EXPERIMENTS IN SQUARE LATTICE WITH A COMMON TREATMENT IN ALL BLOCKS*

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SUMMARY

This paper deals with a generalization of square lattice designs, with k^2 treatments in blocks of $k + 1$ plots, the extra plot in each block receiving a standard treatment, the same for all blocks. The new design leads to lower variances for contrasts between adjusted treatment means

INTRODUCTION

Since a long time ago, the **Instituto Agronômico de Campinas**, a research institute located in Campinas, São Paulo, Brasil, uses for its experiments with corn (maize) varieties and hybrids, square lattices with k^2 treatments and blocks of $k + 1$ plots, the extra plot in each block receiving a standard variety or hybrid, the same for all blocks, not included among the k^2 original treatments. It is clear, therefore, that these square lattice experiments include, on the whole, $k^2 + 1$ treatments in blocks of $k + 1$ plots. For example, in a 3^2 lattice, with 2 orthogonal replications, and treatments $1, 2, \ldots, 9$, plus treatment A (standard variety), the blocks would be as follows:

Block 1: 1 2 3 A
Block 2: 4 5 6 A
Block 3: 7 8 9 A
Block 4: 1 4 7 A
Block 5: 2 5 8 A
Block 6: 3 6 9 A

This paper deals with the intrablock analysis of these designs.

1. INTRABLOCK ANALYSIS

In these designs we have $v = k^2 + 1$ treatments in a square lattice with **m** orthogonal replications, $b = mk$ blocks, of $k + 1$ plots. There are k^2 regular treatments $(1, 2, \ldots, k^2)$, plus a common treatment A. The parameter λ_{ij} is equal to 1 for regular treatments

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that appear in the same block (first associates), equal to zero for regular treatments that do not appear in the same block (second associates). But when one of the treatments in the common one, the λ_{ij} = m. S_o, the normal equations (KEMPTHORNE, 1952; PIMENTEL GOMES, 1969) will have coefficients

$$c_{ii} = m(1 - \frac{1}{k+1}) = \frac{mk}{k+1},$$

$$c_{ij} = -\frac{1}{k+1} \quad \text{(first associates)},$$

$$c_{ij} = 0 \quad \text{(second associates)},$$

$$c_{iA} = \frac{m}{k+1} \quad \text{(a regular treatment and the common treatment)},$$

$$c_{AA} = mk(1 - \frac{1}{k+1}) = \frac{mk^2}{k+1}$$

For the example above, the equation corresponding to treatment **1** is:

$$\frac{mk}{k+1} t_1 - \frac{1}{k+1} t_2 - \frac{1}{k+1} t_3 - \frac{1}{k+1} t_4 - \frac{1}{k+1} t_7 - \frac{1}{k+1} t_A = Q_1$$

and for treatment A (common) it is:

$$-\frac{m}{k+1} t_1 - \frac{m}{k+1} t_2 - \ldots - \frac{m}{k+1} t_9 + \frac{mk^2}{k+1} t_A = Q_A.$$

Since matrix C = (c_{ij}) is singular, we introduce a restriction for the treatments effects, which can be

$$\sum_{i=1}^{k^2} t_i + k t_A = 0 .$$

The solution of the system of normal equation gives then:

$$\hat{t}_A = \frac{1}{mk} Q_A ,$$

$$\hat{t}_i = \frac{1}{m} Q_i + \frac{1}{k(mk - k + m)} Q_A + \frac{1}{m(mk - k + m)} [S_1(Q_i) + S_2(Q_i) + \ldots + S_m(Q_i)] ,$$

where $S_j(Q_i)$ is the sum of Q's in the j^{th} replication, in the block where the i^{th} treatment appears.

The sum of squares for treatments (adjusted) is:

$$SST \text{ (adjusted)} = \frac{1}{m} \Sigma Q_i^2 + \frac{m-1}{m(mk-k+1)} Q_A^2 + \frac{1}{m(mk-k+m)} \sum_{j,j'} S_{jj'}^2(Q),$$

where $S_{jj'}(Q)$ is the sum of Q's in the j'th block of the j^{th} replication.

So the analysis of variance is obtained as explained in table 1.

TABLE 1 – Analysis of variance.

Source of variation	D.F.	S.S.
Replications	$m-1$	As usual
Blocks within replications (unadjusted)	$m(k-1)$	As usual
Treatments (adjusted)	k^2	By formula
Residual	By subtraction	By subtraction
Total	$mk(k+1)-1$	As usual

The adjusted treatment means are:

$$\hat{m}_i = \frac{G}{mk(k+1)} + \hat{t}_i \quad , \quad \hat{m}_A = \frac{G}{mk(k+1)} + \hat{t}_A ,$$

where G is the grand total of all plots.

2. CONTRASTS BETWEEN TREATMENT MEANS

There are 3 cases to be studied.

1st associates: Two regular treatments occurring in the same block, for instance, treatments 1 and 2 in the example above:

$$V(\hat{m}_i - \hat{m}_j) = \frac{2\sigma^2}{m} \left[1 + \frac{m-1}{mk-k+m}\right].$$

2nd associates: Two regular treatments which do not occur in the same block, for instance treatments 1 and 6 in the example above:

$$V(\hat{m}_i - \hat{m}_j) = \frac{2\sigma^2}{m} \left[1 + \frac{m}{mk-k+m}\right].$$

3rd associates: A regular treatment and the common treatment:

$$V(\hat{m}_i - \hat{m}_A) = \sigma^2 \left[\frac{1}{m} + \frac{1}{mk} + \frac{k-1}{k(mk-k+m)} \right].$$

3. EXAMPLE OF ANALYSIS

We shall take as example a 5 x 5 square lattice with 4 orthogonal replications and a common treatment A present in all blocks. The experiment was carried out with corn (maize), and harvest expressed in kg/ha (Table 2).

For treatments 1, 2, . . . , 25 and A we compute now the totals T_i (i = 1, 2, . . . , 25 , A) and the adjusted treatment totals

$$Q_i = T_i - \Sigma_j \frac{n_{ij}}{k+1} B_j \quad ,$$

where N = (n_{ij}) is the **incidence matrix**, and B_j is the total of block j. It is known that the incidence matrix is obtained from elements n_{ij}, with $n_{ij} = 1$ if treatment i occurs in block j, and $n_{ij} = 0$, if it does not occur.

However, it is easier to calculate $Q'_i = (k + 1)Q_i$, as done in Table 3. We have:

$$T_1 = 22{,}194$$

$$Q'_1 = 6Q_1 = 6T_1 - (B_4 + B_6 + B_{11} + B_{17}) =$$

$$= 6 \times 22{,}194 - (35{,}150 + 37{,}957 + 34{,}078 + 33{,}002) = 7{,}023$$

$$Q'_A = 6Q_A = 6 \times 113{,}169 - 678{,}476 = 538 \quad .$$

TABLE 2 – Yields, in kg/ha, or corn (maize) in the quadruple square lattice, with a common treatment in all blocks, used as example.

Block Nº							Block Totals
1st replicate							
1	6,126 (5)	6,497 (11)	6,309 (8)	6,271 (19)	5,743 (22)	6,602 (A)	37,548
2	6,809 (2)	6,642 (10)	5,111 (13)	4,646 (24)	5,240 (16)	6,173 (A)	34,621
3	3,670 (4)	6,899 (7)	5,770 (21)	4,167 (18)	4,195 (15)	6,430 (A)	31,131
4	6,610 (12)	7,166 (20)	5,925 (23)	5,332 (1)	5,509 (9)	4,608 (A)	35,150
5	6,175 (14)	7,413 (3)	5,768 (6)	6,059 (17)	5,704 (25)	7,202 (A)	38,321
							176,771
2nd replicate							
6	6,218 (1)	6,692 (18)	6,621 (10)	6,321 (14)	6,318 (22)	5,787 (A)	37,957
7	5,580 (2)	5,586 (15)	4,682 (23)	6,155 (19)	8,237 (6)	5,487 (A)	35,727
8	6,199 (25)	3,844 (4)	5,549 (12)	5,550 (16)	6,480 (8)	4,844 (A)	32,466
9	5,960 (11)	5,019 (24)	6,300 (7)	6,993 (20)	5,996 (3)	6,280 (A)	36,548
10	6,986 (17)	5,191 (9)	7,204 (5)	6,999 (21)	6,394 (13)	6,001 (A)	38,775
							181,473
3rt replicate							
11	6,350 (25)	6,519 (1)	5,195 (13)	6,187 (19)	5,412 (7)	4,415 (A)	34,078
12	4,542 (8)	4,330 (20)	5,580 (2)	3,847 (21)	4,339 (14)	5,273 (A)	27,911
13	4,491 (4)	6,285 (17)	4,927 (11)	3,998 (23)	3,846 (10)	6,044 (A)	29,591
14	5,374 (22)	5,690 (15)	4,230 (3)	4,177 (9)	5,416 (16)	5,165 (A)	30,052
15	6,064 (6)	5,992 (18)	5,780 (5)	5,102 (24)	4,692 (12)	4,887 (A)	31,817
							153,449
4th replicate							
16	6,052 (7)	6,439 (6)	6,600 (10)	5,855 (8)	7,160 (9)	6,687 (A)	38,793
17	4,125 (1)	5,822 (2)	2,956 (4)	7,022 (5)	7,804 (3)	5,273 (A)	33,002
18	4,235 (13)	4,867 (11)	4,734 (15)	6,342 (14)	7,691 (12)	5,744 (A)	33,613
19	5,199 (16)	3,985 (20)	5,029 (19)	4,998 (18)	6,223 (17)	4,823 (A)	30,257
20	5,129 (29)	5,880 (25)	3,609 (24)	5,718 (23)	5,538 (22)	6,044 (A)	31,118
							166,783

Formulas given in section 2 can be easily changed to use Q' = values instead of Q – values. We obtain:

$$\hat{t}_A = \frac{1}{m\,k\,(k+1)}\, Q'_A = (1/120)\ 538 = 4.5 \approx 4 .$$

$$\hat{t}_1 = \frac{1}{m(k+1)} Q'_j + \frac{1}{k(k+1)(mk-k+m)} Q'_A +$$

$$+ \frac{1}{(k+1)\,m(mk-k+m)} [S_1(Q'_i) + S_2(Q'_i) + \ldots + S_m(Q'_i)] =$$

$$= (1/24)(-7{,}023) + (1/570)(538) + (1/456)(-20{,}403) = -336 .$$

TABLE 3 – Data analized, with values of T_i, Q_i and m_i for each treatment.

Treat. nº	1st rep.	2nd rep.	3rd rep.	4th rep.	T_i	Q'_i	m_i
1	5,332	6,218	6,519	4,125	22.194	- 7,023	5,318
2	6,809	5,580	5,580	5,822	23.791	11,485	6,146
3	7,413	5,996	4,230	7,804	25.443	14,735	6,369
4	3,670	3,844	4,491	2,956	14.961	-36,424	3,948
5	6,126	7,204	5,780	7,022	26.132	15,650	6,406
6	5,768	8,237	6,064	6,439	26.508	14,390	6,466
7	6,899	6,300	5,412	6,052	24.663	7,428	5,899
8	6,309	6,480	4,542	5,855	23.186	2,398	5,846
9	5,509	5,191	4,177	7,160	22.037	-10,548	5,241
10	6,642	6,621	3,846	6,600	23.709	1,292	5,606
11	6,497	5,960	4,927	4,867	22.251	- 3,794	5,440
12	6,610	5,549	4,692	7,691	24.542	14,206	6,211
13	5,111	6,394	5,195	4,235	20.935	-15,477	4,927
14	6,175	6,321	4,339	6,342	23.177	1,260	5,829
15	4,195	5,586	5,690	4,734	20.205	- 9,293	5,175
16	5,240	5,550	5,416	5,199	21.405	1,034	5,660
17	6,059	6,986	6,285	6,223	25.553	16,374	6,451
18	4,167	6,692	5,892	4,998	21.749	- 668	5,636
19	6,271	6,155	6,187	5,029	23.642	4,242	5,941
20	7,166	6,993	4,330	3,985	22.474	4,978	5,948
21	5,770	6,999	3,847	5,129	21.745	1,535	5,642
22	5,743	6,318	5,374	5,538	22.973	1,163	5,673
23	5,925	4,682	3,998	5,718	20.323	- 9,648	5,130
24	4,646	5,019	5,102	3,609	18.376	-23,848	4,589
25	5,704	6,199	6,350	5,080	23.333	4,015	5;827
A	6,602	5,787	4,415	6,687			
	6,173	5,487	5,273	5,273			
	6,430	4,844	6,044	5,744			
	4,608	6,280	5,165	4,823			
	7,202	6,001	4,287	6,044	113,169	538	5,658
	176,771	181,473	153,449	166,783			

On the other hand, we have:

$$\hat{m}_1 \triangleq 5{,}654 - 336 = 5{,}318 \ ,$$

$$\hat{m}_A = 5{,}654 + 4 = 5{,}658 \ .$$

The sum of squares for treatment (adjusted) is:

$$\text{SST (adjusted)} = \frac{1}{m\,(k+1)^2}\,\Sigma\, Q_i'^2 + \frac{m-1}{m\,(k+1)^2\,(mk-k+m)}\,Q_A'^2 +$$

$$+ \frac{1}{m\,(k+1)^2\,(mk-k+m)}\ {}_{i}\Sigma_{j'}\ S^2_{jj'}\ (Q'_i)$$

$$\text{SST (adjusted)} = (1/144)\ 3{,}888{,}633{,}692 + (1/912)\ 289{,}444 +$$

$$+ (1/2{,}736)\ 9{,}235{,}411{,}576 = 30{,}380{,}234$$

The analysis of variance obtained is given in Table 4.

TABLE 4 – Analysis of variance of data in table 2.

Source of variation	D.F.	S.S.	M.S.	F
Blocks	19	35,779,509		
Treatments (adjusted)	25	30,380,234	1,215,209	1.67*
Error	75	54,631,221	728,416	

For first associates we have:

$$\hat{V}\,(\hat{m}_i - \hat{m}_j) = \frac{2(728{,}416)}{4}\,(1 + 3/19) = (0.5789)\ 728{,}416 = 421{,}680 \ .$$

For second associates the estimate of variance is:

$$\hat{V}\,(\hat{m}_i - \hat{m}_u) = \frac{2(728{,}416)}{4}(1 + 4/19) = (0.6053)\ 728{,}416 = 440{,}910 \ .$$

Finally, for a contrast between the common treatment and any other treatment we have:

$$\hat{V}\,(\hat{m}_i - \hat{m}_A) = 728{,}416\,(\frac{1}{4} + \frac{1}{20} + \frac{1}{76}) = 247{,}278 \ .$$

For the usual 5 x 5 square lattice we should obtain:

$$\hat{V}(\hat{m}_i - \hat{m}_u) = 0.6000\ s^2$$

for first associates, and

$$\hat{V}(\hat{m}_i - \hat{m}_u) = 0.6333\ s^2$$

for second associates. We conclude, therefore, that the new design gives lower estimates for these variances.

RESUMO

O Instituto Agronômico de Campinas vem, há muitos anos, utilizando, nos seus ensaios de milho, reticulados quadrados com k^2 tratamentos em blocos de k + 1 parcelas, sendo a parcela extra de cada bloco cultivada com um cultivar padrão (variedade ou híbrido), não incluído entre os k^2 tratamentos originais. Conclui-se, pois, que esses delineamentos incluem $k^2 + 1$ tratamentos, em blocos de k + 1 parcelas.

O presente trabalho deduz fórmulas para a análise da variância desses delineamentos, e para a estimação das médias ajustadas de tratamentos. Fórmulas para a variância de diversos contrastes são deduzidas. Finalmente, apresenta-se um exemplo, detalhadamente analisado, de um ensaio em reticulado quadrado com $k^2 = 25$, e 4 repetições ortogonais, instalado com 26 cultivares, em blocos de 6 parcelas.

LITERATURE CITED

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