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Fuzzy Boundary for Beams Vibrations in Vehicles

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ABSTRACT. This work aims to study the transversal motion of a car's chassis, modeled by a beam and supported at its ends by two sets of spring-damper elements. A finite differences method is utilized along with Robin boundary conditions to solve the mathematical modeling. The constants of the boundary conditions are modeled by a fuzzy parameter obtained through a Fuzzy Rule-Based System (FRBS) acting on the different vibrations of the model, depending on the number of spirals and time of use of the spring. The contributions of this study is based on considering the spring's elastic constant as a relevant instrument acting in the system, due to the fact that analytic formula to perform these calculations, does not consider the time of use. Indeed, the vehicle's mechanical components might have their properties altered due to the fatigue process, for example. The impacts of the car dynamics's property evolution might induce undesirable passenger's discomfort up to the vehicle performance loss. The results showed that the time of use of the spring influences the vehicle's displacement due to the loss of stiffness, thus affecting its stability.

Keywords: Mechanical Vibrations, Fuzzy Robin Boundary Condition, Fuzzy Rule-Based System.

1 INTRODUCTION

The modeling and vibration control of flexible structures have been the target of several researcher's target due to structural systems' high-performance requirements. According to the authors in [18], it is necessary to correctly design dynamic structural systems to guarantee movement stability and precision. The dynamic response of a structure to harmonic excitations depends on stiffness, mass, and damping that influence the natural frequency and vibration modes. These properties result, in turn, from the geometry, materials, and conditions of binding to the external environment.

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Singiresu S. Rao (1995) deduces the equation of motion for a beam and the following boundary condition for a beam connected to masses, springs, and dampers at its ends:

$$\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right) = a\left[kw + c\frac{\partial w}{\partial t} + m\frac{\partial^2 w}{\partial t^2}\right] \tag{1.1}$$

$w(x,t)$ corresponds to vertical displacement, $a = -1$ for the left end and $a = +1$ for the right end of the beam, m corresponds to the mass connected to the ends, E is Young modulus, I is the moment of inertia and k is the spring elasticity constant. These conditions are also known as the Robin boundary.

Thimoshenko (1953) [17] states that beam theories began to be studied from the 17th century. In the middle of the 18th century, Bernoulli and, mainly, Euler presented works that can be considered as the threshold of the theory of beams with elastic material. However, theories were not as rigorous in conceptualizations as is currently the case. Several authors have studied beam vibration analysis due to its importance in mechanical engineering [3].

The authors in [1] have presented a study of a beam's dynamic behavior of an Euler-Bernoulli model with damped supports and variable stiffness. In conclusion as the elasticity of the supports varies, the values of the natural frequencies change, becoming different from the frequencies with the condition of classical supports. As the damping increases, the natural damped frequencies decrease, causing attenuation in the amplitude of the system's movement.

Mechanical vibrations are undoubtedly the main focus to be studied in this work. Unlike other works in the literature, this article presents the novelty of modeling the spring elasticity constant k as the output variable of an FRBS.

The fuzzy set theory was presented in 1965 by Lotfi A. Zadeh [19] when he worked with set classification problems that had no well-defined boundaries. According to [14], fuzzy logic is little known by the vast majority of physicists and relatively few physicists work using this theory. The motivation for studies of this kind arises from the fact that physical measurements are subject to errors and that often these errors are not controllable. The growing influence of fuzzy logic in physics and nuclear engineering becomes evident when consulting a virtual library where the records of a few hundred articles using fuzzy logic are displayed. In several works on differential equations that model physical or biological phenomena, it is considered fuzzy parameters in the equations or in the initial conditions [12], [11], [7], [4], [2], [5], among others. In this research, it has been chose to work with a fuzzy boundary due to the uncertainty at the ends of the beams in the model studied.

The aim of the work is to analyze the transversal vibrations of a beam connected to masses, shock absorbers, and springs at its ends as in the equation (1.1), simulating the chassis of a car. In this case, the different vibration responses will be evaluated by considering the time of use of a spring and the number of spirals, so the elasticity constant k, is obtained through the FRBS. One of the challenges encountered when working with fuzzy boundary problems in mechanical vibrations is the need for the collaboration of experts of the area for the construction of the FRBS. In this work, the mechanical engineering expert participated in elaborating the fuzzy rules and providing the information for the construction of the membership functions of the input and output variables. The partnership with the professional in the area was fundamental to verify the model.

Considering this context, the main contributions of this work is the evaluation of the proposed problem with application of the fuzzy set theory, through the results, one can notice its influence on the control and stability of the vehicle. These issues are not always taken into account, and there are no works in the literature that address the issue.

The work is organized as follows: in Section 2 is introduced the problem and its discretization based on the finite differences method. In Section 3 is presented the results of the deterministic solution to the problem. The results of the fuzzy model for lateral beam vibration are exhibited. Finally, in Section 5 is stated the conclusions of the work.

2 EQUATION OF TRANSVERSAL VIBRATION OF BEAM

One of the most important elements in civil and mechanical engineering is the beam, an element subject to transversal loads. Beams in civil engineering can be used in conjunction with pillars and slabs to transfer the vertical loads received to the pillar. Concerning their mechanical engineering application, beams can be used to model shafts, among several components in complex mechanical structures. For instance, the chassis of a car can be modeled as a beam for vibration analysis purposes.

2.1 Discretization of the Equation of Motion

Consider the free body diagram of a beam element, which is shown in Figure 1, where $M(x,t)$ is the bending moment, $V(x,t)$ is the shear force, $f(x,t)$ is the external force per unit length of the beam and $w(x,t)$ corresponds to transversal displacement [16].

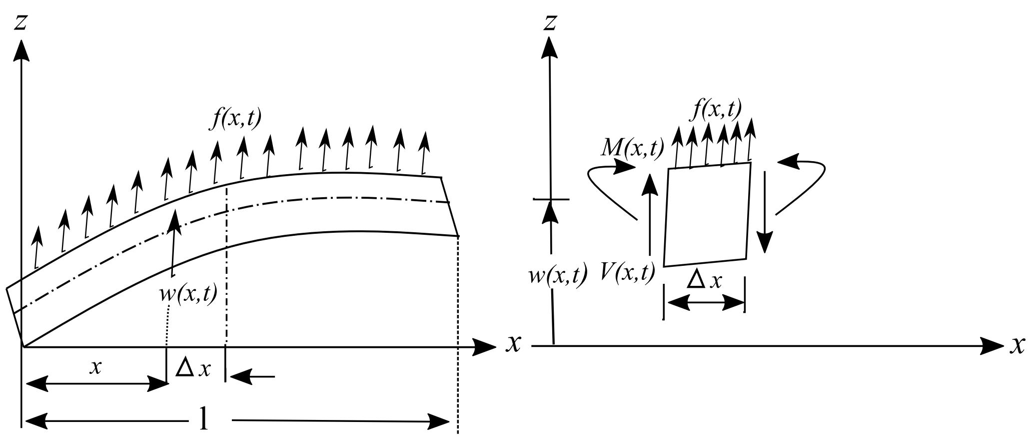

Figure 1: A bending beam. Adapted from [16].

Considering the relationship between the bending moment and the deflection of a slender beam under elastic deformation and assuming a system with free vibration, where $f(x,t) = 0$, the equation of motion becomes:

$$c^2 \frac{\partial^4 w}{\partial x^4}(x,t) + \frac{\partial^2 w}{\partial t^2}(x,t) = 0, \tag{2.1}$$

whereby

$$c = \sqrt{\frac{EI}{\rho A}},$$

being ρ is the mass density, $A(x)$ is the cross-sectional area of the beam, E is Young modulus and I is the transverse moment of inertia.

2.1.1 Initial Conditions

Because the equation of motion [16] involves a second-order derivative to time and a fourth-order derivative to x, two initial conditions and four boundary conditions are required to determine a single solution for $w(x,t)$. The lateral displacement and velocity values are specified as $w_0(x)$ and $w_0^*(x)$ in $t = 0$, so the initial conditions are:

$$w(x,t=0) = w_0(x) \text{ and}$$

$$\frac{\partial w}{\partial t}(x,t=0) = w_0^*(x).$$

2.1.2 Boundary Conditions

For the boundary conditions, the beam is connected to springs, shock absorbers and masses at its ends. The inertial, elastic and damping efforts of the mechanical components are balanced by the shear force of the beam. For this case, consider the equation (1.1), two others complementary boundary conditions are obtained by stating the null bending moments at the ends of the beam. It can be stated that:

$$EI\frac{\partial^2 w}{\partial x^2} = 0. \tag{2.2}$$

The Figure 2 represents a beam connected to springs, shock absorbers and masses at the ends. In this study it is considered $m = m_1 = m_2$, $c = c_1 = c_2$ and $k = k_1 = k_2$.

2.2 Discretization of the Motion Equation

To obtain the numerical solution of the equation (2.1) using the finite difference method, it is considered the range of $[0,l]$ and divide it into $n-1$ equal parts, as follows, $[x_i, x_{i+1}]$ to $i = 1, \cdots n-1$, being $x_1 = 0$ and $x_n = l$.

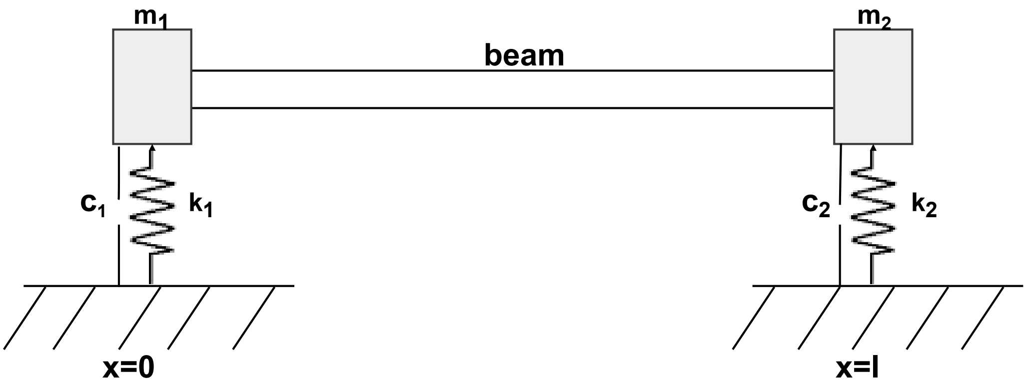

Figure 2: Beam connected to springs, shock absorbers and masses [16].

The discretization performed in the equation (2.1) is valid only for the points $i = 3$ to $i = n-2$, that is, for the internal points of the mesh. Therefore, the discretization for the other points will be performed using other procedures.

The focus of the work is to solve the equation (2.1) numerically, then replacing the derivatives that appear in (2.1) with the approximations by centered finite differences [10], it is obtained:

$$\frac{w_i^{j+1} - 2w_i^j + w_i^{j-1}}{\Delta t^2} = -c^2 \left(\frac{w_{i+2}^j + w_{i-2}^j + 6w_i^j - 4w_{i+1}^j - 4w_{i-1}^j}{\Delta x^4} \right).$$

Since the goal of this study is to find the unknowns in the next step in time where j is the index of the time variable, then it is isolated w_i^{j+1} in the following manner:

$$w_i^{j+1} - 2w_i^j + w_i^{j-1} = \frac{-c^2 \Delta t^2}{\Delta x^4} \left(w_{i+2}^j + w_{i-2}^j + 6w_i^j - 4w_{i+1}^j - 4w_{i-1}^j \right),$$

and considering $\lambda = \dfrac{-c^2 \Delta t^2}{\Delta x^4}$, it is obtained:

$$w_i^{j+1} = \lambda w_{i+2}^j + \lambda w_{i-2}^j + (6\lambda + 2) w_i^j - 4\lambda w_{i+1}^j - 4\lambda w_{i-1}^j - w_i^{j-1}. \tag{2.3}$$

NOTE: Studies regarding the convergence of explicit methods are performed in [8], in which it is shown that the explicit method is unstable for certain λ values. To ensure its stability and acceptable results one should use $\lambda < 1$.

Thus, the following matrix system is obtained:

$$W^{n+1} = AW^n - W$$

whereby $A = \begin{bmatrix} \lambda & -4\lambda & 6\lambda+2 & -4\lambda & \lambda & 0 & 0 & 0 & \dots & 0 \\ 0 & \lambda & -4\lambda & 6\lambda+2 & -4\lambda & \lambda & 0 & 0 & \dots & 0 \\ 0 & 0 & \lambda & -4\lambda & 6\lambda+2 & -4\lambda & \lambda & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \lambda & -4\lambda & 6\lambda+2 & -4\lambda & \lambda \\ 0 & 0 & 0 & 0 & \dots & 0 & \lambda & -4\lambda & 6\lambda+2 & -4\lambda \end{bmatrix}$

$W^{n+1} = \begin{bmatrix} w_3^{j+1} \\ w_4^{j+1} \\ w_5^{j+1} \\ \vdots \\ w_{n-4}^{j+1} \\ w_{n-3}^{j+1} \end{bmatrix}$ $W^n = \begin{bmatrix} w_3^{j} \\ w_4^{j} \\ w_5^{j} \\ \vdots \\ w_{n-3}^{j} \\ w_{n-2}^{j} \end{bmatrix}$ and $W = \begin{bmatrix} w_3^{j-1} \\ w_4^{j-1} \\ w_4^{j-1} \\ \vdots \\ w_{n-4}^{j-1} \\ w_{n-3}^{j-1} \end{bmatrix}$

For the $i = 1$ and $i = n$ border points, it has been used the boundary condition (1.1), to get:

- For $i = 1$:

$$w_1^{j+1} = \frac{w_1^j\left(-\alpha ak\Delta t^2 + \alpha ac\Delta t + 2\alpha am - 1\right) + 3w_2^j - 3w_3^j + w_4^j - \alpha amw_1^{j-1}}{\alpha ac\Delta t + \alpha am}; \tag{2.4}$$

- For $i = n$, one has

$$w_n^{j+1} = \frac{w_n^j\left(-\alpha ak\Delta t^2 + \alpha ac\Delta t + 2\alpha am + 1\right) + 3w_{n-2}^j - 3w_{n-1}^j - w_{n-3}^j - \alpha amw_n^{j-1}}{\alpha ac\Delta t + \alpha am}. \tag{2.5}$$

where $\alpha = \dfrac{\Delta x^3}{\Delta t^2 EI}$.

The approximations for the point $i = 2$ and $i = n-1$ were obtained using the boundary condition (2.2):

- For $i = 2$:

$$\begin{aligned} w_2^{j+1} &= \left(\frac{q}{2y} + \frac{\lambda}{2}\right) w_1^j + \left(\frac{3}{2y} - 2\lambda\right) w_2^j + \left(\frac{-3}{2y} + \frac{(6\lambda+2)}{2}\right) w_3^j + \left(\frac{1}{2y} - 2\lambda\right) w_4^j \\ &+ \frac{\lambda}{2} w_5^j - \frac{\alpha am}{2} w_1^{j-1} \frac{w_3^{j-1}}{2} \end{aligned} \tag{2.6}$$

where $q = -\alpha ak\Delta t^2 + \alpha ac\Delta t + 2\alpha am - 1$ e $y = \alpha ac\Delta t + \alpha am$;

- For $i = n-1$ the following approach:

$$\begin{aligned} w_{n-1}^{j+1} &= \left(\frac{s}{2y} + \frac{\lambda}{2}\right) w_n^j + \left(\frac{3}{2y} - 2\lambda\right) w_{n-1}^j + \left(\frac{3}{2y} + \frac{(6\lambda+2)}{2}\right) w_{n-2}^j \\ &+ \left(\frac{-1}{2y} - 2\lambda\right) w_{n-3}^j + \frac{\lambda}{2} w_{n-4}^j - \frac{\alpha am}{2y} w_n^{j-1} - \frac{w_n^{j-1} - 2}{2}. \end{aligned} \tag{2.7}$$

where $s = -\alpha ak\Delta t^2 + \alpha ac\Delta t + 2\alpha am + 1$ e $y = \alpha ac\Delta t + \alpha am$.

The next section details the numerical deterministic solution of the equation (2.1), using the discretization performed.

3 DETERMINISTIC MODEL FOR TRANSVERSAL BEAM VIBRATIONS

The beam's model in this case is the chassis of a car, as shown in Figure 3.

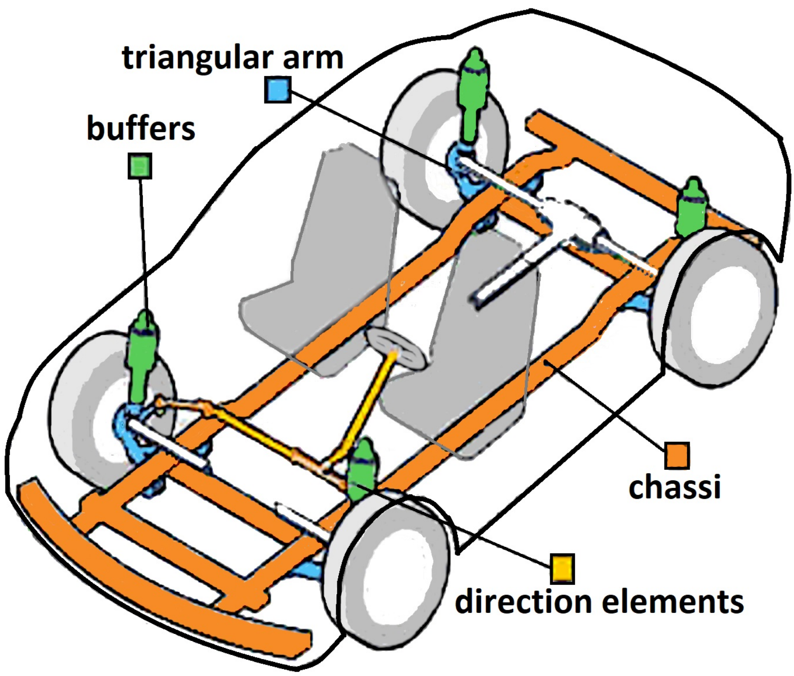

Figure 3: Beam model. Adapted from [6].

In the deterministic model, the value of the spring stiffness used in the boundary condition (1.1) is constant value; the time of use and the quantity of spirals of the spring are not considered.

The analytical formula for the spring stiffness is given by:

$$k = \frac{G \times d^4}{8 \times D^3 \times N_a}, \tag{3.1}$$

in which G describes the shear modulus of the spring's material, d corresponds to the diameter of the spring wire, D is the diameter of the spring and N_a equals the quantity of spirals. The following values were used to calculate the value of $k = 1.2917 \times 10^6\ N/m$ for nine spirals: $G = 547.1 \times 10^9\ N/m^2$, $d = 17 \times 10^{-3}m$ and $D = 17 \times 10^{-2}m$ for the specific spring of a vehicle. The parameter values are $c = 3.14$, $\Delta t = 1 \times 10^{-6}$, $\Delta x = 0.01m$, $EI = 2800N/m^2$ and $m = m_1 = m_2 = 590kg$. These values were calculated according to the analyzed application and with the help of the expert. For the free vibration response, an initial velocity condition is considered in the first iteration due to an impulse (e.g.: Dirac effort) applied at the point $\left(\frac{n+1}{2}\right)$, as follows: $w^1_{\frac{n+1}{2}} = 4 \times \Delta t$. The displacement and velocity for the remaining points are zero.

Figure 4 shows the transversal displacement of the beam after 2000 iterations in time. In this analysis is considered a spring with the quantity of nine spirals .

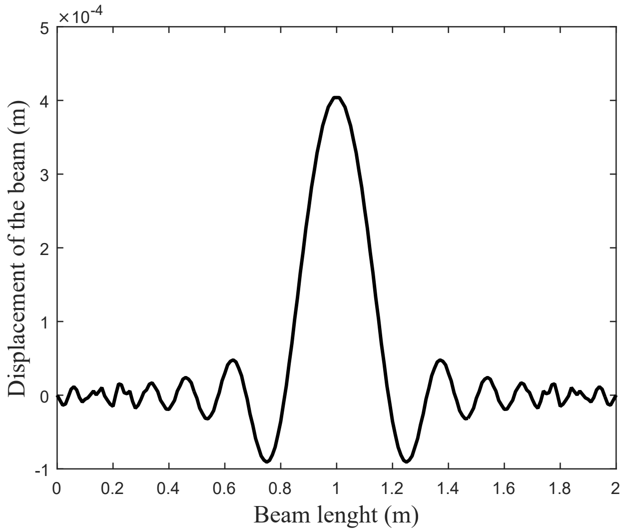

Figure 4: Numerical solution of the equation (2.1) for $k = 1.2917 \times 10^6\ N/m$.

Figure 5 presents the graph of the propagation of the waves in the beam over time for $k = 1.2917 \times 10^6\ N/m$ with the quantity of nine spirals.

To check the stability of the method used for discretization of the problem, it is shown the displacement of the beam with 10^5 iterations in time for the last point ($x = n$), with the quantity of nine spirals $k = 1.2917 \times 10^6$, see Figure 6. Note that the oscillations do not diverge in time.

In the next section, there is an explanation of fuzzy set theory and a model for beam vibrations with fuzzy boundary.

4 FUZZY MODEL FOR BEAM VIBRATIONS

This section is divided into two subsections. First is introduction of some important definitions of fuzzy sets theory, and in the second the model studied in this work is presented.

4.1 Fuzzy Set Theory

First, it is defined a *fuzzy subset D* of X given $\mu_D : X \to [0,1]$, called *the membership function* which is associated to the fuzzy set D, where X is a non-empty set. The value $\mu_D(x)$ represents

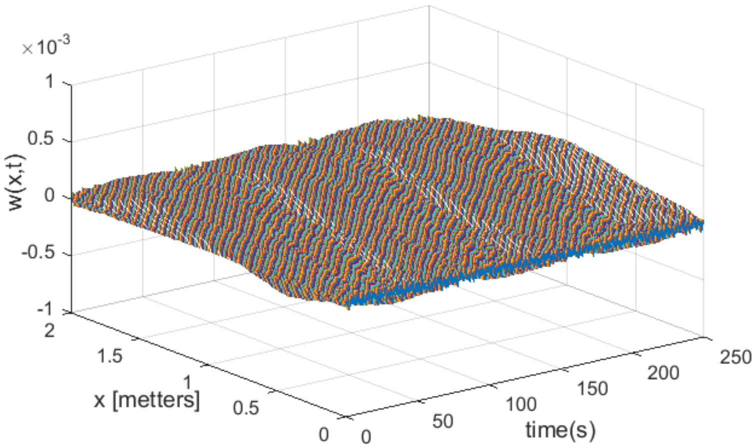

Figure 5: Wave propagation in the beam.

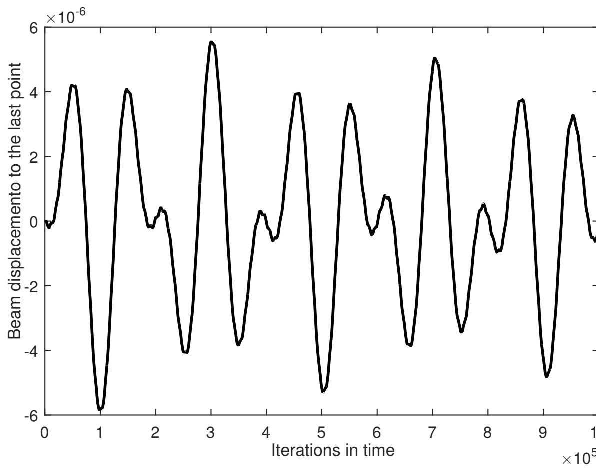

Figure 6: Stability of the method.

the membership degree of $x \in D$. A *classic subset* D of X is a particular fuzzy set for which the membership function is the characteristic function $\mathscr{X}_D : X \to \{0,1\}$ [19].

There is a qualification in any fuzzy set given by the α-cuts of a fuzzy set D, denoted by $[D]^\alpha$ and defined as $[D]^\alpha = \{x \in X, \mu_D(x) \geq \alpha\}$, $0 < \alpha \leq 1$; $[D]^0 = \overline{supp(D)}$, where $supp(D) = \{x \in X, \mu_D(x) > 0\}$ is the support of D. A fuzzy set D is called a *fuzzy number* when $X = \mathbb{R}$ and all α-cuts of D are non empty, all α-cuts of D are closed intervals of $\mathbb{R}$, and the support of D is bounded.

Second, a key role in fuzzy set theory is the Fuzzy Rule-Based System (FRBS) [15]. Such systems contain four components: an input processor that makes the fuzzification of the input data, a collection of fuzzy rules called rule base, an inference machine, and an output processor that provides a real number as an output. Once the rule base is established, i.e., how it is related the fuzzy set as follows, "If...then..." an FRBS can be understood as a mapping between an input and an output of the formula $y = f(x)$, $x \in R^n$ and $y \in R^m$. This system class is widely used in model problems, control, and classification.

4.2 Fuzzy Model for Beam Vibrations

From the **boundary condition** represented by the equation (1.1), the spring stiffness k of the chassis of a car is modeled as the fuzzy parameter. Where r classifies the spring by measuring the thousands of kilometers driven with the car, and N_a is the number of spirals in the spring. The input variables of the FRBS are:

- Classification of the spring measured in thousand kilometers driven (r): domain of the membership functions is [0 , 80] and the linguistic terms are new, semi-new and old.
- Quantity of spirals (N_a): domain of the membership functions is [3 , 15] and the linguistic terms are small, medium and big.

The output variable of the FRBS is the constant k:

- Spring stiffness constant k: domain of membership functions is $[0.775 \times 10^6 , 3.8750 \times 10^6]$, measured in Newtons per meter. The linguistic terms are very small, small, medium, high medium and big.

The membership functions and rule base that encodes the relationships between r, N_a and k was constructed using the knowledge of an area expert. The nine fuzzy rules are summarized in Table 1.

Table 1: Fuzzy rules of the FRBS.

Classification of the spring (r) / Quantity of spirals (N_a)	*new*	*semi-new*	*old*
small	*big*	*high medium*	*small*
medium	*medium*	*small*	*very small*
big	*very small*	*small*	*medium*

The support of each membership functions output variable k is calculated using the expression (3.1), taking into account the classification of the spring into new, semi-new and old and the quantity of spirals, in each case.

Figures 7, 8 and 9 represent graphs of the membership functions for the input and output variables of the FRBS, respectively.

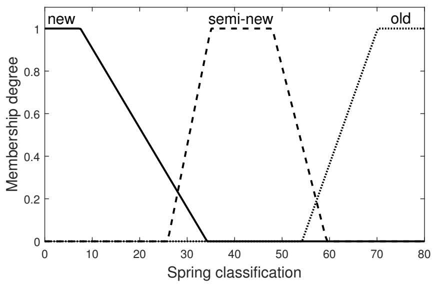

Figure 7: Membership functions for the classification spring.

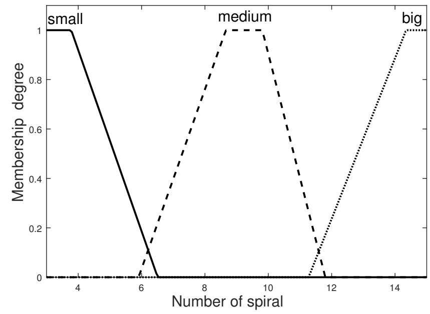

Figure 8: Membership functions for the quantity of spirals.

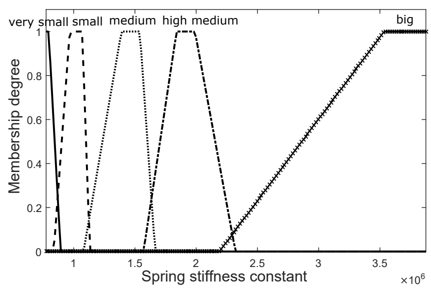

Figure 9: Membership functions for spring elasticity constant.

The fuzzy inference method used is the Mamdani method and the defuzzification method is the center of gravity, as presented in [15]. The parameters used are all realistic, defined as $\Delta x = 0.001m$, $\lambda = 5$, $m = m_1 = m_2 = 590$ kg, $\Delta t = 1 \times 10^{-6}$ and $c = 3.14$. To evaluate the numerical procedure, it is considered a total of three scenarios, in which the parameters are kept fixed in each analysis, with modifications only in the number of spirals. A choice is made to represent in three scenarios, with the number of intermediate spirals in the range from three to fifteen. In each scenario appear a deterministic k, obtained by the equation (3.1) for the number of spirals analyzed, and three k obtained through FRBS, investigating the spring's displacement considered new, semi-new, and old.

The first scenario, according to Figure 10 represents the transversal displacements of the beam with the three values of k obtained through the FRBS, considering a new, semi-new and old spring (Table 2) and $k = 3.8750 \times 10^6$ N/m for quantity of three spirals. Figure 11 shows the absolute error between the displacements of the beam with the values of k obtained by the FRBS and k deterministic. This error is obtained for each value of the fixed beam length discretization by calculating the difference in modulus between the displacement value of the beam obtained by the FBRS (for each linguistic term) and considering $k = 3.8750 \times 10^6$ N/m.

Table 2: Spring constant values k for three spirals.

Spring classification in 10^3 km driven	20	50	80
value of k in N/m	$3.20 \times 10^6 N/m$	$1.93 \times 10^6 N/m$	$1 \times 10^6 N/m$

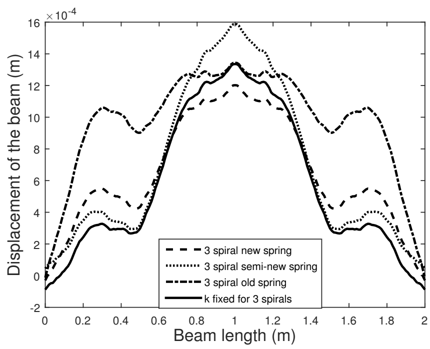

Figure 10: Numerical solutions of the equation (2.1) for the k from FRBS.

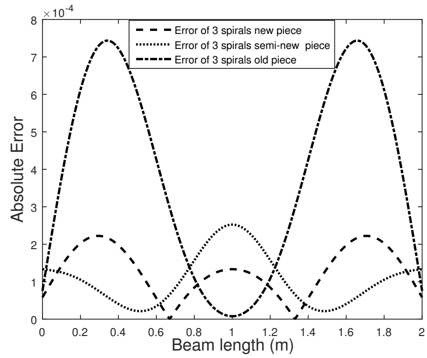

Figure 11: Absolute error between the displacements of the beam with k from FBRS and k deterministic.

The second scenario, according to Figure 12 represents the displacements of the beam with the three values of k obtained through the FRBS (Table 3) and $k = 1.9375 \times 10^6$ N/m for seven spirals. In Figure 13 the absolute error between the displacements of the beam with the values k obtained by the FRBS and deterministic k are represented.

The third scenario, according to Figure 14 shows the different displacements of the beam with the three values of k obtained through the FRBS, considering a new, semi-new and old spring (Table 4), and $k = 0.775 \times 10^6$ N/m for the quantity of fifteen spirals. In Figure 15 the absolute error between the displacements of the beam with the values of the k obtained through the FRBS and deterministic k are presented.

Notice that the different displacements of the beam, with the influence of the spring classification acting on the system, since 2×10^6 iterations in time is considered in each case. As seen in Figures 10, 12 and 14 there is a maximum displacement of 16×10^{-4} m, in all simulations. Nonetheless, notice the importance of the initial velocity condition given through $w^1_{\frac{(n+1)}{2}} = 4 \times \Delta t$. The initial velocity located at a point corresponds to a small Dirac effort over the 1,5 ton of the vehicle mass.

Therefore, this small initial condition might also explain the small amplitude displacement achieved. The vehicle represented has a natural frequency of vertical rigid body displacement of around 5.5Hz, while in automotive application this frequency is recommended to be placed

Table 3: Spring constant values k for seven spirals.

Spring classification in $10^3\,km$ driven	20	50	80
value of k in N/m	$1.39\times10^6 N/m$	$9.93\times10^5 N/m$	$8.17\times10^5 N/m$

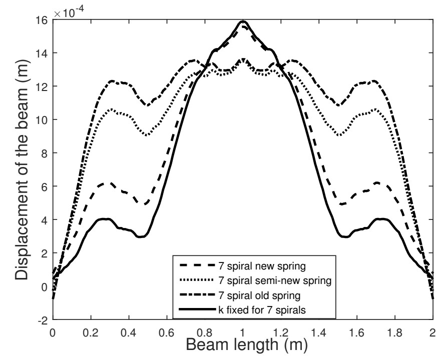

Figure 12: Numerical solution of the equation (2.1) for the k from FBRS.

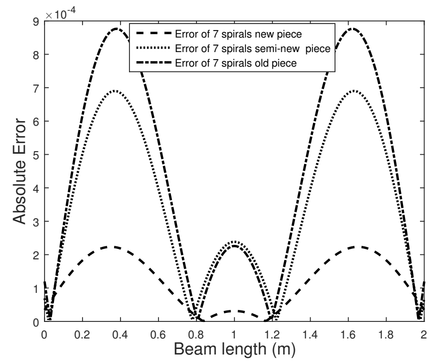

Figure 13: Absolute error between the displacements of the beam with k from FBRS and k deterministic.

Table 4: Spring constant values k for fifteen spirals.

Spring classification in $10^3\,km$ driven	20	50	80
value of k in N/m	$8.15\times10^6 N/m$	$1.41\times10^6 N/m$	$9.98\times10^5 N/m$

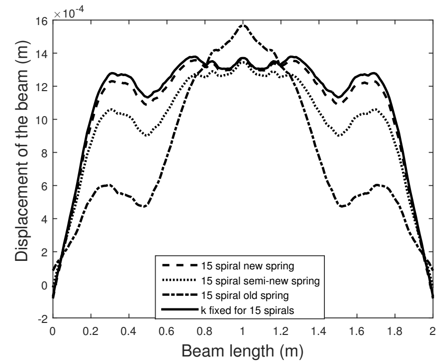

Figure 14: Numerical solution of the equation (2.1) for the k from FBRS.

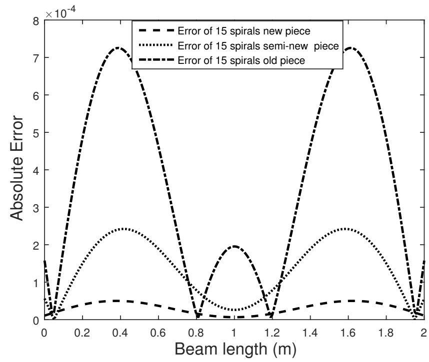

Figure 15: Absolute error between the displacements of the beam with k from FBRS and k deterministic.

between 0.5Hz and 3Hz [9]. Despite this fact, it is believed by the authors that the problem can be still representative of a real problem.

The highest value of k was obtained with a new spring with three spirals (Figure 10), the displacement of the beam presents different frequencies compared to the displacement with a semi-new or old spring, Figures 12 and 14. The quantity of spirals strongly influences the displacement of the beam and its behavior. In all scenarios, when having an old spring, due to the loss of its stiffness, the beam presents more difficulty in completing its displacement, directly influencing the vehicle's displacement.

5 CONCLUSION

In this work, the values adopted are consistent with realistic values for a car, except for the elastic suspension at the extremities of the chassis. According to [13], the springs adopted are closest to truck suspensions.

The transversal oscillations of the chassis of a car modeled by a beam with two sets of spring-damper elements at each extremity are addressed. The present paper's main contribution consists of considering the time of use of these springs for their elastic model through the fuzzy set theory. Indeed, the calculation of the spring stiffness constant does not take into account the time of use, and it might affect the dynamic behavior of the car. This influence should be considered, and it is treated in the paper. The problem addressed gives us the possibility of obtaining various solutions for the model, considering whether the spring is new, semi-new, or old. The different vibration level of the vehicle is observed depending on the type of spring considered.

The model approached is composed of a fourth-order partial differential equation. For the discretization of the problem, the finite differences method was used, in which further studies were needed for the discretization of higher-order derivatives. Therefore, this work provides detailed approximations of higher-order derivatives as a means of numerical solutions to problems of this nature.

Thus, through the graphs, it is noticed that the spring classification is a relevant factor in the system because it has the function of absorbing the impacts that the car suffers, and is also responsible for maintaining the correct height of the vehicle. The constant k of the spring determines the relationship between the load applied and the deformation of the spring, influencing the conditions of comfort and stability of the vehicle.

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