.; Huizinga, M.; Bayrak, O.; Wood, S.; Jirsa, J. (2009): Strength and serviceability design of reinforced concrete deep beams. CTR Technical Report 0-5253-1, Center for Transportation Research, The University of Texas at Austin, April 2009. pp. 400

15. Bresler, B.; Scordelis, A.C. (1963): Shear strength of reinforced concrete beams. ACI-Journ. V.60 (1963), No.1, 51-74

16. Cladera Bohigas, A. (2002): Shear design of reinforced high-strength concrete beams. Dr.-thesis, Dep. dÉnginyeria de la Construcció, Universitat Politécnica de Catalunya, Barcelona, December 2002. pp. 168 and pages A.1-A.96, B.1-B3, C.1-C12, D.1-D.2; E.1-E.6

17. Cladera, A.; Marí, A.R. (2002): Shear Strength of Reinforced High –Strength and Normal –Strength Concrete Beams. A New Simplified Shear Design Method. Universitat Politécnica de Catalunya, 1-19, 2002

18. Cao, Shen (2000): Size Effect and the Influence of Longitudinal Reinforcement on the Shear Response of Large Reinforcement Concrete Members. A Thesis in conformity with the requirements for the degree of Master of Applied Science Graduate Department of Civil Engineering University of Toronto, 2000

19. Cederwall, K.; Hedman, O.; Losberg, A. (1970): Shear strength of partially prestressed beams with pretensioned reinforcement of high-grade deformed bars. Division of concrete structures, Chalmers University of Technology, Gothenburg, Sweden, Publikation 70/6

20. Cederwall, K.; Hedman, O.; Losberg, A. (1974): Shear strength of partially prestressed beams with pretensioned reinforcement of high-grade deformed bars. SP 42 - 9

21. Chana, P.S. (1981): Some aspects of modelling the behaviour of reinforced concrete under shear loading. Techn. Report 543, Cement and Concrete Association, Wexham Springs, 1981

22. Chang, T.S.; Kesler, C.E. (1958): Static and fatigue strength in shear of beams with tensile reinforcement, ACI- Journal, June 1958

23. Diaz de Cossio, R.; Siess, C.P. (1960): Behavior and strength in shear of beams and frames without web reinforcement. ACI-Journ. V.31 (1960), No.8, Feb., 695-735

24. Drangsholt, G.; Thorenfeldt, E. (1992): High Strength Concrete. SP2 – Plates and Shells. Report 2.1, Shear Capacity of High Strength Concrete Beams. SINTEF Structural Engineering – FCB, August 1992, STF70 A92125

25. Elzanaty, A.H.; Nilson, A.H; Slate, F.O. (1986): Shear Capacity of Reinforced Concrete Beams Using High-Strength Concrete. ACI-Journ.V.83 (1986), No.2, March-April, 290-296

26. Feldman, A.; Siess, C.P. (1955): Effect of Moment Shear Ratio on Diagonal Tension Cracking and Strength in Shear of Reinforced Concrete Beams. Univ. of Illinois Civil Eng. Studies, Struct. Research Series No. 107, 1955

27. Ferguson, P.M.; Thompson, J.N. (1953): Diagonal Tension in T-Beams without stirrups. Journal of the American Concrete Institute (Mar. 1953), S. 655-676

28. Ferguson, P. M. (1956): Some Implications of Recent Diagonal Tension Tests. ACI Journal V. 28, No. 2, Aug. 1956

29. Ghannoum, W.M. (1998): Size Effect on Shear Strength of Reinforced Concrete Beams. Department of Civil Engineering and Applied Mechanics, McGill University Montréal, Canada, November 1998

30. Grimm, R. (1996/97): Einfluß bruchmechanischer Kenngrößen auf das Biege- und Schubtragverhaltenhochfester Betone. Diss., Fachb. Konstr. Ingenieurbau der TH Darmstadt, 1996 und DafStb H.477, Beuth Verlag GmbH, Berlin 1997

31. Haddadin, M.J., Hong, S., Mattock, A.H. (1971): Stirrup Effectiveness in Reinforced Concrete Beams with Axial Force. Proceedings ASCE; V.97 ST9 (Sept. 1971), 2277-2297

32. Hallgren, M. (1994): Flexural and Shear Capacity of Reinforced High Strength Concrete Beams without Stirrups. KTH, Stockholm, TRITA-BKN. Bull.9, 1994, 1-49

33. Hallgren, M. (1996): Punching shear capacity of reinforced high strength concrete slabs. Doctoral thesis, KTH Stockholm und TRITA-BKN: Bulletin 23, Stockholm, 1996

34. Hamadi, Y.D. (1976): Force transfer across cracks in concrete structures. PhD-thesis, Polytechnic of Central London, 1976

35. Hanson, J.A. (1958): Shear Strength of Ligthweight Reinforced Concrete Beams. ACI, Title No. 55-24, (1958), 387- 403

36. Hanson, J.A. (1961): Tensile Strength and Diagonal tension resistance of Structural lightweight Concrete. ACI-Journ. V.58 (1961), No.1, July, 1-39

37. Injaganeri, S. S. (2007): Studies on size effect on design strength and ductility of reinforced concrete beams in shear. Thesis Doctor of Philosophy, Indian Institute of Technology, April 2007

38. Islam, M.S.; Pam, H.J.; Kwan, A.K.H. (1998): Shear Capacity of high-strength concrete beams with their point of inflection within the shear span. Proc. Instn Civ. Engrs Structs & Bldgs. 128 (Feb. 1998), 91-99

39. Johnson, M.K.; Ramirez, J.A. (1989): Minimum Shear Reinforcement in Beams with Higher Strength Concrete. ACI-Struct. Journ. V.86 (1989), No.4, 376-382

40. Kani, G.N.J. (1967): How safe are our large reinforced concrete beams? ACI-Journ. V.64 (1967), No.3, 128-141. Disc. in ACI-Journ., Sept. 1967, 602-613

41. Kani, M.W.; Huggins, M.W.; Wittkopp, R.R. (1979): Kani on shear in reinforced concrete. Dep. of Civil Engineering, Univ.of Toronto 1979

42. Kawano, H.; Watanabe, H. (1998): Shear strength of reinforced concrete columns – Effect of specimen size and load reversal. 141-154 in: Concrete Under Severe Conditions 2. Proceedings of the 2nd International Conference on CONSEC '98. Vol. III. Editors: Gjørv, O.E.; Sakai, K., Banthia, N. (1998), E & FN Spon, London and New York

43. Kim, J.-K.; Park, Y.-D. (1994): Shear strength of reinforcement high strength concrete beams without web reinforcement. Magazine of Concrete research 46 (1994), No. 166, 7-16

44. Krefeld, W.J.; Thurston, Ch.W. (1966): Studies of the shear and diagonal tension strength of simply supported r.c.-beams. ACI-Journ. V.63 (1966), April, 451-475

45. Kuhlmann, U. Ehmann, J. (2001): Versuche zur Ermittlung der Querkrafttragfähigkeit von Verbundplatten unter Längszug ohne Schubbewehrung- Versuchsbericht. Institut für Konstruktion und Entwurf Stahl-, Holz-, und Verbundbau, Universität Stuttgart, Nr. 2001- 6X, Februar 2001

46. Kuhlmann, U.; Zilch, K.; Ehmann, J.; Jähring, A.; Spitra, F. (2002): Querkraftabtragung in Verbundträgern mit schlaff bewehrter und aus Zugbeanspruchung gerissener Stahlbetonplatte ohne Schubbewehrung- Mitteilungen. Institut für Konstruktion und Entwurf Stahl-, Holz-, und Verbundbau, Universität Stuttgart, Nr. 2002- 2

47. Kulkarni, S.M.; Shah, S.P. (1998): Response of reinforced Concrete Beams at High Strain Rates. ACI Structural Journal V.95 (Nov.-Dec. 1998), No. 6, 705-714

48. Laupa, A.; Siess, C.P.; Newmark, N.M. (1953): The shear strength of simple-span reinforced concrete beams without web reinforcement. Univ. of Illinois, Struct. Research Series, No.52, 1953

49. Leonhardt, F.; Walther, R. (1962): Schubversuche an einfeldrigen Stahlbetonbalken mit und ohne Schubbewehrung. DAfStb H.151, Berlin, 1962

50. Lubell, A.; Sherwood, T.; Bentz, E.; Collins, M. (2004): Safe Shear Design of Large Wide Beams. Concrete International 26, January (2004), No.1, 62-78

51. Lubell, A. (2006): Shear in wide reinforced concrete beams. Dr.-thesis, Graduate Dep. of Civil Engineering, University of Toronto, 2006. pp. 455

52. Marti, P.; Pralong, J.; Thürlimann, B. (1977): Schubversuche an Stahlbeton-Platten. IBK-Bericht Nr. 7305-2, ETH Zürich, Sept. 1977

53. Mathey, R.G.; Watstein, D. (1963): Shear strength of beams without web reinforcement containing deformed bars of different yield strengthes. ACI-Journ. V.60 (1963), No.2, Febr. 1963, 183-207

54. Moayer, M.; Regan, P.E. (1974): Shear strength of prestressed and reinforced concrete T-beams. 183-214 in: Shear in reinforced concrete. Vol. 1, Publ. SP 42, ACI Detroit. 1974

55. Moody, K.G.; Viest, I.M.; Elstner, R.C.; Hognestad,E. (1954/1955): Shear strength of r.c.-beams. Part 1 – Tests of Simple Beams. ACI-Journal V.26, (1954), No.4, Dec. 1954, 317-332 Part 2 – Tests of Restrained Beams without Web Reinforcement. ACI-Journal V.26, (1955), No.5, Jan. 1955, 417-434 Part 3 – Tests of Restrained Beams with Web Reinforcement. ACI-Journal V.26, (1955), No.6, Febr. 1955, 525-539

56. Morrow, J.D.; Viest, F.M. (1957): Shear strength of r.c.-frame members without web reinforcement. ACI-Journ. V.28 (1957), No.9, March, 833-869

57. Mphonde, A.G.; Frantz, G.C. (1984): Shear tests of high- and low- strength concrete beams without stirrups. ACI-Journ. V.81 (1984), July-Aug., 350-357

58. Niwa, J.; Yamada, K.; Yokozawa, K.; Okamura, M. (1987): Revaluation of the equation for shear strength of r.c.-beams without web reinforcement. Proc. JSCE No.372/V-5 1986-8 Translation in: Concrete Library of JSCE, No. 9, June 1987

59. Podgorniak-Stanik, B.A. (1998): The Influence of Concrete Strength, Distribution of Longitudinal Reinforcement, Amount of Transverse Reinforcement and Member Size on Shear Strength of Reinforced Concrete Members. University of Toronto (1998)

60. Rajagopalan, K.S.; Ferguson, P.M. (1968): Exploratory shear tests emphasizing percentage of longitudinal reinforcement. ACI-Journ. V.65 (1968), No.8, 634-638

61. Regan, P.E. (1971 a): Shear in Reinforced Concrete – an analytical study. CIRIA-Report, a report to the Construction Research and Information Association. Imperial College of Science and Technology, Dep. of Civil Engineering, Concrete Section. April 1971

62. Regan, P.E. (1971 b): Shear in Reinforced Concrete – an experimental study. CIRIA-Report, a report to the Construction Research and Information Association. Imperial College of Science and Technology, Dep. of Civil Engineering, Concrete Section. April 1971

63. Regan, P.E. (1971 c): Behaviour of reinforced and prestressed concrete subjected to shear forces. Institution of civil engineers, Proceedings, Paper 7441S

64. Reineck, K.-H.; Koch, R.; Schlaich, J. (1978): Shear Tests on Reinforced Concrete Beams with axial compression for Offshore Structures – Final Test Report. Stuttgart, July 1978, Institut für Massivbau, Univ. Stuttgart (unveröffentlicht)

65. Remmel, G. (1991): Zum Zugtragverhalten hochfester Betone und seinem Einfluß auf die Querkrafttragfähigkeit von schlanken Bauteilen ohne Schubbewehrung. Diss., TH Darmstadt, 1992

66. Rosenbusch, J.; Teutsch, M. (2002): Trial Beams in Shear. Brite/Euram project 97-4163, Final Report Sub task 4.2, Institut für Baustoffe, Massivbau und Brandschutz, TU Braunschweig, January, 2002

67. Rosenbusch, J.; Zur Querkrafttragfähigkeit von Balken aus stahlfaserverstärkten Stahlbeton. Diss., Fachbereich Bauingenieurwesen, TU Braunschweig, Institut für Baustoffe, Massivbau und Brandschutz, Juni 2003. pp. 199

68. Rüsch, H.; Haugli, F.R.; Mayer, H. (1962): Schubversuche an Stahlbeton-Rechteckbalken mit gleichmäßig verteilter Belastung. DafStb H.145, W. Ernst & Sohn, Berlin, 1-30

69. Salandra, M.A.; Ahmad, S.H. (1989): Shear Capacity of reinforced Lightweight High-Strength Concrete Beams. ACI Structural Journal V86, (Nov-Dec. 1989), No. 6, 697-704

70. Scholz, H. (1994):Ein Querkrafttragmodell für Bauteile ohne Schubbewehrung im Bruchzustand aus normalfestem und hochfestem Beton. Berichte aus dem Konstruktiven Ingenieurbau Heft 21, Technische Universität Berlin 1994

71. Sherwood, E. G. (2008): One-way shear behaviour of large, lightly-reinforced concrete beams and slabs.Dr.-thesis, Dep. of Civil Engineering, University of Toronto, 2008. pp. 547

72. Shin, S-W.; Lee, K-S; Moon, J-I; Ghosh, S. K. (1999): Shear Strength of Reinforced High-Strength Concrete Beams with Shear Span-to-Depth Ratios between 1.5 and 2.5. ACI-Struct. Journ. V.96-S61 (1999), July-August, 549-556

73. Sneed, L.H. (2007): Influence of member depth on the shear strength of concrete beams. Dr. Dissertation, Fac. of Civil Engineering of Purdue University, West Lafayette, Indiana. December 2007. pp. 258

74. Sneed, L.H.; Ramirez, J.A. (2008): Effect of depth on the shear strength of concrete beams without shear reinforcement - experimental study. PCA Research and Development Information, PCA R&D SN2921. Portland cement Assoc., Skokie, Illinois, 2008. pp. 173

75. Sneed, L.H.; Ramirez, J.A. (2010): Influence of effective depth on shear strength of concrete beams - experimental study. ACI Structural Journal 107 (2010), Sept.-Oct., 554-562

76. Swamy, N.; Qureshi, S.A. (1971): Strength, cracking and deformation similitude in reinforced T-beams under bending and shear. ACI-Journ. V.68 (1971), March, 187-195 the Railway Technical Research Institute, V. 46 (2005), No. 1, Feb., 53-58

77. Tanimura, Y.; Sato, T. (2005): Evaluation of Shear Strength of Deep Beams with Stirrups. Quarterly Report of the Railway Technical Research Institute, V. 46 (2005), No. 1, Feb., 53-58

78. Taylor, H.P.J. (1968): Shear stress in reinforced concrete beams without shear reinforcement. Cement and Concrete Ass., Techn. Rep. No.407, Febr. 1968

79. Taylor, H.P.J. (1972): Shear strength of large beams. ASCE-Journ. of the Struct. Div., V.98 (1972), ST11, 2473-2490

80. Thiele, C. (2010): Bemessung von Stahlbetondecken ohne Querkraftbewehrung mit integrierten Leitungsführungen. 12 pp. in: Doktorandensymposium 2010 - 51. Forschungskolloquium des DAfStb, 11. und 12. November 2010, Kaiserslautern. DAfStb e.V., Berlin

81. Tureyen, A.K. (2001): Influence of longitudinal reinforcement type on the shear strength of reinforced concrete beams without transverse reinforcement. Diss. Purdue University, West Lafayette, IN. December 2001

82. Tureyen, A.K.; Frosch, R.J. (2002): Shear tests of FRP-reinforced concrete beams without stirrups.ACI Structural Journal 99 (2002), July-August, 427-434

83. Van den Berg, F.J. (1962): Shear Strength of Reinforced Concrete Beams without Web Reinforcement. ACI-Journ. V.59 (1962) 1467-1477, 1587-1600 u. 1849-1862

84. Walraven, J.C. (1978): The influence of depth on the shear strength of lightweight concrete beams without shear reinforcement. Report S-78-4, Stevin Lab., Delft Univ., 1978

85. Winkler, K. (2011): Querkraftversuche an maßstäblich skalierten, schubunbewehrten Stahlbetonbalken.Versuchsbericht 2011-1. Lehrstuhl für Massivbau, Ruhr Universität Bochum. 3. Feb. 2011. pp. 105

86. Xie, Y.; Ahmad S.H.; Yu, T.; Hino, S.; Chung, W. (1994): Shear Ductility of reinforced Concrete Beams of normal and high-strength concrete. ACI Structural Journal V.91 (Mar.-Apr. 1994), No. 2, 140-149

87. Yoon, Y.-S.; Cook, W.D.; Mitchell, D. (1996): Minimum Shear Reinforcement in Normal, Medium and High-Strength Concrete Beams. ACI-Journ., V.93 (1996), No.5, Sept.-Oct., 576-584

88. Yoshida, Y.; Bentz, E.; Collins, M.P. (2000): Results of Large Beam Tests. University of Toronto (2000)

ANNEX B – CONCLUSION SUMMARY

Parameter	Design Formulation			
	ABNT:NBR6118 (2014)	**ACI 318 (2014)**	**ACI 318 (2019)**	**Frosch et al. (2017)**
f_c'	There is a decrease trend over the dataset, slightly more prominent	A strong trend of decreasing in S/R mean as compressive strength increases	Decreasing in S/R mean as compressive strength increases	No trends are noted, and the approach is the more conservative
a/d	No trends concerning the mean, most of the results with S/R<1 is located next to inflection point of Kani's valley	No trends concerning the mean, most of the results with S/R<1 is located next to inflection point of Kani's valley	a slight decrease trend	No strong trends are present
ρ_L	Most of the values to which S/R<1 to light reinforced beams, with a strong increase trend	Most of the values to which S/R<1 to light reinforced beams, with a strong increase trend	Presents no notable trends regarding this parameter,	No trends are present, and the results are dispersed in a more uniform manner
d	A strong decrease trends occurs, with excessively conservative design for shallow beams, decreasing until non conservative results to higher beam depth	A strong decrease trends occurs, with excessively conservative design for shallow beams, decreasing until non conservative results to higher beam depth	Significantly corrects the model error in shallow beams, leading to smaller S/R, and increasing its value as the beam depth increases	This dispersion is like ACI 318:2019 but is more conservative, mainly to the smaller beam depths