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HRL-Local Infinite Triangular Array Languages

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ABSTRACT

A new subclass of infinite triangular arrays called hrl-local infinite triangular arrays is introduced. We introduce infinite triangular domino systems to recognize the infinite triangular picture language. Also we introduce strictly domino testable ωω*-triangular array languages.*

Key words: hrl-local infinite triangular arrays, infinite triangular domino systems, strictly dommo testable ωω-triangular array languages.

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INTRODUCTION

Infinite triangular pictures are the digitized images which occur in the triangular grid of two dimensional plane. Infinite triangular picture p is a triangular array of elements of the terminal alphabet. It is useful to introduce the notation $\Sigma_T^{\omega\omega}$ for the set of all infinite triangular pictures over the same alphabet Σ. Infinite triangular picture has infinite number of rows and infinite number of right slanting lines and infinite number of left slanting lines. The size $|p|$ of a picture p is specified by the pair $(|p|_{row}, |p|_{rsline}, |p|_{lsline})$ of its number of rows and right slanting lines and left slanting lines. A pixel p(i, j, k), $1 \leq i \leq |p|_{row}$, $1 \leq j \leq |p|_{rsline}$, $1 \leq j \leq |p|_{lsline}$ is the element at position (i, j, k) in the triangular array P. Conventionally the indices grow from bottom to top for the rows and from right to left for left slanting lines and from left to right for right slanting lines.

For convenience we usually consider the bordered version of picture p obtained by surrounding the picture with the special boundary symbol # which is assumed not to be in the alphabet. In [2] domino recognizability of triangular picture languages and hrl-domino systems are defined. Also in [2] we define the overlapping of iso-triangular pictures.

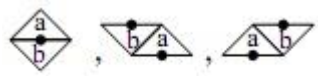
In [1] we learn about recognizable infinite array languages. In [6] we study the recognizability of infinite arrays and in [3] we define the recognizability of infinite triangular pictures.

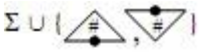
In this paper we consider another formalism hrl-domino system to recognize infinite triangular picture languages.

INFINITE TRIANGULAR PICTURE AND DOMINO SYSTEMS

Infinite triangular picture has infinite number of rows and infinite number of right slanting and left slanting lines. The set of all infinite triangular arrays over Σ is denoted by $\Sigma_{IT}^{\omega\omega}$.

In [4,5] hv-local picture languages are defined, where the square tiles of side 2 are replaced by dominoes that correspond to two kinds of tiles (i) horizontal dominoes of size (1,2) and (ii) vertical dominoes of size (2, 1). As in [2] here we consider dominoes of the following types



Definition 1. $L \subseteq \Sigma_{IT}^{\omega\omega}$ is hrl-local if there exists a finite set Δ of dominoes over  such that $L = \{p \in \Sigma_{IT}^{\omega\omega} \,/\, B_{2,1}(\hat{p}) \subseteq \Delta\}$. In this case we write L = L(Δ).

Definition 2. An infinite triangular domino system is a 4-tuple ITD = (Σ, Γ, Δ, π) where Σ and Γ are two finite alphabets, Δ is a finite set of dominoes over the alphabet $\Gamma \cup$  and $\pi : \Gamma \rightarrow \Sigma$ is a projection.

An infinite triangular domino system recognizes an infinite triangular picture language L over the alphabet Σ and is defined as $L = \pi(L')$ where $L' = L(\Delta)$ is the hrl-local infinite triangular picture language over Γ. The family of infinite triangular picture languages recognizable by infinite triangular domino systems is denoted by L(ITDS).

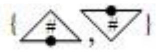
Proposition 1. If $L \subseteq \Sigma_{IT}^{\omega\omega}$ is a hrl-local infinite triangular picture language then L is a local infinite triangular picture language. That is L(ITDS) $\subseteq$ L(ITTS).

Proof. Let $L \subseteq \Sigma_{IT}^{\omega\omega}$ is a hrl-local infinite triangular picture language. Then L = L(Δ) where Δ is a finite set of dominoes. We will construct a finite set θ of $\omega\omega$-triangular arrays of size 2 and show that L = L(θ).

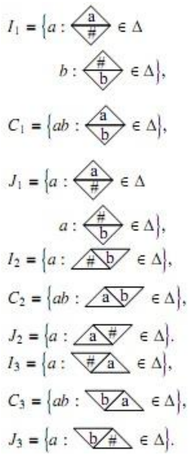
We now show that L′ = L. Let p $\in$ L′. Then by definition $B_{1,2}(\hat{p}) \in \theta$. This implies that
$B_{2,1}(\hat{p}) \subseteq B_{2,1}(B_{2,1}(\hat{p})) \subseteq B_{2,1}(\theta) \subseteq \Delta$. Hence p $\in$ L.

Conversely let p $\in$ L and q $\in$ $B_{1,2}(\hat{p})$. Then $B_{2,1}(q) \subseteq B_{2,1}(\hat{p}) \subseteq \Delta$. Therefore q $\in$ θ and p $\in$ L′.
Hence L = L′.
□

Lemma 1. Let L be a local $\omega\omega$-triangular picture language over an alphabet Σ. Then there exists an hrl local language L′ over an alphabet and a mapping $\pi : \Gamma \to \Sigma$ such that L = π(L′).

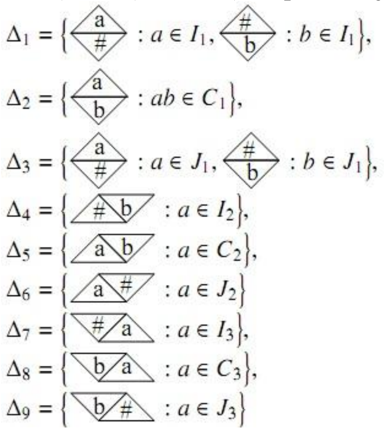
Proof. Let L = L(θ) where θ is a finite set of $\omega\omega$-triangular arrays over $\Sigma \cup$


Let S_1, S_2, $S_3 \subseteq \Sigma_T^{**}$ be three strings languages over Σ. The hrl-combination of S_1, S_2, S_3 is an triangular array language $L_T = S_1 \oplus S_2 \oplus S_3 \subseteq \Sigma_T^{**}$ such that p $\in$ L if and only if the strings corresponding to the hrl overlapping of p belong to S_1, S_2 and to S_3 respectively. Also we have to prove that the class of all hrl-local triangular array languages is the hrl combination of the class of all local string languages.
□

Theorem 1. hrl-TLOC = TLOC $\oplus$ TLOC $\oplus$ TLOC.
Proof. Let L $\in$ hrl-TLOC. Then $L = L_T^{**}(\Delta)$, for a finite set of dominoes over $\Sigma \cup$ {#}. Let


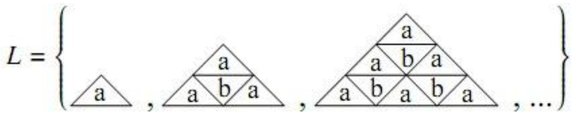
Let L_1, L_2 and L_3 be the hrl-local string languages generated by the local sys-tems (I_1, C_1, J_1), (I_2, C_2, J_2) and (I_3, C_3, J_3) respectively. Then $L = L_1 \oplus L_2 \oplus L_3 \in \text{TLOC} \oplus \text{TLOC} \oplus \text{TLOC}$.

Conversely let $L \in \text{TLOC} \oplus \text{TLOC} \oplus \text{TLOC}$. Let $L = L_1 \oplus L_2 \oplus L_3$ where L_1, L_2 and L_3 are the local string languages generated by the local systems (I_1, C_1, J_1), (I_2, C_2, J_2) and (I_3, C_3, J_3) respectively. Let

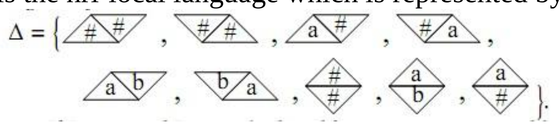


Let $\Delta = \Delta_1 \cup \Delta_2 \cup \Delta_3 \cup \Delta_4 \cup \Delta_5 \cup \Delta_5 \cup \Delta_6 \cup \Delta_7 \cup \Delta_8 \cup \Delta_9$. Then $L = L_T^{**}(\Delta)$ and therefore $L \in$ hrl-TLOC.

Example 1.



is the hrl-local language which is represented by the following set of dominoes.



If L_1, L_2 and L_3 are the local languages generated by the local systems $S_1 = \{I_1, C_1, J_1\}$ and $S_2 = \{I_2, C_2, J_2\}$ respectively where

$I_1 = \{a\}$, $C_1 = \{ab\}$, $J_1 = \{a\}$,

$I_2 = \varphi$, $C_2 = \{ab\}$, $J_2 = \{a\}$,

$I_3 = \{a\}$, $C_3 = \{ab\}$, $J_3 = \varphi$,

$L_1 = \{(ab)^*\}$, $L_2 = \{(ab)^*a\}$. Then $L = L_1 \oplus L_2 \oplus L_3$.

Definition 3. Let $L \subseteq \Sigma_{IT}^{\omega\omega}$, we denote by $B_{h,k,l}(p)$, domino testable, if there exists a finite set of dominoes of size $(1, 1, l + 1)$, $(1, k + 1, 1)$ and $(h + 1, 1, 1)$ such that

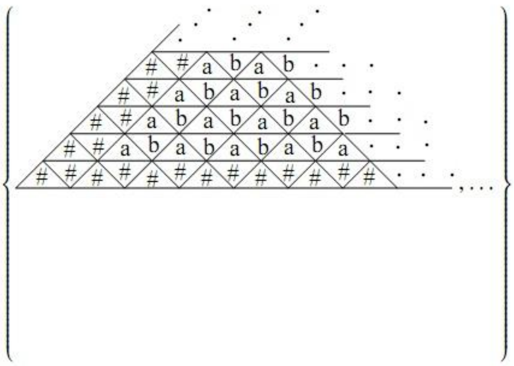
$L = \{p \in \Sigma_{TT}^{\omega\omega} : B_{1,1,l+1}(\hat{p}) \cup B_{1,k+1,1}(\hat{p}) \cup B_{h+1,1,1}(\hat{p}) \subseteq \Delta\}$

and we write $L = L_T^{\omega\omega}(\Delta)$. The family of all strictly (h, k, l)-domino testable ωω-triangular array languages is denoted by $\text{SDTT}_{h,k,l}^{\omega\omega}$. Let $L \subseteq \Sigma_{IT}^{\omega\omega}$ is strictly domino testable, if $L \subseteq \text{SDTT}_{IT}^{\omega\omega}$, for some h, k > 0. The family of all strictly domino testable ωω-triangular languages is denoted by $\text{SDTT}^{\omega\omega}$. Here we notice that

$$\mathrm{SDTT}^{\omega\omega}_{h,k,l} \subseteq \mathrm{SDTT}^{\omega\omega}_{h+1,k,l}$$
$$\mathrm{SDTT}^{\omega\omega}_{h,k,l} \subseteq \mathrm{SDTT}^{\omega\omega}_{h,k+1,l}$$
and
$$\mathrm{SDTT}^{\omega\omega}_{h,k,l} \subseteq \mathrm{SDTT}^{\omega\omega}_{h,k,l+1}$$

Example 2. Let $\Sigma = \{a, b\}$ and

$L_{h,k,l} =$



In $L_{h,k,l}$ the entries in the $(i, j, k)^{th}$ position for $i > h$ and $j > k$ and $k > l$ are a and b and other entries are a_1 and b_1 then

$$L_{h+1,k,l} \in \mathrm{SDTT}^{\omega\omega}_{h+1,k,l} \setminus \mathrm{SDTT}^{\omega\omega}_{h,k,l},$$
$$L_{h,k+1,l} \in \mathrm{SDTT}^{\omega\omega}_{h,k+1,l} \setminus \mathrm{SDTT}^{\omega\omega}_{h,k,l},$$
$$L_{h,k,l+1} \in \mathrm{SDTT}^{\omega\omega}_{h,k,l+1} \setminus \mathrm{SDTT}^{\omega\omega}_{h,k,l}.$$

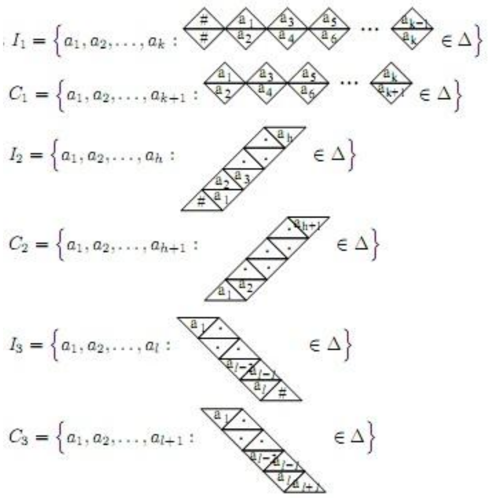
Thus we have

$$\begin{array}{ccccccc}
\vdots & & \vdots & & \vdots & & \\
\cup & & \cup & & \cup & & \\
\mathrm{SDTT}^{\omega\omega}_{2,1,1} & \subset & \mathrm{SDTT}^{\omega\omega}_{2,2,1} & \subset & \mathrm{SDTT}^{\omega\omega}_{2,3,1} & \subset & \cdots \\
\cup & & \cup & & \cup & & \\
\mathrm{SDTT}^{\omega\omega}_{1,1,1} & \subset & \mathrm{SDTT}^{\omega\omega}_{1,2,1} & \subset & \mathrm{SDTT}^{\omega\omega}_{1,3,1} & \subset & \cdots
\end{array}$$

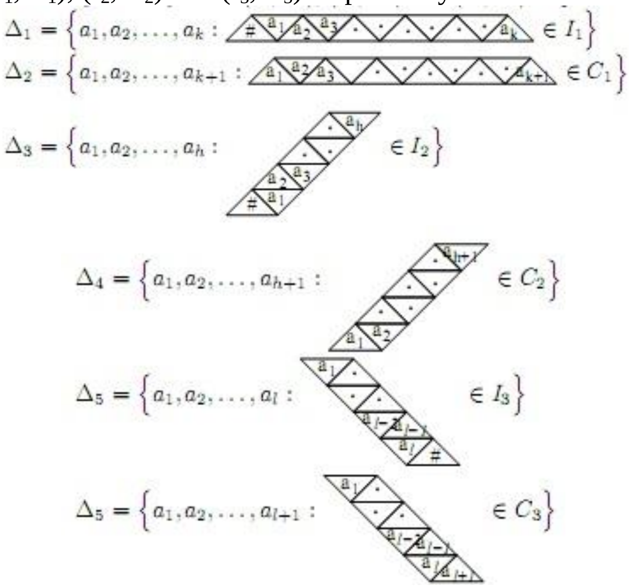
Theorem 2.

$$\mathrm{SDTT}^{\omega\omega}_{h,k,l} = \mathrm{SFLT}^{\omega}_{k+1} \oplus \mathrm{SFLT}^{\omega}_{h+1} \oplus \mathrm{SFLT}^{\omega}_{l+1} \quad \text{and}$$
$$\mathrm{SDTT}^{\omega\omega} = \mathrm{SFLT}^{\omega} \oplus \mathrm{SFLT}^{\omega} \oplus \mathrm{SFLT}^{\omega}.$$

Proof. Let



Let L_1, L_2 and L_3 be the members of $\mathrm{SFLT}^{\omega}_{k+1}$, $\mathrm{SFLT}^{\omega}_{h+1}$ and $\mathrm{SFLT}^{\omega}_{l+1}$ generated by local systems (I_1, C_1), (I_2, C_2) and (I_3, C_3) respectively. Let



Let $\Delta = \Delta_1 \cup \Delta_2 \cup \Delta_3 \cup \Delta_4 \cup \Delta_5 \cup \Delta_6$. Then $L_1 \oplus L_2 \oplus L_3 = L^{\omega\omega}_{T}(\Delta)$ where $L_1 \oplus L_2 \oplus L_3 \in \mathrm{SDTT}^{\omega\omega}_{h,k,l}$. Thus we proved.

□

CONCLUSION

In this paper the notion of recognizability of infinite triangular pictures by a new formalism called hrl-domino systems have been investigated. Learning algorithm and automata characterization of hrl-local ωω-triangular array languages will be studied. The learning of infinite triangular pictures and unary infinite triangular picture languages and their complexity deserve to be studied further.

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