Using multiple random walk simulation in short-term grade models

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Abstract

Geostatistical simulation comprises a variety of techniques which can help on the decision-making process for uncertainties. They allow the uncertainty assessment of function responses (which depend on the simulated inputs) commonly through a non-linear relationship (net present value, interest tax return, geometallurgical ore recovery…). However, one of their limitations is that running the simulation can take considerable processing time to be executed in large deposits or large grids. Herein is presented an attempt to solve this problem in short-term modeling cases, via the use of Multiple Random Walk Simulation. This algorithm combines kriging with the simulation of independent random walks in order to generate simulated scenarios much faster than via traditional simulation algorithms. A case study is presented to illustrate the application of the method in an iron mine. The Multiple Random Walk Simulation models were properly built, respecting the reproduction of both histogram and variograms. Also, the speed-up was compared with standard methods of geostatistical simulation and there was a considerable speed gain with Multiple Random Walk Simulation (3.39 to 5.65 times faster than the others).

Keywords: geostatistics; conditional simulation; mining; short-term modeling.

1. Introduction

Geostatistical simulations can be very useful in mining planning and decision-making about uncertainties. They are mostly used in geological/grade modeling and long-term mine planning, providing several practical benefits. The simulations generate several possible models, all of them respecting both spatial continuity and global distribution derived from the data. And by considering all these simulated scenarios, it is possible to assess the uncertainty of the local values and, moreover, to apply non-linear transfer functions to the model, which allow the calculation of geometallurgical responses and net present value (NPV), for example (Coward and Dowd, 2015).

Although the simulations provide a lot of information to help in decision-making, it is still scarcely used in mining. Its use in short-term geological modelling and mine planning is minimal, despite the fact that it can help, for example, in estimating the grade variability of the blocks and to update the monthly plans (by incorporating recently collected data).

One of the main reasons why simulations are rarely used in daily routines is due to the fact that these algorithms can be very time-consuming. High-performance processors may be more affordable today and can solve this problem, but most algorithms are not designed to take full advantage of these multi-processors.

In order to fulfill this gap, the Multiple Random Walk Simulation (MRWS) was developed to run efficiently in multiprocessors and is now a new alternative that attempts to make 3D conditional simulations more viable for short-term applications. The method is presented in the next section and, after that, a case study illustrates its use.

2. Methodology

The main characteristics of the method are presented next. More specific details can be found in Caixeta (2015), Ribeiro *et al.* (2012) and Ribeiro *et al.* (2013). The MRWS is a parametric method. According to Rossi and Deutsch (2014, p. 123), a value $Y_s(u)$ is conditionally simulated in a Gaussian space as a composition of an estimated value $Y^*(u)$ and a residual $R(u)$ with the correct covariance (Eq. 1):

$$Y_s(u) = Y^*(u) + R(u) \qquad \text{(Eq. 1)}$$

The estimated value Y*(u) can be obtained by applying simple kriging to the Gaussian sample values, which will reproduce the covariance between kriged nodes and samples. By extending the same rationale to the residuals, R(u), i.e. kriging them with the same set of weights used for kriging the Gaussian scores, the correct covariance between residuals at data locations (n) to the node being simulated (u) is reproduced (Journel and Huijbregts, 1978).

Nevertheless, kriging does not reproduce the targeted Gaussian histogram since the interpolation smoothes the original variance. In order to do that, MRWS estimates the residual error using random walks. A random walk (see Lawler and Limic, 2010, for more details) can be described as a vector of random numbers defining a random path by incremental values at every separation lag δh. Each increment value i is defined by f along n number of lags. At each lag distance, an independent increment of $+f$ or $-f$ is drawn, with 50% chance each. The process continues until it reaches the number of lags stipulated. The process restarts at the same origin ($i = 0$, initial point along the random walk) generating another random path, like the ones in Figure 1. Their interesting feature is that, in a set with several random walks, the corresponding values for any chosen lag distance follow a normal distribution with an expected value of zero and a variance proportional to that distance h divided by the size of the increments and to the square of the scaling factor f (Eq. 2).
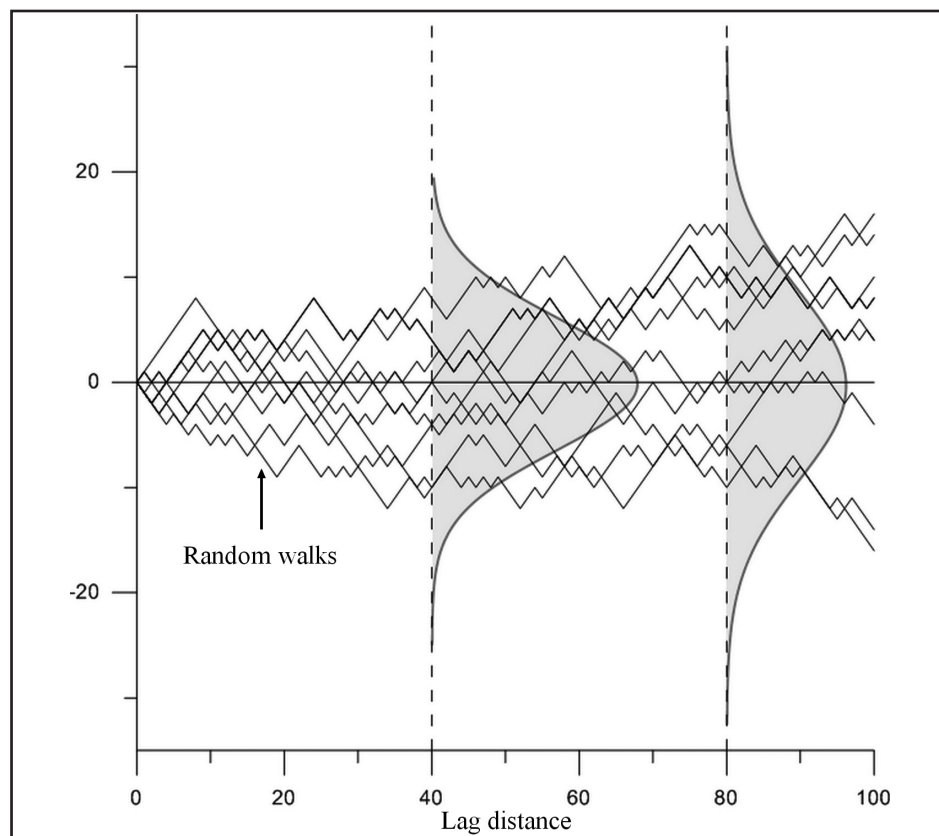

Figure 1
Illustration of multiple random walk paths and their Gaussian properties, in which the variance of the histogram formed by the multiple path values increases with h.

$$\sigma^2 = \frac{h}{\delta h} \cdot f^2 \qquad \text{(Eq. 2)}$$

where σ^2 represents the variance of the random walk values at distance h; δh is the lag separation between each increment f. So, by considering the residual part $R(u)$ of Eq. 1 as a linear combination of a set of random walks and the simple kriging weights, $R(u)$ will have an expected value of zero and will reproduce the desired covariance. And since both $Y*(u)$ and $R(u)$ from Eq. 1 are in Gaussian units, it is possible to sum them up to generate an appropriate Gaussian simulated value.

The workflow for the algorithm can be summarized in five parts:

(i) perform a normal score transformation on the original data.

(ii) simulate a sufficient number of random walks and associate them to each conditioning sample data. Each new realization randomly picks one of these random walk paths.

(iii) solve a simple kriging system at each grid node to define the weights associated with the local conditioning data surrounding this node. These kriging weights associated to each datum are used to interpolate the normal scores and the values from the random walk paths picked at each data location found in the local neighborhood of the grid node being simulated according to:

$$Y_s^{(l)}(u) = \underbrace{\left[\sum_{i=0}^{n} \lambda_i [Y_{ui}] \right]}_{Y*(u)} + \underbrace{\left[GWN + \sum_{i=0}^{n} \lambda_i [RW_{ui}(h)] \right]}_{R(u)} = \sum_{i=0}^{n} \lambda_i [RW_{ui}(h) + Y_{ui}] + GWN \qquad \text{(Eq. 3)}$$

where $Y_s^{(l)}(u)$ is the simulated value for the realization (l) at location u; n is the number of conditioning data in the neighborhood of u; λi is the weight calculated by simple kriging for datum u_i; Y_{ui} is the conditioning data value in Gaussian units; $RW_{ui}(h)$ is the value drawn from the random walk path associated to that conditioning data Y_{ui} at the distance h from the grid node at u; h is the distance between data and the node to be simulated; GWN is a Gaussian white noise added to re-establish the nugget effect smoothed by kriging, similarly to what is used in the Turning Bands Simulation algorithm (Emery and Lantuéjoul, 2006). This last is randomly drawn from a Gaussian distribution of zero mean and variance equal to the nugget effect.

(iv) perform another realization only by randomly picking another random walk path generated at each datum. Retain the new $RW_{ui}(h)$ read at the picked path. The new simulated value uses the same simple kriging

weights and data values.

(v) calibrate the variance of the random walks value to match the unit variance and back transform the simulations to restore the original units.

One remark is that the MRWS approach only guarantees the covariance reproduction between simulated value and sample value; the covariance between two simulated points is only well reproduced when a reasonable amount of conditioning data is available (which is the common case in short-term applications). But when there is a large spaced dataset (say 3 to 6 times grid nodes separation) or there is a great extrapolation on the model, the method still can be used, but it is recommended to simulate auxiliary conditioning data. The idea is to infill a large spaced initial grid, similarly to the multigrid concept (Tran, 1994) used in sequential simulation algorithms to reproduce large scale spatial continuity. It is done by initially simulating the variable where

auxiliary points are necessary, conditioned to the original data. After this, the MRWS will proceed as previously explained.

The great advantage of this procedure is that it is highly parallelizable. In other words, this workflow can be divided into several independent parts, which can be run simultaneously in different processors, providing a considerable acceleration when multiple processors are available. A version for 3D simulations was written for that and is being made available to use into SGeMS (Remy *et al.*, 2009). The algorithm was written in C++ as a plugin for the software. All the Random Walk parameters, calibration issues and simulation of auxiliary points (using Sequential Gaussian Simulation in this version) are totally automated. The parameters required are the same from kriging, plus a reference distribution, number of realizations and seed value. If desired, the automatic parameters can also be customized by the user.

3. Case study

The case study consisted in applying MRWS in a short-term context, where a great amount of samples are available. It was made using data from an iron mine located in the Quadrilátero Ferrífero, state of Minas Gerais, Brazil.

Detailed information about the mine location was omitted due to confidentiality issues. Data from 1 378 drill holes were used (490 from long-term exploration and 888 from short-term exploration). Since both exploration

campaigns collected data via the same method (diamond drill hole) and passed through the quality control program, the samples were used all together. After proper compositing, 8 310 samples were available to be used (Figure 2).
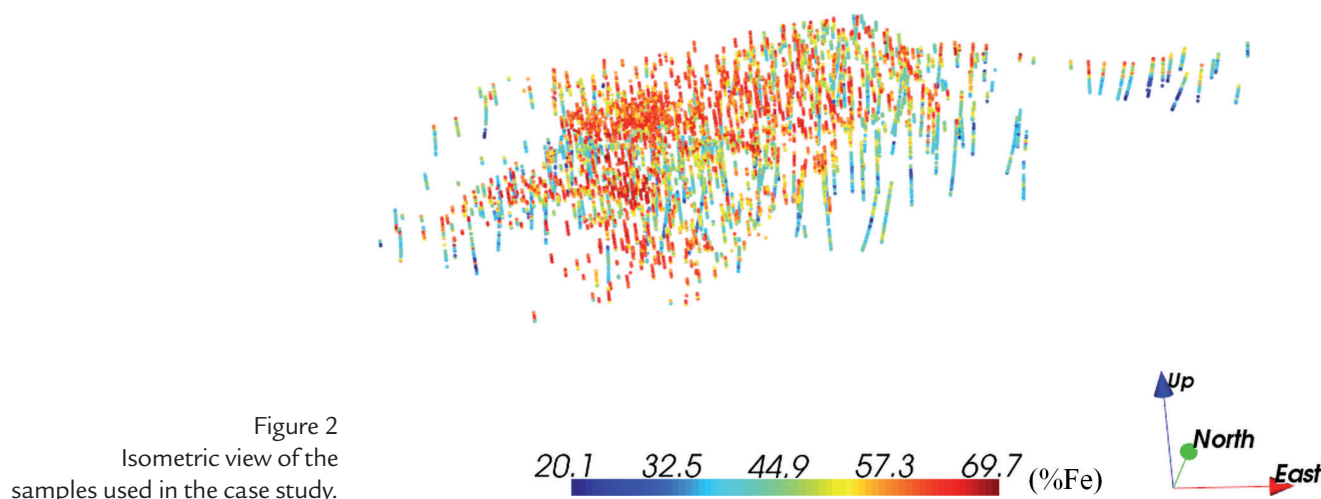

Figure 2
Isometric view of the samples used in the case study.

20.1   32.5   44.9   57.3   69.7 (%Fe)

Up
North
East

These samples comprise different lithologies. Because of that, four stationary domains were defined based on similar rock types and global iron content. An estimated geological model was provided and used to define the regions of each domain.

The simulation of the global iron content was made separately

in each domain. The block size was 12.5m x 12.5m x 10m, which was discretized in 8 points per block to run the simulations. The variograms were modeled with the data from each one of the four domains and used for the simulations. A hundred realizations were performed using MRWS and auxiliary points were initially simulated where necessary.

After the simulations, the points were upscaled to the standard mine block size.

The simulated models of the four domains are presented in Figures 3 and 4. They show a consistent spatial distribution of the global iron content, with the higher grades on the upper part of the deposit and a high variability in the itabirite domains.
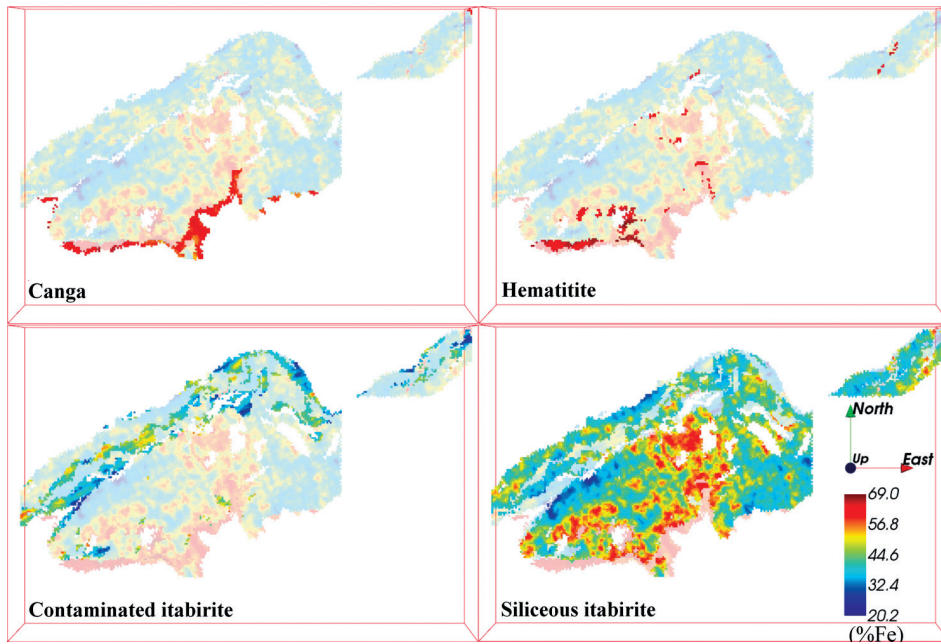
Figure 3
Horizontal sections (at 875 m of elevation) of one simulated realization, highlighting the regions related to each domain.
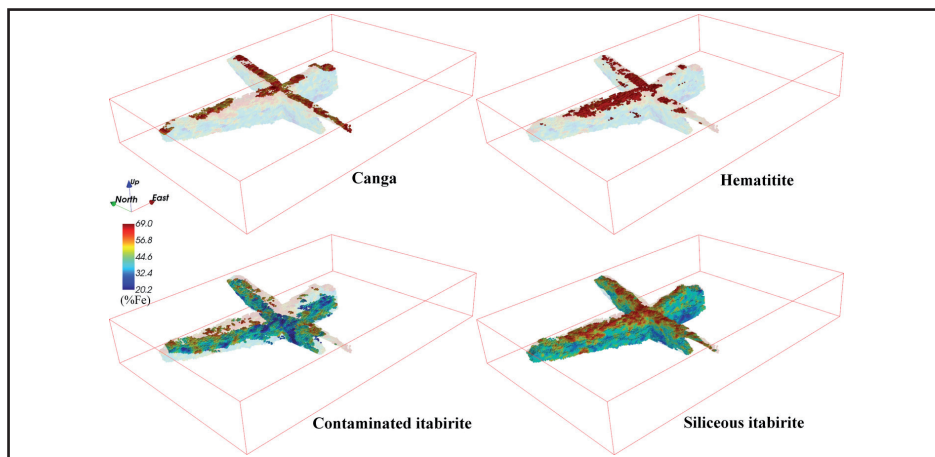

Figure 4
Isometric view of vertical sections of one simulated realization, highlighting the regions related to each domain.

The reproduction of variograms and histograms were checked in order to verify if the models are statistically coherent. As is shown in Figures 5 and 6, the results were good, allowing the further use of these models to assess uncertainty and other derived parameters.
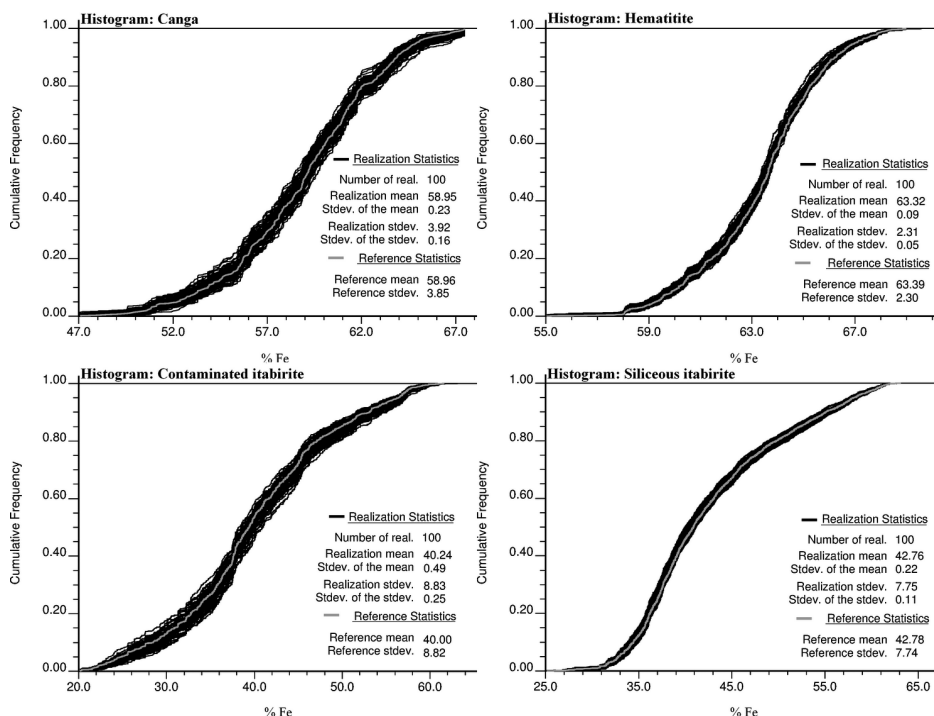

Figure 5
Check for the histogram and basic statistics reproduction of each domain simulated.
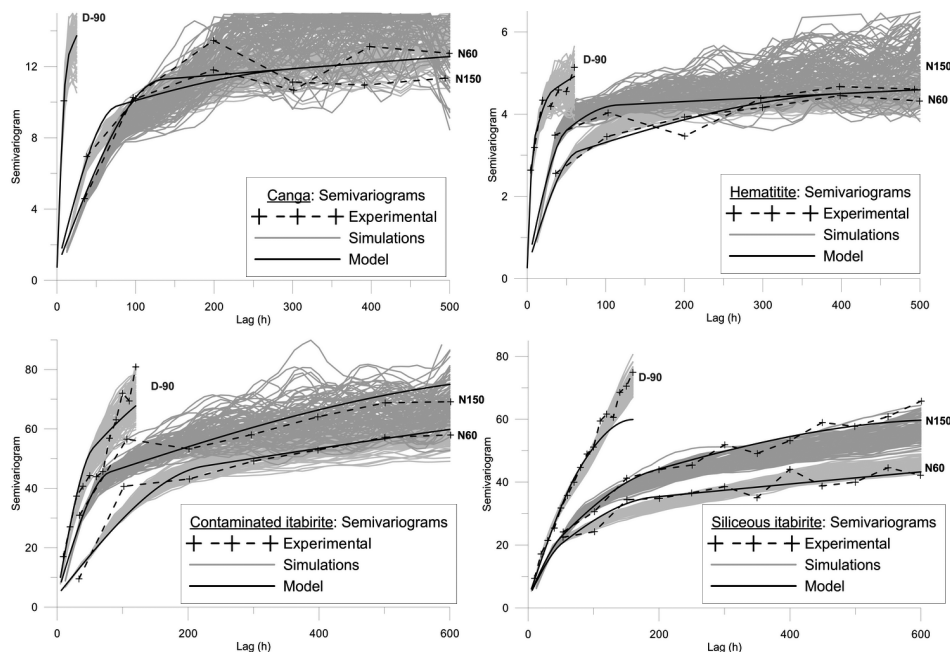
Figure 6
Check for the variogram reproduction of each domain simulated.

And to highlight the speed gain by using the MRWS, the time elapsed for the simulation was measured and compared with the ones from two established simulation methods: Sequential Gaussian Simulation (SGS) and Turning Bands Simulation (TBS). All methods used similar parameters to simulate the same number of nodes. SGS was run into SGeMS and TBS was run into Isatis® (Geovariances, 2014) with 1 000 lines. The results are shown in Table 1.

Table 1
Elapsed time for each simulation method to perform 100 realizations. Hardware comprises 20 processors of an Intel(R) Xeon(R) CPU E5-2630 0 @ 2.30GHz 2x Hexacore.

	MRWS	SGS	TBS
Total time elapsed	45min 58s	2h 35min 57s	4h 19min 32s
Speed-up MRWS	1.00	3.39	5.65

As noted, the MRWS promoted a remarkable speed-up in the simulation process. In terms of absolute time, it saved almost 2h in comparison with SGS and more than 3.5h in comparison with TBS.

4. Discussion and conclusions

In summary, these results show the MRWS works very well in a real iron 3D short-term mining application. The visual aspects of the simulated models are very coherent and the variograms and histogram are well reproduced by the methodology. Moreover, the speed gain provided by the method makes its use very appealing.

It is essential to highlight that the speed gain in using MRWS will vary depending on the amount of memory available, number of realization, number of processors, structure of the algorithms and number of auxiliary points simulated. And the most favorable conditions to use the method are exactly the ones of short-term mining, where many data are available.

Another appealing aspect of the method is that it can be run efficiently in SGeMS, which is a good open-source software. Also, this methodology can be used to update grade control models within a local window (e.g. a planned dig line to define a monthly production) as more infill samples are made available without the need to re-simulate the entire deposit. A local window is herein referred to as a subpart of a larger grid; this last possibly including the entire deposit (or domain).

Acknowledgments

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