

A Multiobjective Optimization Application to Control the *Aedes Aegypti* Mosquito using a Two-Dimensional Diffusion-Reaction Model

J. S. LIMA^{1*}, A. S. V. VASCONCELOS² and R. T. N. CARDOSO³

Received on January 6, 2023 / Accepted on September 20, 2023

ABSTRACT. *Aedes aegypti* is the main vector of multiple diseases, such as dengue, yellow fever, Zika, and chikungunya. Diseases associated with mosquitoes have been growing in recent years, with more than one-third of the world population at risk. Control techniques have been studied to prevent the spread of the *Aedes aegypti*, such that new formula and how to use adulticides and larvicides, among others. This paper proposes a novel approach in the field of partial differential equations and optimization. We consider a two-dimensional diffusion-reaction model that describes the interaction between aquatic and adult female stages spreading across a domain with parameters dependent on rainfall and temperature. We also formulate a mono-objective and multiobjective optimization problem to minimize the *Aedes aegypti* populations and the control, considering the application of adulticides and larvicides, using actual data from the city of Lavras/Brazil. The operator splitting technique is used to solve the diffusion-reaction system, coupling finite difference and the fourth-order Runge-Kutta method and optimal solutions were searched by using the Real-Biased Genetic Algorithm and Non-dominated Sorting Genetic Algorithm -II. Numerical results show significant reduction of the *Aedes aegypti* population.

Keywords: *Aedes aegypti*, partial differential equations, optimization.

*Corresponding author: Josenildo Silva de Lima – E-mail: nildo2802@gmail.com

¹Postgraduate Program in Mathematical and Computational Modeling, PPGMMC, Federal Center of Technological Education of Minas Gerais, Avenida Amazonas 47675, 30510-000 Belo Horizonte, MG, Brazil – E-mail: nildo2802@gmail.com <https://orcid.org/0000-0003-2229-0806>

²Postgraduate Program in Mathematical and Computational Modeling, PPGMMC, Federal Center of Technological Education of Minas Gerais, Avenida Amazonas 47675, 30510-000 Belo Horizonte, MG, Brazil – E-mail: amaliasv@hotmail.com <https://orcid.org/0000-0002-8789-614X>

³Department of Mathematics, Federal Center of Technological Education of Minas Gerais, Avenida Amazonas 47675, 30510-000, Belo Horizonte, MG, Brazil – E-mail: rodrigocardoso@cefetmg.br <https://orcid.org/0000-0002-6784-2929>

1 INTRODUCTION

Recent research on the *Aedes aegypti*¹ population genetics and its relationship with epidemiological records of occurrences of diseases transmitted by this species allowed reconstruction of its history in the last 261 years [24]. The problem of how and when this mosquito came to occupy its distribution is summarized by Powell et al. [28]. There is no specific treatment for most diseases caused by *Ae. aegypti* [24]. Several advances in science aim at approaches to control vector-borne diseases, such as biological control with adulticides, larvicides, curtains treated with insecticides and traps [29].

Adequate mathematical modeling is a crucial factor in understanding the dynamics of the mosquito population and devising strategies, using appropriate control techniques to reduce this population. Among such studies can be highlighted the proposal of Dias et al. [9], that aim to control the vector in the spread of dengue fever, using an approach of multiobjective optimization, whose intention was to minimize the social and economic costs for the use of chemical (insecticides) and biological (insertion of males sterilized by radiation) control. Florentino et al. [11] developed a study using multiobjective optimization techniques to help solve problems involving *Ae. aegypti* control. The population dynamics of the mosquito was studied in order to understand the epidemic phenomenon and suggest strategies of multiobjective programming for mosquito control.

Diffusion-reaction² equations arise naturally in modeling chemical reactions, engineering, physical phenomena and are widely used in biological systems to describe interaction and diffusion between species [15, 22]. In 2018, Zhu et al. [47] presented a mathematical diffusion-reaction model with a free boundary to describe dengue transmission. Furthermore, the authors obtained the basic reproduction number, determining the persistence and eradication of the disease. That same year, Yamashita et al. [40] presented a diffusion-reaction-advection model to describe the mosquito *Ae. aegypti* population dynamics in different cities.

In 2005, Takahashi et al. [31] presented a model that shows the dynamics of *Ae. aegypti* dispersion intending to highlight procedures to minimize the dengue vector. In 2019, Carvalho et al. [5] proposed a mathematical model that evaluates control strategies, which aim to eliminate the *Ae. aegypti* mosquito, as well as proposals for the vaccination campaign along with mechanical and chemical control, carried out with insecticides and larvicides. In 2019, Multerer et al. [21] used partial differential equations to describe *Ae. aegypti* mosquito population dynamics to release sterile males. In addition, they applied optimal control theory to identify the release strategy that most effectively eliminates mosquitoes.

In 2019, Silva et al. [30], presented an entomological model of two populations that describes the dynamics of *Ae. aegypti* in the aquatic and adult phases of females with parameters dependent on

¹A taxonomic rule in Biology determines that abbreviation *Ae. aegypti* for the specie name *Aedes aegypti* must be used, except for the first appearance, tables, figures, and abstract.

²Reaction-diffusion equations arise naturally in systems consisting of many interacting components, and are widely used to describe pattern-formation phenomena in variety of biological, chemical and physical systems [16].

precipitation. They also showed the sensitivity analysis and the local stability of the model and used mono-objective optimization to control *Ae. aegypti*.

In 2022, Vasconcelos [35] they described a mathematical model to represent the population of *Aedes* life stages with parameters dependent on temperature and precipitation. Real data from female mosquitoes captured by traps in the city of Lavras (Brazil) was considered to calibrate the model parameters. In addition, an optimal control problem was formulated to evaluate the costs of control actions with adulticides and larvicides using Pontryagin's Maximum Principle.

This paper propose a diffusion-reaction model for the dynamic and spreading of *Ae. aegypti*, and an effective control strategy to reduce its density, avoiding the occurrence of dengue and other epidemics. The dynamic of mosquitoes considers the aquatic phase and adult females. The main reason for the dispersal of *Ae. aegypti* mosquitoes is the search for a source of human blood or places for oviposition. Furthermore, the present work develops an optimization scheme that minimizes both the population of *Ae. aegypti* in time, space, and financial costs. Regarding the financial costs, the control should be applied as few as possible to reduce the costs involved in the purchase of adulticides and larvicides and the social costs such as the number of female mosquitoes. The resulting system is solved numerically using the operator splitting technique, which is well-known in the resolution of issues arising in large systems of partial differential equations, as well as to problems involving chemical reactions and dispersal of the mosquito [13, 19, 25, 39].

Given its easy implementation, we used the Non-dominated Sorting Genetic Algorithm II (NSGA-II) and Real-Biased Genetic Algorithm (RBGA) to find solutions to the multiobjective and mono-objective optimization problems [8, 32]. For comparison purposes, numerical simulations explored two different scenarios for two months of the summer season: considering the dispersion starting with one point of the domain, the simulation considering with and without control. Results showed the effectiveness of control to reduce the *Ae. aegypti* population both in time and space.

This work presents the following novelty: a mono-objective and multiobjective optimization approach to minimize the mosquito population and the control, considered as the application of adulticides and larvicides restricted to the two-dimensional diffusion-reaction model. Regarding the literature, the main differences with the works of [30, 35] are: i) a numerical study of the proposed two-dimensional mathematical diffusion-reaction model that describes the interaction between aquatic and adult *Ae. aegypti* female stages; ii) a mono-objective and multiobjective optimization problem is used. Concerning the work of [30], the temperature is inserted in addition to rainfall.

2 MATHEMATICAL MODEL

The proposed mathematical model considers that the life cycle of mosquitoes comprises two phases: aquatic (immature) phase consisting of the eggs, larvae, and pupae, and adult phase consisting of the adult females mosquitoes, as can be seen in Figure 1. The female mosquitoes lay eggs in containers of standing water [20,38].

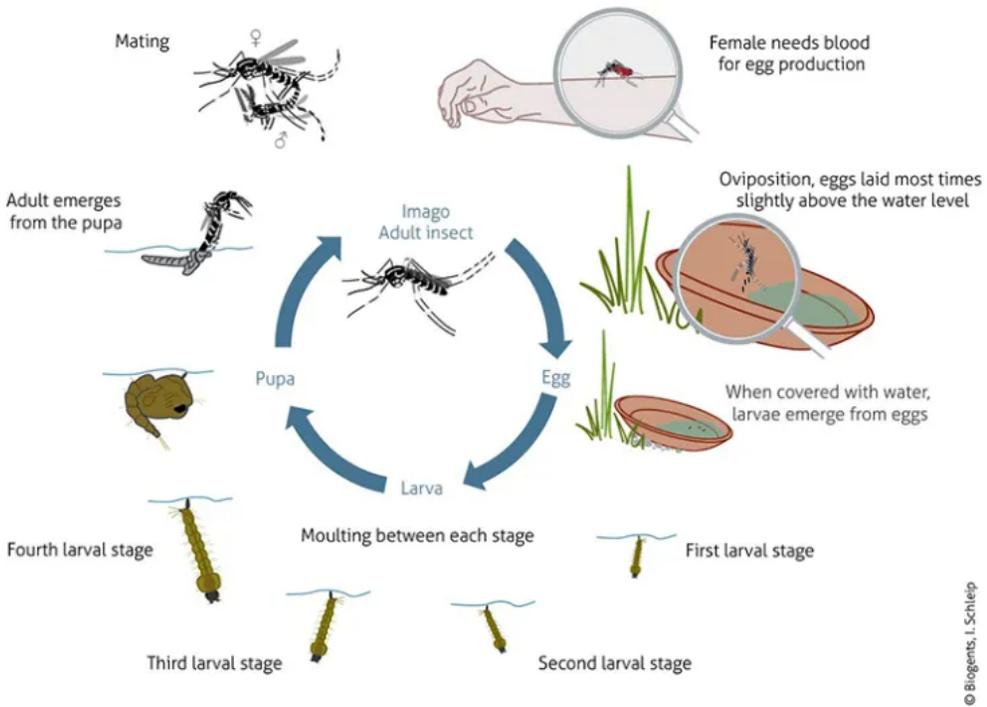


Figure 1: The life cycle of *Aedes* mosquitoes [3].

To describe the space-time dynamics of the mosquitoes it will be proposed a two-dimensional diffusion-reaction system, where the diffusion follows the Fick's Classical Law. The variable $A(x,y,t)$ represents the immature mosquitoes population at time t occupying the reference position (x,y) and $F(x,y,t)$ represents the adult females population at time t occupying the reference position (x,y) . These two populations are in a region $\Omega \equiv [0, L] \times [0, L]$ from space, 0 and L are the initial and final positions, respectively, and $(x,y) \in \Omega$. These dynamics occur during the time interval $I = [0, T]$, where 0 and T are the initial and final time, respectively, and $t \in I$.

Define $\{\varepsilon, \phi, C, \gamma, \alpha, \mu_A, \mu_F, u_A, u_F, \kappa_1, \kappa_2, L, T\} \in \mathbb{R}^+$. The problem is to find $A(x, y, t) \in \mathbb{R}^+$ and $F(x, y, t) \in \mathbb{R}^+$ for all $x \in \Omega$ and $t \in I$ satisfying

$$\begin{cases} \frac{\partial A(x, y, t)}{\partial t} = \kappa_1 \left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} \right) + \varepsilon \phi(d) \left(1 - \frac{A}{C(r)} \right) F - (\alpha(d) + \mu_A(r, d) + u_A)A, \\ \frac{\partial F(x, y, t)}{\partial t} = \kappa_2 \left(\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} \right) + \gamma \alpha(d)A - (\mu_F(d) + u_F)F, \end{cases} \quad (2.1)$$

with Neumann boundary conditions

$$\begin{aligned} A_x(0, 0, t) = 0, & \quad A_x(L, L, t) = 0, & \quad t > 0. \\ A_y(0, 0, t) = 0, & \quad A_y(L, L, t) = 0, & \quad t > 0. \\ F_x(0, 0, t) = 0, & \quad F_x(L, L, t) = 0, & \quad t > 0. \\ F_y(0, 0, t) = 0, & \quad F_y(L, L, t) = 0, & \quad t > 0. \end{aligned}$$

and initial conditions:

$$\begin{aligned} A(x, y, 0) &= g_1(x, y), \\ F(x, y, 0) &= g_2(x, y). \end{aligned}$$

Functions $g_1(x, y)$ and $g_2(x, y)$ will be defined in section 6. In this model, described by Eq.(2.1), each population, A and F , spread out in the domain according to their respective diffusion coefficients κ_1 and κ_2 . Thus, here the variable parameters are based on the temperature d and precipitation r expressions that favor vector growth and development in time and space, as shown in the section below. The immature forms are generated from the fraction ε of viable eggs from the daily oviposition $\phi(d)$ that adult female mosquitoes F deposit in potential breeding. The density of immature forms is regulated by the carrying capacity $C(r)$, which is limited by environmental conditions, space, and availability of organic matter found in water for food. Immature forms evolve to adult mosquitoes at a rate $\alpha(d)$, resulting in $\gamma\alpha(d)$ females and $(1 - \gamma)\alpha(d)$ males. For our proposal, only females are considered, so that female mosquitoes are vector-borne diseases. The death rate of immature (A) and adult female mosquitoes F are given by $\mu_A(r, d)$, and $\mu_F(d)$, respectively. The control functions u_A and u_F represent additional death for immature and adult female mosquitoes, respectively.

3 RAINFALL AND TEMPERATURE DATA

The study area includes the city of Lavras in the state of Minas Gerais, Brazil. In this region, the climate is subtropical, which is excellent for *Ae. aegypti* proliferation [18]. Figure 2 shows the location of the municipality of Lavras.

Total annual precipitation is about 1237 mm, and temperatures generally range between 11 °C in winter and 29 °C in summer. Daily data for 15 years of rainfall and temperature recorded in



Figure 2: Location of Lavras [6].

Lavras were obtained from the Brazilian National Institute of Meteorology (INMET). The data were then converted into a representative year formed by the average of these 15 years, as Figure 3 shows. Figure 4 shows that the average is a good measure of the central tendency for precipitation and temperature. We noticed similar behavior in the city's temperature and precipitation data, in the sense that the average of the 15-year period is representative of a general context for these data. It should be better to consider the average of the last few years than to simply use the data from the previous year, for example.

There is no controversy regarding the importance of temperature and rainfall in the development of *Ae. aegypti*, based on studies [33, 34]. In fact, the entomological parameters in several models are assumed to be considered as dependent on temperature and rainfall [7, 26, 35, 45]. Thus, here the variable parameters are based on temperature d and precipitation r expressions that favor vector growth and development in time and space, as shown below.

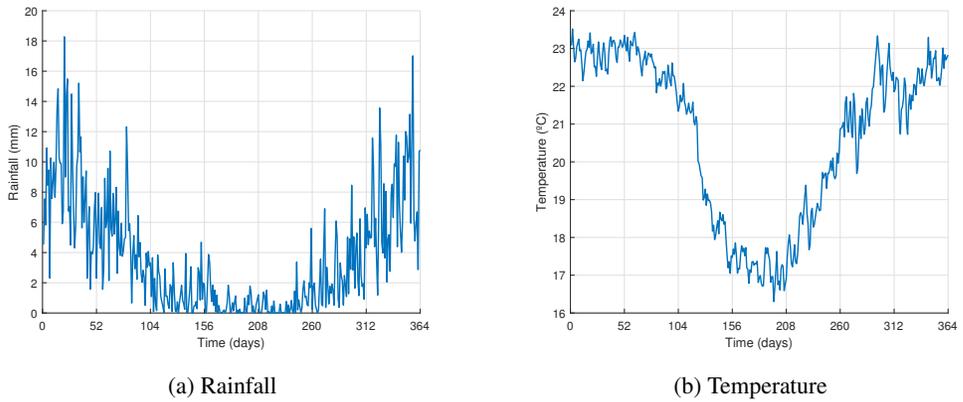


Figure 3: Average rainfall and temperature - Lavras/MG.

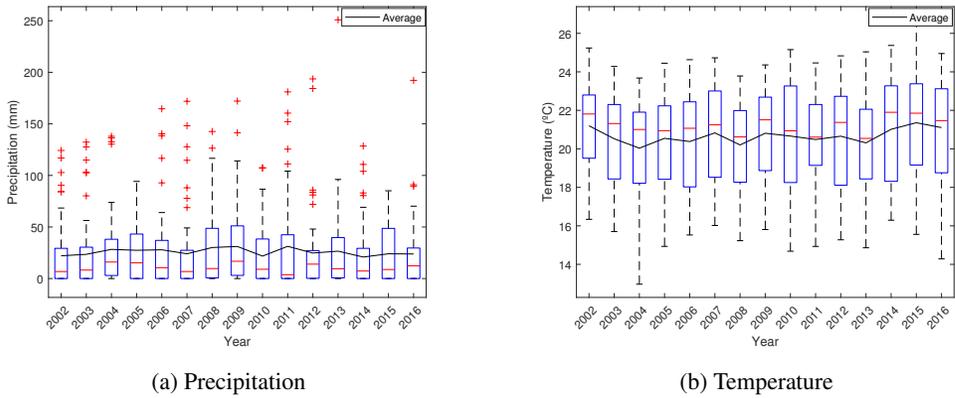


Figure 4: Average precipitation and temperature - Lavras/MG.

1. Carrying capacity adapted [27,30]

$$C(r) = \min C + \left(\frac{\max C - \min C}{\max r - \min r} \right) \cdot (r - \min r).$$

2. Oviposition rate [41]

$$\phi(d) = -5.400 + 1.800d - 2.124 \cdot 10^{-1}d^2 + 1.015 \cdot 10^{-2}d^3 - 1.515 \cdot 10^{-4}d^4.$$

3. Development rate of immature to adult [41]

$$\alpha(d) = 1.310 \cdot 10^{-1} - 5.723 \cdot 10^{-2}d + 1.164 \cdot 10^{-2}d^2 - 1.341 \cdot 10^{-3}d^3 + 8.723 \cdot 10^{-5}d^4 - 3.017 \cdot 10^{-6}d^5 + 5.153 \cdot 10^{-8}d^6 - 3.420 \cdot 10^{-10}d^7.$$

4. Natural death rate of female mosquitoes [41]

$$\mu_F(d) = 8.692 \cdot 10^{-1} - 1.590 \cdot 10^{-1}d + 1.116 \cdot 10^{-2}d^2 - 3.408 \cdot 10^{-4}d^3 + 3.809 \cdot 10^{-6}d^4.$$

5. Natural death rate of eggs, larvae and pupae [41]

$$\mu_A(d) = 2.130 - 3.797 \cdot 10^{-1}d + 2.457 \cdot 10^{-2}d^2 - 6.778 \cdot 10^{-4}d^3 + 6.794 \cdot 10^{-6}d^4. \quad (3.1)$$

$$\mu_A(r) = \min \mu_A + \left(\frac{\max \mu_A - \min \mu_A}{\max \text{rainfall} - \min \text{rainfall}} \right) \cdot (\text{rainfall} - \min \text{rainfall}). \quad (3.2)$$

The mortality in the aquatic phase, μ_A , is influenced by temperature and rainfall [7, 44]. The dependence can be considered assuming a function, so that $\mu_A(r, d) = a_0 + a_1\mu_A(r) + b_1\mu_A(d) + O(r^2, d^2)$. The linear part is considered to reinforce the association and fulfill the values of the literature. Therefore, $\mu_A(r, d) = \frac{\mu_A(d)}{2} + \frac{\mu_A(r)}{2}$ is defined by the average values obtained in the Eq. 3.1 and 3.2.

4 NUMERICAL FORMULATION

Equation (2.1) is composed of two naturally distinct operators: diffusion and reaction. Its numerical solution can be obtained from the sequential operator splitting technique [25] [13], in which each one of the processes is independently solved, and these individual results are coupled in each time step of the method. It is interesting because problems of different mathematical nature, in this case, diffusion and reaction systems, can be solved separately with proper numerical techniques.

The algorithm to solve the system (2.1) was based on the development presented by Wyse et al. [39] and Lima et al. [19]. In this methodology it is necessary to decompose the system (2.1) into two problems and proceed with the sequential operator splitting: a system of partial differential equations, more specifically the diffusion system, and ordinary differential equations associated with the reactive term. The diffusion problem is solved using the finite difference method, and the reaction problem is solved using the fourth-order Runge-Kutta method.

Introducing the temporal discretization $I = [0, T] = \bigcup_{n=0}^N I_n$, with $I_n = [t_n, t_{n+1}]$ being a I partition, and $N = T/\Delta t$ being the number of I partitions, such that $\Delta t = t_{n+1} - t_n$ is a uniform time step, we proceed with the algorithm:

Step 1: For $t = 0$, initialize the variables $A(x, y, 0) = g_1(x, y)$, and $F(x, y, 0) = g_2(x, y)$.

Step 2: For some fixed $n \geq 0$, given the initial conditions $A(x, y, t_n)$, and $F(x, y, t_n)$ and defining $\check{A}(t_n) = A(x, y, t_n)$, and $\check{F}(t_n) = F(x, y, t_n)$, calculate $\tilde{A}(x, y, t_n)$, and $\tilde{F}(x, y, t_n)$ at time t_{n+1} through the following problem:

Problem A: Find $\tilde{A}(x, y, t)$, and $\tilde{F}(x, y, t)$, defined in $(x, y) \in \Omega$, and $t \in I_n$, satisfying the system:

$$\begin{cases} \frac{\partial \tilde{A}(x, y, t)}{\partial t} = \kappa_1 \left(\frac{\partial^2 \tilde{A}(x, y, t)}{\partial x^2} + \frac{\partial^2 \tilde{A}(x, y, t)}{\partial y^2} \right), \\ \frac{\partial \tilde{F}(x, y, t)}{\partial t} = \kappa_2 \left(\frac{\partial^2 \tilde{F}(x, y, t)}{\partial x^2} + \frac{\partial^2 \tilde{F}(x, y, t)}{\partial y^2} \right), \end{cases}$$

with Neumann boundary conditions

$$\begin{aligned} \tilde{A}_x(0, 0, t_n) = \tilde{A}_x(L, L, t_n) = 0, \\ \tilde{F}_x(0, 0, t_n) = \tilde{F}_x(L, L, t_n) = 0, \end{aligned}$$

and initial conditions

$$\begin{aligned} \tilde{A}(x, y, t_n) &= \check{A}(t_n), \\ \tilde{F}(x, y, t_n) &= \check{F}(t_n). \end{aligned}$$

Step 3: In the same time interval I_n , we use the solution of Problem A as the initial condition to obtain the solution of the system of nonlinear ordinary differential equations associated with the reaction term of (2.1), which is given by the following problem:

Problem B: Find $A(t)$ and $F(t)$, defined in $t \in I_n$, satisfying the system:

$$\begin{cases} \frac{dA}{dt} = \varepsilon \phi \left(1 - \frac{A}{C} \right) F - (\alpha + \mu_A + u_A)A, \\ \frac{dF}{dt} = \gamma \alpha A - (\mu_F + u_F)F, \end{cases}$$

with initial conditions

$$\begin{aligned} A(t_n) &= \tilde{A}(x, y, t_{n+1}), \\ F(t_n) &= \tilde{F}(x, y, t_{n+1}), \end{aligned}$$

where $\tilde{A}(x, y, t_{n+1})$ and $\tilde{F}(x, y, t_{n+1})$ are the solutions obtained from Problem A.

Step 4: The solution for Problem B is the approximate solution of the model at time $t_{n+1} \in I_n \subset I$. If $t_{n+1} < T$, increment n , return to Step 2 and repeat the process until equality occurs.

The solution for Problem A was obtained using the explicit finite difference method [17]. To find this numerical solution with finite difference methods, we first need to define a set of grid points in the domain $\Omega \equiv [0, L] \times [0, L]$ choosing a uniform state step size $\Delta x = L/(M_1 + 1)$ and $\Delta y = L/(M_2 + 1)$, where M_1 and M_2 represents the number of mesh nodes (M_1 and M_2 is an integer) and a time step size $\Delta t = t_{n+1} - t_n$, which was previously defined at the beginning of the algorithm. We introduce an explicit finite difference method for solving the following two-dimensional time-dependent diffusion equation described in Problem 1, with the following approximation:

$$A_{i,j,n+1} = A_{i,j,n} + k_1 (A_{i-1,j,n} - 2A_{i,j,n} + A_{i+1,j,n} + A_{i,j-1,n} - 2A_{i,j,n} + A_{i,j+1,n}), \tag{4.1}$$

$$F_{i,j,n+1} = F_{i,j,n} + k_2 (F_{i-1,j,n} - 2F_{i,j,n} + F_{i+1,j,n} + F_{i,j-1,n} - 2F_{i,j,n} + F_{i,j+1,n}), \quad (4.2)$$

in which $A_{i,j,n}$ and $F_{i,j,n}$ are the values of populations A and F at a grid point (i, j, n) and $k_t = \frac{\kappa_t \Delta t}{h^2}$ with $h = \Delta x = \Delta y$ and $t = 1, 2$. The difference equations (4.1) and (4.2), $i = 1, \dots, M_{(1,2)} - 1$ and $j = 1, \dots, M_{(1,2)} - 1$, together with the initial and boundary conditions, defined in Problem A, were solved using the Gauss-Seidel algorithm.

Concerning Problem B given by the system of ordinary differential equations, we use the standard fourth-order Runge-Kutta method [17].

5 MULTI-OBJECTIVE OPTIMIZATION

In this section, we will present the mono-objective and multiobjective optimization problems.

5.1 Mono-objective optimization

This section deals with an optimization design to control *Ae. aegypti* mosquitoes, considering both immature and adult females to be controlled. In the first moment, we will address the mono-objective optimization problem. The goal is to minimize costs with larvicides and adulticides and reduce social costs by combating *Ae. aegypti* females. Thus, mono-objective optimization techniques are appropriate for this problem.

The decision variables are the control u_A and u_F , which correspond to the percentage control rates applied in the immature and adult female phases during the time interval t_A and t_F , respectively. The objective function J is a quadratic function that depends on the control and also on the number of mosquitoes, constrained to the model (2.1), and constrained to appropriate control according to their physical representation.

Consider $t_{max} = \max\{t_{A_1}, t_{A_2}, t_{F_1}\}$ as a month during 60 days of the summer. Thus, by the equation of a line, the descending control, in which the residual effect of the control decreased at each instant of time, is $u_A = -\frac{u_A}{\tau} \cdot t + \frac{u_A}{\tau} \cdot (t_0 + \tau)$, in which t_0 is the first day of control, chosen by the optimization algorithm, and τ is the number of days of the control application. In parallel, the way to apply the adulticides, u_F , at time t_F follows the step size control, in which there is no residual effect [36].

Therefore, control actions in practice must occur in places with a high rate of *Ae. aegypti* infestation. Consider two larvicides applications in the aquatic phase of the vector using a maximum of 50% in each application. The use of adulticide in the adult phase of females is used by observing the region of the city with the highest rate of *Ae. aegypti* infestation, which may reach 100% control.

These intervals were calculated based on the basic offspring number $Q_0 = \frac{\gamma\alpha}{(\alpha + \mu_A + u_A)} \frac{\epsilon\phi}{(\mu_F + u_F)}$ [35]. The interpretation of Q_0 in practice is that if $Q_0 > 1$, female mosquitoes manage to establish their population in a particular region of the domain, this region is classified as infested by the vector. If $0 < Q_0 < 1$, female mosquitoes cannot establish their population. Therefore,

the region is considered free of vector infestation. Figure 5 shows the regions where we express $Q_0(u_A, u_F) < 1$ and $Q_0(u_A, u_F) > 1$. With this, we can obtain the upper bounds to apply control in the aquatic and adult phases of *Ae. aegypti*.

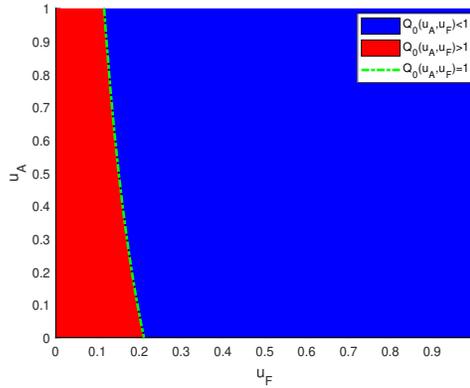


Figure 5: The region in blue color indicates the ordered pair (u_A, u_F) , where $Q_0(u_A, u_F) < 1$, while the region in red color indicates the ordered pair (u_A, u_F) , where $Q_0(u_A, u_F) > 1$.

Formally, the optimization problems are defined as follows:

$$\min_{u_{A_1}, u_{A_2}, u_{F_1}, t_{A_1}, t_{A_2}, t_{F_1}} J = C_1 \iiint_{\Omega \times I} u_A dx dy dt + C_2 \iiint_{\Omega \times I} u_F dx dy dt + C_3 \iiint_{\Omega \times I} F dx dy dt, \tag{5.1}$$

$$\text{subject to: } \left\{ \begin{array}{l} \frac{\partial A(x,y,t)}{\partial t} = \kappa_1 \left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} \right) + \varepsilon \phi(d) \left(1 - \frac{A}{C(r)} \right) F - (\alpha(d) + \mu_A(r,d) + u_A) A, \\ \frac{\partial F(x,y,t)}{\partial t} = \kappa_2 \left(\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} \right) + \gamma \alpha(d) A - (\mu_F(d) + u_F) F, \\ A_x(0,0,t) = 0, \quad A_x(L,L,t) = 0, \quad t > 0, \\ A_y(0,0,t) = 0, \quad A_y(L,L,t) = 0, \quad t > 0, \\ F_x(0,0,t) = 0, \quad F_x(L,L,t) = 0, \quad t > 0, \\ F_y(0,0,t) = 0, \quad F_y(L,L,t) = 0, \quad t > 0, \\ A(x,y,0) = g_1(x,y), \\ F(x,y,0) = g_2(x,y) \\ 0 \leq u_{A_1} \leq 0.50, \\ 0 \leq u_{A_2} \leq 0.50, \\ 0 \leq u_F \leq 0.21, \\ 2 \leq t_{A_1} \leq 20, \\ 30 \leq t_{A_2} \leq 50, \\ 2 \leq t_{F_1} \leq 50, \end{array} \right. \tag{5.2}$$

$$\begin{aligned}
 A_x(0, 0, t) = 0, & & A_x(L, L, t) = 0, & & t > 0. \\
 A_y(0, 0, t) = 0, & & A_y(L, L, t) = 0, & & t > 0. \\
 F_x(0, 0, t) = 0, & & F_x(L, L, t) = 0, & & t > 0. \\
 F_y(0, 0, t) = 0, & & F_y(L, L, t) = 0, & & t > 0.
 \end{aligned}$$

and initial conditions:

$$\begin{aligned}
 A(x, y, 0) &= g_1(x, y), \\
 F(x, y, 0) &= g_2(x, y).
 \end{aligned}$$

where C_1 is the relative cost to control the immature stage, C_2 is the relative cost to control adult females, C_3 is the social cost related to the number of female mosquitoes (hospital cost, medicines cost, personal cost), respectively. The parameters u_{A_1} and u_{A_2} are the intensity of larvicides applied on the immature stage until the time t_{A_1}, t_{A_2} . Finally, u_{F_1} is the intensity of adulticides applied on the adult female stage until the time t_{F_1} .

Therefore, we first numerically solve the dynamical system (2.1) (as explained in previous sections). From there, the optimization algorithm generates values for the decision variables $u_{A_1}, u_{A_2}, u_F, t_{A_1}, t_{A_2}, t_F$ and then uses the solution to evaluate the objective function of (5.1). The optimal values of the decision variables were obtained by a Real-Biased Genetic Algorithm (RBGA). For RBGA, the population size and the number of generations were considered 40. The individuals are represented by arrays in the computational simulations, which are decoded by the algorithm, producing a fitness value for each. The biased operator produces new solutions close to the best parent, with a particular perturbation and distance between the parents. The mutation causes small perturbations in the fitness values of some individuals, given a certain probability, to generate diversity in the population. The selection operator consists of randomly selecting individuals from the population, removing the ones with the lower fitness value [14, 32, 37].

5.2 Multiobjective optimization

From the mono-objective problem approach, it is pertinent to show the multiobjective optimization problem, showing the public health manager several possibilities for decision-making. The multiobjective optimization problem presents conflicting objectives: to reduce social costs with the control of *Ae. aegypti* females; minimize costs with larvicides and adulticides. We consider the same decision variables as in mono-objective problem $u_{A_1}, u_{A_2}, u_F, t_{A_1}, t_{A_2}, t_F$, as well as the control intervals, which are shown in Figure 5.

The multiobjective optimization problem is then stated in the following way: to minimize the adult female population in time and space, also considering the social costs and the control costs in immature and adult phases. Formally, the optimization problem is defined as follows:

$$\begin{cases}
 \min_{u_A, u_F, t_A, t_F} J_1 = C_1 \iiint_{\Omega \times I} u_A dx dy dt + C_2 \iiint_{\Omega \times I} u_F dx dy dt \\
 \min_{u_A, u_F, t_A, t_F} J_2 = C_3 \iiint_{\Omega \times I} F dx dy dt
 \end{cases} \tag{5.3}$$

$$\begin{aligned}
 & \left. \begin{aligned}
 & \frac{\partial A(x,y,t)}{\partial t} = \kappa_1 \left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} \right) + \varepsilon \phi(d) \left(1 - \frac{A}{C(r)} \right) F - (\alpha(d) + \mu_A(r,d) + u_A)A, \\
 & \frac{\partial F(x,y,t)}{\partial t} = \kappa_2 \left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} \right) + \gamma \alpha(d)A - (\mu_F(d) + u_F)F, \\
 & A_x(0,0,t) = 0, \quad A_x(L,L,t) = 0, \quad t > 0, \\
 & A_y(0,0,t) = 0, \quad A_y(L,L,t) = 0, \quad t > 0, \\
 & F_x(0,0,t) = 0, \quad F_x(L,L,t) = 0, \quad t > 0, \\
 & F_y(0,0,t) = 0, \quad F_y(L,L,t) = 0, \quad t > 0, \\
 & A(x,y,0) = g_1(x,y), \\
 & F(x,y,0) = g_2(x,y) \\
 & 0 \leq u_{A_1} \leq 0.50, \\
 & 0 \leq u_{A_2} \leq 0.50, \\
 & 0 \leq u_F \leq 0.21, \\
 & 2 \leq t_{A_1} \leq 20, \\
 & 30 \leq t_{A_2} \leq 50, \\
 & 2 \leq t_{F_1} \leq 50,
 \end{aligned} \right\} \text{subject to :} \tag{5.4}
 \end{aligned}$$

When dealing with a dynamic-population-based approach, NSGA-II is suitable for solving similar problems of multiobjective optimization [9, 11]. In multiobjective optimization problems, we are searching for a solution that is the best concerning all the objectives considered since the objectives are usually conflicting. The result of a multiobjective optimization algorithm is a set of solutions organized in the Pareto front. The NSGA-II algorithm has a fast procedure to classify the best individuals of this front, which are called non-dominated points, resulting in a combined non-dominated front. Furthermore, the NSGA-II has: *i*) elitism preservation approach to guarantee the best possible solutions to be present in the non-dominated front; *ii*) crowding distance comparison operator, for diversity preservation of the solutions [8].

Moreover, none of these solutions is better than any other. So, the advantage of the multiobjective optimization approach is that it corresponds to the variation of mono-objective scenarios. It means that the Pareto front shows several available solutions for the decision-maker. When analyzing such solutions, the decision-maker can prioritize the criterion that he considers the most important according to the objective functions.

6 RESULTS AND DISCUSSION

This section reports the results obtained when we apply computational techniques to solve numerically the proposed diffusion-reaction model coupled with multiobjective and mono-objective optimization. We implemented the methods using the programming language C version C90 and compiled them with GCC 4.9.2. The simulations were performed on a computer with an Intel® Core™ i7-8550 processor, with 8GB RAM, and Windows 10 Home Single Language 64bit operating system.

All numerical experiments consider that mosquitoes occupy a spatial region $\Omega = [0, 20] \times [0, 20]$ m, which is equivalent to an area of 400 square meters in a neighborhood, during the time interval $I = [0, 60]$ days, referring to a period in the summer season. In practice, this region has two residences within one block. This band is large enough that mosquitoes are not affected by the boundary conditions imposed by the mathematical model. This work performed an extensive search of the model (2.1) parameters' values: ε , ϕ , C , γ , α , μ_A , μ_F , to better fit the obtained results to the real-world conditions regarding the *Ae. aegypti* dynamics. Table 1 shows these values.

Table 1: Entomological parameters for the *Aedes aegypti* model (2.1).

Parameters	Description	Values	Unity	Reference
ε	Fraction of eggs hatching	0.5	day ⁻¹	[1]
ϕ	Oviposition rate	1.060327 – 8.294997	day ⁻¹	[42]
C	Carrying capacity	50 – 200	dimensionless	[45]
γ	Fraction of female mosquitoes	0.5	day ⁻¹	[1, 44]
α	Development rate of immature to adult	0.02615 – 0.11612	day ⁻¹	[42]
μ_A	Natural death rate of eggs, larvae and pupae	0.01397 – 0.06001	day ⁻¹	[42]
μ_F	Natural death rate of female mosquitoes	0.028773 – 0.035859	day ⁻¹	[43]

The aquatic phase of *Ae. aegypti* can be transported in a region as dispersion (flower pots, domestic animal feeding dishes, and other unattended wet or rain-filled containers) provide breeding sites for mosquitoes. Therefore, the diffusion coefficient assumed as $\kappa_1 = 3.1 \times 10^{-9}$ km²/day [10] [2].

Additionally, we consider that the females of *Ae. aegypti* are dispersed for 100 meters per day [12]. Therefore, the diffusion coefficient can be obtained from $\kappa_2 = \langle x \rangle^2 / q_i t$, with $\langle x \rangle^2$ the mean square displacement and q_i a numerical constant related to dimensionality ($q_i = 2, 4$, or 6 , for 1, 2, or 3-dimensional diffusion, respectively), resulting in $\kappa_2 = 3.1 \times 10^{-6}$ km²/day [2] [23].

The additional death of immature and adult mosquitoes u_A and u_F are obtained via optimization, as well as the control duration times t_A and t_F . The relative costs are considered as $C_1 = 10$, $C_2 = 100$ [4], and $C_3 = 0.01$ [30, 36]. The optimization procedure used the Real-Biased Genetic Algorithm and Non-dominated Sorting Genetic Algorithm II, considering the parameters in Tables 2 and 3, respectively.

Table 2: Parameters of the RBGA.

Parameters	Values	Reference
Mutation rate	5%	[32] [36]
Recombination rate	90%	[32] [36]
Biased crossover probability	30%	[32] [36]
Simulations	30	[32] [36]

Table 3: Parameters of the NSGA-II.

Parameters	Values	Reference
Mutation rate	5%	[8]
Recombination rate	90%	[8]
Simulations	30	[36]

Mono-objective optimization results

For comparison purposes, in the numerical simulations, we use two scenarios in which we consider an arbitrary urban 20 meters of a neighborhood composed of houses and a street:

- Scenario 1 shows the diffusion-reaction with a mosquito focus and without control in 20 meters;
- Scenario 2 shows the diffusion-reaction model with only one mosquito focus. In this case, the descending control is considered for the immature phase with two control applications and the step size control for the adult phase. The reason for considering this scenario is that, in practice, instead of the public health manager planning actions to carry out the control in the considered region, the control will only be carried out in places where there is a large infestation of *Ae. aegypti*.

In partial differential equation problems, it is common to use trigonometric functions as initial conditions [46, 47]. Scenarios 1 and 2 consider the same amount of aquatic and adult female stages as an initial condition (6.1). This initial condition is chosen since it is considered that the breeding sites are distributed in a region of the study area. The same parameters are considered for x and y . Furthermore, a region in the middle of the domain is considered and the effect of mosquito dispersal from a neighboring area is analyzed. The following initial condition describes this initial distribution of aquatic forms and adult females. Eq. (6.1) shows these initial conditions.

$$\begin{cases}
 A(x,y,0) = 10 \left(\sin \left(\pi \left(\frac{x-1}{19-1} \right) \left(\frac{y-1}{19-1} \right) \right) \right)^{100}, & \text{if } 1 \leq (x,y) \leq 19, \\
 F(x,y,0) = 10 \left(\sin \left(\pi \left(\frac{x-1}{19-1} \right) \left(\frac{y-1}{19-1} \right) \right) \right)^{100}, & \text{if } 1 \leq (x,y) \leq 19, \\
 A(x,y,0) = F(x,y,0) = 0, & \text{if } 0 \leq (x,y) < 1 \text{ or } 19 < (x,y) \leq 20.
 \end{cases} \tag{6.1}$$

Scenario 1

The population concentrations for both mosquito phases (aquatic and winged females) are shown for the same time in Figure 6 (a) and (b) and when $t = 0$. Note that in the study region, the amount of *Ae. aegypti* varies from 0 to 10 depending on the sinusoid. In practice, one can consider a residence in the center of the study region. Scenario 1 considers the numerical simulation of model (2.1) without additional control, with initial conditions (6.1) over a two-dimensional spatial domain $\Omega = [0, 20] \times [0, 20]$ meters and 60 days in the summer. Figure 7 shows the distribution in time and two-dimensional population space in the aquatic stage and mosquitoes in the female adult phase. In Figure 7, it can be seen the growth in the habitat of the adult female and aquatic populations over time, according to temperature and rainfall data in the city.

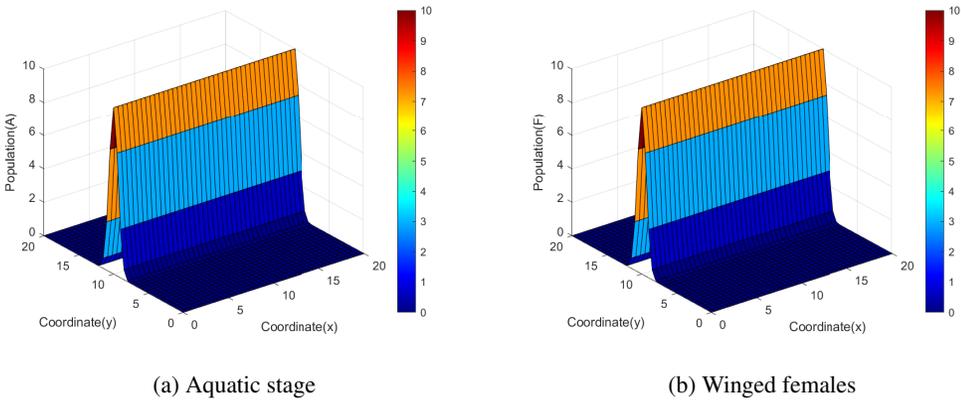


Figure 6: Mosquitoes initial population dispersion for Scenario 1.

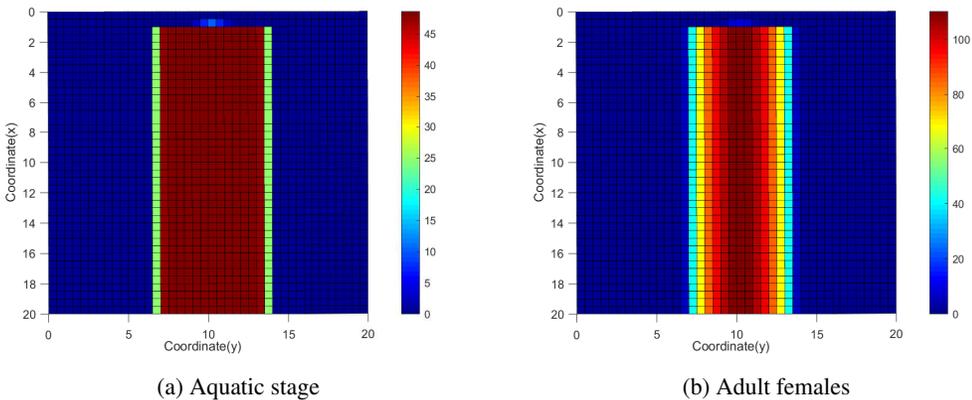


Figure 7: Mosquitoes population dispersion for Scenario 1.

Scenario 2

We are considering the same initial condition (6.1) for this numerical experiment to better compare with the previous Scenario 1, but taking into account significant constant control parameters. Scenario 2 considers the decrease in control in the aquatic phase during two 10-day applications and one control application in the adult phase of females during one day. In practice, this idea is to control the spreading of *Ae. aegypti* in both aquatic and female adult phases. For this, we obtained $u_{A_1} + u_{A_2} = 93\%$ and $u_F = 3\%$, resulting from the optimization problem, and considering $t_{A_1} + t_{A_2} = 20$ days, and $t_F = 1$ day almost nine epidemiological weeks during the summer season. These results show that the well-executed control in the aquatic phase prevents the development of eggs in new adult females, collaborating to reduce control in the adult phase. Table 4 summarizes the values obtained for the mono-objective optimization problem.

Table 4: Optimal control variables for Scenario 2.

Parameters	u_{A_1}	t_{A_1}	u_{A_2}	t_{A_2}	u_F	t_{F_1}	J	Ef.
Values	47%	7	46%	33	3%	31	2906	13%

Note that there is a significant decrease in the number of mosquitoes during the simulation, as shown in Figure 8, and ratified by the value of the objective function described in Table 4. When the mono-objective optimization algorithm is applied in the model (2.1), where the value of the objective function is $J = 2906$, and we have an efficiency of 13% in the control of the *Ae. aegypti* mosquito in 60 days, with a control application for around 20 days for aquatic phase mosquitoes and 1 day for adult females.

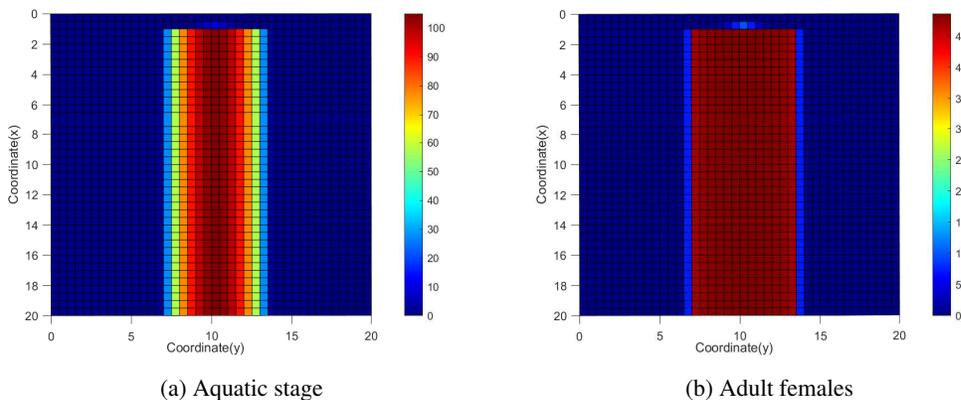


Figure 8: Mosquitoes population dispersion for Scenario 2.

Figure 9 shows the convergence of the optimal functional Eq. (5.1) value using the RBGA algorithm for Scenario 2. It can be seen when analyzing Figure 9 that as the number of generations increased, there was an improvement in the value of the objective function. There was also a reduction in its variability, causing the size of the boxplot box to decrease. The quality of the

optimal values is satisfactory. The best value was that which, among the 30 executions of the genetic algorithm, had the lowest objective function value.

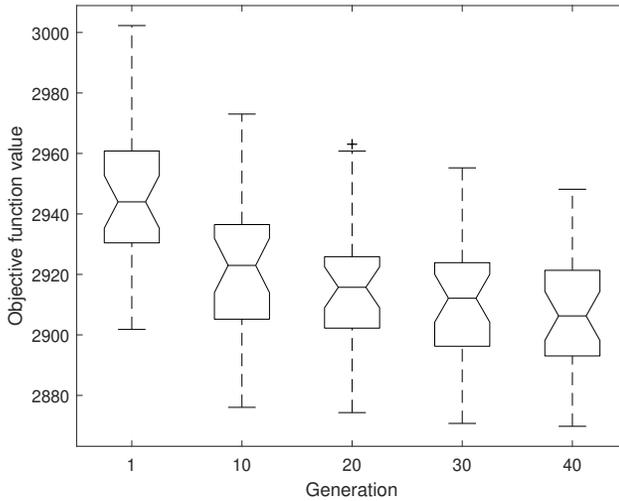


Figure 9: Evolution of the objective function value for Scenario 2 over the generations in the 30 simulations.

Multiobjective optimization results

Figure 10 (a) and (b) show the decision variable spaces for the two larvicide control applications in the aquatic phase. The red dots represent the optimal values found in the mono-objective problem $u_{A_1} = 0.47$ and $u_{A_2} = 0.46$ at times $t_{A_1} = 7$ and $t_{A_2} = 33$, respectively. In the same way, the mono-objective optimal point for adulticide control in adult females, $u_F = 0.03$ in $t_F = 31$, is present in the decision variable space shown in 10. In the three figures, it is possible to verify that the optimal points found are present in the entire decision space, focusing on the upper bounds of time and the amount of descending control and stepping to apply.

The problem is to minimize two functions: cost with control on the x-axis and cost with the treatment of females on the y-axis. As this is a multiobjective problem, improving one of the objectives means making the other worse. Figure 11 shows the Pareto front found. Let us analyze three points: i) point M represents a higher cost with the treatment of females and a low cost with control; ii) point P represents a more significant amount of control, lower cost of treating females; iii) when we take the knee point N, we are trying to make a balance between the two objectives, that is, low cost of control and low cost of treating females. Furthermore, the value of the mono-objective function represented by the red dot is dominated by the optimal solutions. Thus, the results indicate that it is possible to find optimal control policies balancing the costs of objective function 5.3. Therefore, it is up to the public health manager to make the decision that best meets the needs concerning the costs involved with *Ae. aegypti* control policies.

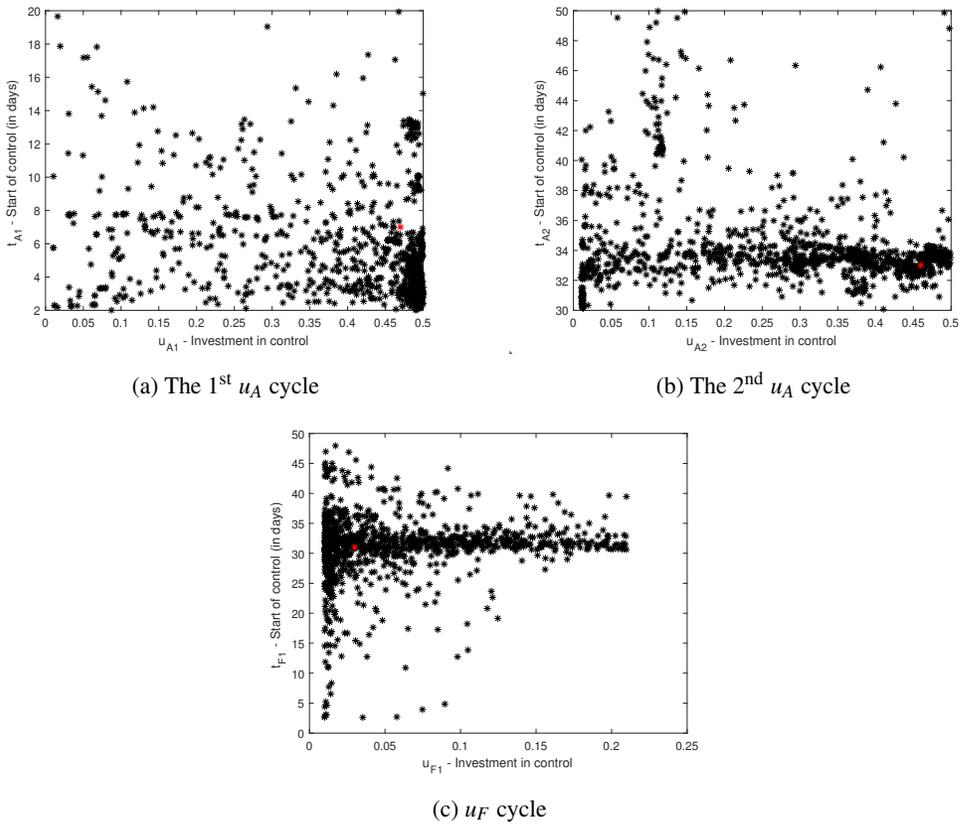


Figure 10: Decision variable space of investment in larvicide and adulticide control. In red, is the respective mono-objective optimal point.

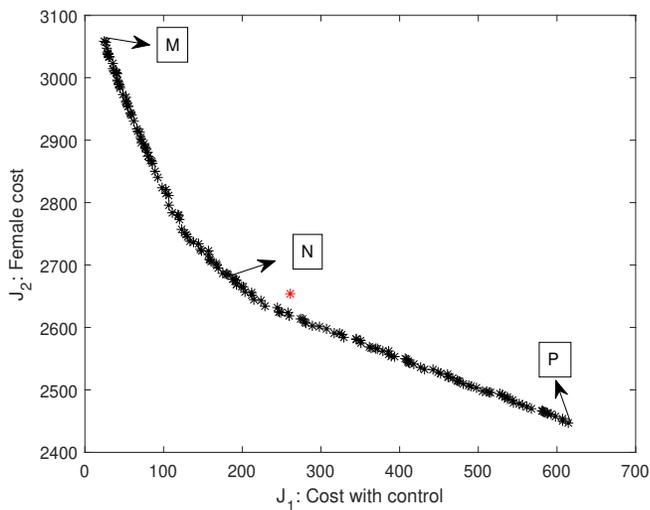


Figure 11: Dominance of Pareto-optimal.

7 CONCLUSION

This paper describes a mono-objective and multiobjective optimization approach to reduce mosquito intensity and *Ae. aegypti* related costs, whose population is described by a two-dimensional diffusion-reaction model that describes the interaction between aquatic and adult female stages spread across a domain. Furthermore, the model parameters depend on real rainfall and temperature data from the case study considering the city of Lavras/Brazil.

The results of the mono-objective optimization indicate a vector reduction in the aquatic and the adult phase of females based on the descending and step control, cost reduction, and efficiency of 13%. That is, the numerical simulations showed that the control interventions successfully reduced the costs and infestation of *Ae. aegypti*. Through multiobjective optimization, the public health manager has a range of possibilities for the public manager to implement a successful control action following his financial and social reality. However, our paper opens up possibilities for future work, such as coupling the model to human populations and considering monetary costs in the optimization process.

REFERENCES

- [1] J.H. Arias, H.J. Martinez, L.S. Sepúlveda-Salcedo & O. Vasilieva. Predator-prey model for analysis of *Aedes aegypti* population dynamics in Cali, Colombia. *International Journal of Pure and Applied Mathematics*, **4**(105) (2015), 561–597. doi:10.12732/ijpam.v105i4.2.
- [2] R. Barrera, M. Amador & G.G. Clark. Use of the pupal survey technique for measuring *Aedes aegypti* (Diptera: Culicidae) productivity in Puerto Rico. *The American journal of tropical medicine and hygiene*, **74**(2) (2006), 290–302.
- [3] Biogents. Life Cycle of *Aedes* Mosquitoes. <https://sea.biogents.com/life-cycle-aedes-mosquitoes/> (2023).
- [4] D. Cardona-Salgado, D.E. Campo-Duarte, L.S. Sepúlveda-Salcedo & O. Vasilieva. Wolbachia-based biocontrol for dengue reduction using dynamic optimization approach. *Applied Mathematical Modelling*, **82** (2020), 125–149.
- [5] S.A. Carvalho, S.O. da Silva & I.d.C. Charret. Mathematical modeling of dengue epidemic: control methods and vaccination strategies. *Theory in Biosciences*, **138**(2) (2019), 223–239.
- [6] W. Commons. File:Minas Gerais Meso Micro Municip. https://commons.wikimedia.org/wiki/File:MinasGerais_MesoMicroMunicip.svg (2006).
- [7] F. Cordeiro, Á. Eiras, F. Silva & J. Acebal. A Model for *Aedes aegypti* Infestation According to Meteorological Variables: case of Caratinga (Minas Gerais-Brazil). *Trends in Computational and Applied Mathematics*, **22** (2021), 61–78.
- [8] K. Deb, A. Pratap, S. Agarwal & T. Meyarivan. A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE transactions on evolutionary computation*, **6**(2) (2002), 182–197.

- [9] W.O. Dias, E.F. Wanner & R.T.N. Cardoso. A multiobjective optimization approach for combating *Aedes aegypti* using chemical and biological alternated step-size control. *Mathematical biosciences*, **269** (2015), 37–47. doi:10.1016/j.mbs.2015.08.019.
- [10] C.P. Ferreira, P. Pulino, H.M. Yang & L.T. Takahashi. Controlling Dispersal Dynamics of *Aedes aegypti*. *Mathematical Population Studies*, **13**(4) (2006), 215–236. doi:10.1080/08898480600950515.
- [11] H.O. Florentino, D.R. Cantane, F.L. Santos & B.F. Bannwart. Multiobjective genetic algorithm applied to dengue control. *Mathematical biosciences*, **258** (2014), 77–84.
- [12] R.M.D. Freitas, C.T. Codeco & R.L.D. Oliveira. Daily Survival Rates and Dispersal of *Aedes aegypti* Females in Rio de Janeiro, Brazil. *The American journal of tropical medicine and hygiene*, **76**(4) (2007), 659–665. doi:10.4269/ajtmh.2007.76.659.
- [13] J. Geiser. “Decomposition Methods for Differential Equations: Theory and Applications”. CRC Press, New York (2009).
- [14] D.E. Goldenberg. “Genetic algorithms in search, optimization and machine learning”. Addison-Wesley Publishing, United States (1989).
- [15] K. Hattaf & N. Yousfi. Global stability for reaction–diffusion equations in biology. *Computers & Mathematics with Applications*, **66**(8) (2013), 1488–1497.
- [16] B.E. Hobbs & A. Ord. “Structural geology: the mechanics of deforming metamorphic rocks”. Elsevier (2014).
- [17] A. Iserles. “A First Course in the Numerical Analysis of Differential Equations”. Cambridge University Press, United Kingdom (2009).
- [18] LAVRASPREV. Características geográficas. <https://lavrasprev.mg.gov.br/Caracter\%C3\%ADsticas+geogr\%C3\%A1ficas/id/50> (2017).
- [19] J. Lima, A. Wyse, A. Santos & R. Cardoso. Reaction-diffusion Model Applied to the Population Dynamics of Wild and Transgenic Mosquitoes. *TEMA (São Carlos)*, **21**(3) (2020), 505–519.
- [20] F.E. Moustaid & L.R. Johnson. Modeling Temperature Effects on Population Density of the Dengue Mosquito *Aedes aegypti*. *Insects*, **10**(11) (2019), 393. doi:10.3390/insects10110393.
- [21] L. Multerer, T. Smith & N. Chitnis. Modeling the impact of sterile males on an *Aedes aegypti* population with optimal control. *Mathematical biosciences*, **311** (2019), 91–102.
- [22] J. Murray. “Mathematical biology: I. An introduction”. Springer, New York (2001).
- [23] J. Murray. “Mathematical Biology II: Spatial Models and Biomedical Applications”. Springer, New York (2003).
- [24] W.H. Organization. Dengue and severe dengue. <https://www.ioc.fiocruz.br/dengue/textos/curiosidades.html>.
- [25] D.W. Peaceman & H.H. Rachford, Jr. The numerical solution of parabolic and elliptic differential equations. *Journal of the Society for industrial and Applied Mathematics*, **3**(1) (1955), 28–41.

- [26] E. Pliego-Pliego, O. Vasilieva, J. Velázquez-Castro & A.F. Collar. Seasonality on the life cycle of *Aedes aegypti* mosquito and its statistical relation with dengue outbreaks. *Applied Mathematical Modelling*, **50** (2017), 484 – 496.
- [27] S. Polwiang. The seasonal reproduction number of dengue fever: impacts of climate on transmission. *PeerJ*, **3** (2015), e1069.
- [28] J.R. Powell, A.G. Soria & P. Kotsakiozi. Recent History of *Aedes aegypti*: Vector Genomics and Epidemiology Records. *Bioscience*, **68**(11) (2018), 854–860. doi:10.1093/biosci/biy119.
- [29] H.S. Rodrigues, M.T.T. Monteiro & D.F.M. Torres. Vaccination models and optimal control strategies to dengue. *Mathematical biosciences*, **247** (2014), 1–12. doi:10.1016/j.mbs.2013.10.006.
- [30] L.S.B. Silva, A.S.V. Vasconcelos, A.L. Sanches, R.T.N. Cardoso, J.L.A. Fernandes & Á.E. Eiras. Otimização Mono-objetivo no Controle do Mosquito *Aedes aegypti* por meio de um Modelo de Duas Populações com Influência da Precipitação. *TEMA (São Carlos)*, **20**(1) (2019), 197–216. doi:10.5540/tema.2019.020.01.0197.
- [31] L.T. Takahashi, N.A. Maidana, W.C. Ferreira, P. Pulino & H.M. Yang. Mathematical models for the *Aedes aegypti* dispersal dynamics: travelling waves by wing and wind. *Bulletin of mathematical Biology*, **67**(3) (2005), 509–528.
- [32] R.H.C. Takahashi, J.A. Vasconcelos, J.A. Ramírez & L. Krahenbuhl. A multiobjective methodology for evaluating genetic operators. *IEEE Transactions on Magnetics*, **39**(3) (2003), 1321–1324.
- [33] B.L. Tran, W.C. Tseng, C.C. Chen & S.Y. Liao. Estimating the threshold effects of climate on dengue: a case study of Taiwan. *International journal of environmental research and public health*, **17**(4) (2020), 1392.
- [34] L.D. Valdez, G.J. Sibona & C. Condat. Impact of rainfall on *Aedes aegypti* populations. *Ecological Modelling*, **385** (2018), 96–105.
- [35] A.S. Vasconcelos, J.S. Lima, R.T. Cardoso, J.L. Acebal & A.M. Loaiza. Optimal control of *Aedes aegypti* using rainfall and temperature data. *Computational and Applied Mathematics*, **41**(3) (2022), 1–24.
- [36] A.S. Vasconcelos, L.S. Silva, R.T. Cardoso & J.L. Acebal. Optimization of a rainfall dependent model for the seasonal *Aedes aegypti* integrated control: a case of Lavras/Brazil. *Applied Mathematical Modelling*, **90** (2021), 413–431.
- [37] J.A. Vasconcelos, J.A. Ramirez, R.H.C. Takahashi & R.R. Saldanha. Improvements in genetic algorithms. *IEEE Transactions on magnetics*, **37**(5) (2001), 3414–3417. doi:10.1109/20.952626.
- [38] J. Wong, S.T. Stoddard, H. Astete, A.C. Morrison & T.W. Scott. Oviposition Site Selection by the Dengue Vector *Aedes aegypti* and Its Implications for Dengue Control. *PLoS Negl Trop Dis*, **5**(4) (2011), e1015. doi:10.1371/journal.pntd.0001015.
- [39] A.P. Wyse, A.J.B. Santos, J.S. Azevedo, J.S. Lima & J.R. Faria. Modeling the spreading and interaction between wild and transgenic mosquitoes with a random dispersal. *PLoS one*, **13**(10) (2018), e0205879. doi:10.1371/journal.pone.0205879.

- [40] W.M.S. Yamashita, S.S. Das & G. Chapiro. Numerical modeling of mosquito population dynamics of *Aedes aegypti*. *Parasites & vectors*, **11**(1) (2018), 245. doi:10.1186/s13071-018-2829-1.
- [41] H. Yang, M. Macoris, K. Galvani, M. Andrighetti & D. Wanderley. Assessing the effects of temperature on dengue transmission. *Epidemiology & Infection*, **137**(8) (2009), 1179–1187.
- [42] H.M. Yang. Assessing the Influence of Quiescence Eggs on the Dynamics of Mosquito *Aedes aegypti*. *Applied Mathematics*, **5**(17) (2014), 2696. doi:10.4236/am.2014.517257.
- [43] H.M. Yang. The transovarial transmission in the dynamics of dengue infection: Epidemiological implications and thresholds. *Mathematical biosciences*, **286** (2017), 1–15.
- [44] H.M. Yang, J.L. Boldrini, A.C. Fassoni, L.F.S. Freitas, M.C. Gomez, K.K.B. de Lima, V.R. Andrade & A.R.R. Freitas. Fitting the Incidence Data from the City of Campinas, Brazil, Based on Dengue Transmission Modelling Considering Time-Dependent Entomological Parameters. *PloS One*, **11**(3) (2016), e0152186. doi:10.1371/journal.pone.0152186.
- [45] H.M. Yang & C.P. Ferreira. Assessing the effects of vector control on dengue transmission. *Applied Mathematics and Computation*, **198**(1) (2008), 401–413.
- [46] M. Zhou, H. Xiang & Z. Li. Optimal control strategies for a reaction-diffusion epidemic system. *Nonlinear Analysis: Real World Applications*, **46** (2019), 446–464.
- [47] D. Zhu, J. Ren & H. Zhu. Spatial-temporal basic reproduction number and dynamics for a dengue disease diffusion model. *Mathematical Methods in the Applied Sciences*, **41**(14) (2018), 5388–5403. doi:10.1002/mma.5085.

