Strings in Flat Space and Plane Waves from $\mathcal{N} = 4$ Super Yang Mills

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We explain how the string spectrum in flat space and plane waves arises from the large N limit of U(N) $\mathcal{N} = 4$ super Yang Mills. We reproduce the spectrum by summing a subset of the planar Feynman diagrams. We also describe some other aspects of string propagation on plane wave backgrounds. This talk based on [1].

1 Introduction

The fact that large N gauge theories have a string theory description was believed for a long time [2] . These strings live in more than four dimensions [3] . One of the surprising aspects of the AdS/CFT correspondence [4-7] is the fact that for $\mathcal{N} = 4$ super Yang Mills these strings move in ten dimensions and are the usual strings of type IIB string theory. The radius of curvature of the ten dimensional space goes as $R/l_s \sim (g_{YM}^2 N)^{1/4}$. The spectrum of strings on $AdS_5 \times S^5$ corresponds to the spectrum of single trace operators in the Yang Mills theory. The perturbative string spectrum is not known exactly for general values of the 't Hooft coupling, but it is certainly known for large values of the 't Hooft coupling where we have the string spectrum in flat space. In these notes we will explain how to reproduce this spectrum from the gauge theory point of view. In fact we will be able to do slightly better than reproducing the flat space spectrum. We will reproduce the spectrum on a plane wave. These plane waves incorporate, in a precise sense, the first correction to the flat space result for certain states.

The basic idea is the following. We consider chiral primary operators such as $Tr[Z^J]$ with large J. This state corresponds to a graviton with large momentum p^+. Then we consider replacing some of the Zs in this operator by other fields, such as ϕ, one of the other transverse scalars. The position of ϕ inside the operator will matter since we are in the planar limit. When we include interactions ϕ can start shifting position inside the operator. This motion of ϕ among the Zs is described by a field in 1+1 dimensions. We then identify this field with the field corresponding to one of the transverse scalars of a string in light cone gauge. This can be shown by summing a subset of the Yang Mills Feynman diagrams. We will present a heuristic argument for why other diagrams are not important.

Since these results amount to a "derivation" of the string spectrum at large 't Hooft coupling from the gauge theory, it is quite plausible that by thinking along the lines sketched in this paper one could find the string theory for other cases, most interestingly cases where the string dual is not known (such as pure non-supersymmetric Yang Mills).

This paper is organized as follows. In section two we describe various aspects of plane waves. We discuss particle and string propagation on a plane wave as well as their symmetries. In section three we describe how plane waves arise from Penrose limits of various spacetimes, concentrating mostly on $AdS_5 \times S^5$. In section 4 we describe the computation of the spectrum from the $\mathcal{N} = 4$ Yang Mills point of view.

2 Plane waves

We will be interested in the following plane wave solution of IIB supergravity [8]

$$ds^2 = -4dx^+dx^- - y^2(dx^+)^2 + dy^i dy^i \tag{1}$$

We also have a constant field strength

$$F = dx^+(dy_1dy_2dy_3dy_4 + dy_5dy_6dy_7dy_8) \tag{2}$$

String propagation on this background can be solved exactly. by choosing light cone gauge in the Green-Schwarz action [9, 10]. The lightcone action becomes

$$S = \frac{1}{2\pi\alpha'}\int dt \int_0^{\pi\alpha'|p_-|} d\sigma \left[\frac{1}{2}\dot{y}^2 - \frac{1}{2}y'^2 - \frac{1}{2}\mu^2 y^2 + i\overline{S}(\partial\!\!\!/ + \mu I)S\right] \tag{3}$$

where $I = \Gamma^{1234}$ and S is a Majorana spinor on the worldsheet and a positive chirality SO(8) spinor under rotations in the eight transverse directions. We quantize this action by expanding all fields in Fourier modes on the circle labeled by σ. For each Fourier mode we get a harmonic oscillator (bosonic or fermionic depending on the field). Then the light cone Hamiltonian is

$$2p^- = -p_+ = H_{lc} = \sum_{n=-\infty}^{+\infty} N_n \sqrt{\mu^2 + \frac{n^2}{(\alpha'|p_-|/2)^2}} \tag{4}$$

Here n is the label of the fourier mode, $n > 0$ label left movers and $n < 0$ right movers. N_n denotes the total occupation number of that mode, including bosons and fermions. Note that the ground state energy of bosonic oscillators is canceled by that of the fermionic oscillators. The constraint on the momentum in the sigma direction reads

$$P = \sum_{n=-\infty}^{\infty} nN_n = 0 \tag{5}$$

In the limit that μ is very small, $\mu\alpha'|p_-| \ll 1$, we recover the flat space spectrum. It is also interesting to consider the opposite limit, where

$$\mu\alpha' p^+ \gg 1 \tag{6}$$

This limit corresponds to strong tidal forces on the strings. It corresponds to strong curvatures. In this limit all the low lying string oscillator modes have almost the same energy. This limit corresponds to a highly curved background with RR fields. In fact we will later see that the appearance of a large number of light modes is expected from the Yang-Mills theory.

3 Plane waves as Penrose limits.

Penrose showed that plane waves can be obtained as limits of various backgrounds [11]. Here we first consider a specific case and then we will say something about the general case.

3.1 Type IIB plane wave from $AdS_5 \times S^5$

In this subsection we obtain the maximally supersymmetric plane wave of type IIB string theory as a limit of $AdS_5 \times S^5$.

The idea is to consider the trajectory of a particle that is moving very fast along the S^5 and to focus on the geometry that this particle sees. We start with the $AdS_5 \times S^5$ metric written as

$$\begin{aligned} ds^2 &= R^2 \left[-dt^2 \cosh^2\rho + d\rho^2 + \sinh^2\rho d\Omega_3^2 + d\psi^2 \cos^2\theta \right. \\ &+ \left. d\theta^2 + \sin^2\theta d\Omega_3'^2 \right] \end{aligned} \tag{7}$$

We want to consider a particle moving along the ψ direction and sitting at $\rho = 0$ and $\theta = 0$. We will focus on the geometry near this trajectory. We can do this systematically by introducing coordinates $\tilde{x}^\pm = \frac{t \pm \psi}{2}$ and then performing the rescaling

$$\begin{aligned} x^+ &= \tilde{x}^+ , \quad x^- = R^2 \tilde{x}^- , \quad \rho = \frac{r}{R} , \\ \theta &= \frac{y}{R} , \quad R \to \infty \end{aligned} \tag{8}$$

In this limit the metric (7) becomes

$$ds^2 = -4dx^+dx^- - (\vec{r}^{\,2} + \vec{y}^{\,2})(dx^+)^2 + d\vec{y}^{\,2} + d\vec{r}^{\,2} \tag{9}$$

where $\vec{y}$ and $\vec{r}$ parametrize points on R^4. We can also see that only the components of F with a plus index survive the limit. The mass parameter μ can be introduced by rescaling (8) $x^- \to x^-/\mu$ and $x^+ \to \mu x^+$. These solutions where studied in [8].

It will be convenient for us to understand how the energy and angular momentum along ψ scale in the limit (8) . The energy in global coordinates in AdS is given by $E = i\partial_t$ and the angular momentum by $J = -i\partial_\psi$. This angular momentum generator can be thought of as the generator that rotates the 56 plane of R^6. In terms of the dual CFT these are the energy and R-charge of a state of the field theory on $S^3 \times R$ where the S^3 has unit radius. Alternatively, we can say that $E = \Delta$ is the conformal dimension of an operator on R^4. We find that

$$\begin{aligned} 2p^- &= -p_+ = i\partial_{x^+} = i\partial_{\tilde{x}^+} = i(\partial_t + \partial_\psi) = \Delta - J \\ 2p^+ &= -p_- = -\frac{\tilde{p}_-}{R^2} = \frac{1}{R^2} i\partial_{\tilde{x}^-} = \frac{1}{R^2} i(\partial_t - \partial_\psi) = \frac{\Delta + J}{R^2} \end{aligned} \tag{10}$$

Configurations with fixed non zero p_- in the limit (8) correspond to states in AdS with large angular momentum $J \sim R^2 \sim (gN)^{1/2}$. It is useful also to rewrite (4) in terms of the Yang Mills parameters. Then we find that the contribution of each oscillator to $\Delta - J$ is

$$(\Delta - J)_n = w_n = \sqrt{1 + \frac{4\pi g N n^2}{J^2}} \tag{11}$$

Notice that gN/J^2 remains fixed in the $gN \to \infty$ limit that we are taking.

When we perform the rescalings (8) we can perform the limit in two ways. If we want to get the plane wave with finite string coupling then we take the $N \to \infty$ limit keeping the string coupling g fixed and we focus on operators with $J \sim N^{1/2}$ and $\Delta - J$ fixed.

On the other hand we could first take the 't Hooft limit $g \to 0$, gN =fixed, and then after taking this limit, we take the limit of large 't Hooft coupling keeping $J/\sqrt{gN}$ fixed and $\Delta - J$ fixed. Taking the limit in this fashion gives us a plane wave background with zero string coupling. Since we will be interested in these notes in the free string spectrum of the theory it will be more convenient for us to take this second limit.

From this point of view it is clear that the full supersymmetry algebra of the metric (7) is a contraction of that of $AdS_5 \times S^5$ [8]. This algebra implies that $p^\pm \geq 0$.

4 Strings from $\mathcal{N} = 4$ Super Yang Mills

After taking the 't Hooft limit, we are interested in the limit of large 't Hooft coupling $gN \to \infty$. We want to consider states which carry parametrically large R charge $J \sim \sqrt{gN}$. [1] This R charge generator, J, is the SO(2) generator rotating two of the six scalar fields. We want to find the spectrum of states with $\Delta - J$ finite in this limit. We are interested in single trace states of the Yang Mills theory on

[1] Since we first took the 't Hooft limit then giant gravitons are not important.

$S^3 \times R$, or equivalently, the spectrum of dimensions of single trace operators of the euclidean theory on R^4. We will often go back and forth between the states and the corresponding operators.

Let us first start by understanding the operator with lowest value of $\Delta - J = 0$. There is a unique single trace operator with $\Delta - J = 0$, namely $Tr[Z^J]$, where $Z \equiv \phi^5 + i\phi^6$ and the trace is over the N color indices. We are taking J to be the SO(2) generator rotating the plane 56. At weak coupling the dimension of this operator is J since each Z field has dimension one. This operator is a chiral primary and hence its dimension is protected by supersymmetry. It is associated to the vacuum state in light cone gauge, which is the unique state with zero light cone hamiltonian. In other words we have the correspondence

$$\frac{1}{\sqrt{J}N^{J/2}}Tr[Z^J] \longleftrightarrow |0, p_+\rangle_{l.c.} \tag{12}$$

We have normalized the operator as follows. When we compute $\langle Tr[\overline{Z}^J](x)Tr[Z^J](0)\rangle$ we have J possibilities for the contraction of the first $\overline{Z}$ but then planarity implies that we contract the second $\overline{Z}$ with a Z that is next to the first one we contracted and so on. Each of these contraction gives a factor of N. Normalizing this two point function to one we get the normalization factor in (12)[2].

Now we can consider other operators that we can build in the free theory. We can add other fields, or we can add derivatives of fields like $\partial_{(i_1} \cdots \partial_{i_n)}\phi^r$, where we only take the traceless combinations since the traces can be eliminated via the equations of motion. The order in which these operators are inserted in the trace is important. All operators are all "words" constructed by these fields up to the cyclic symmetry, these were discussed and counted in [3]. We will find it convenient to divide all fields, and derivatives of fields, that appear in the free theory according to their $\Delta - J$ eigenvalue. There is only one mode that has $\Delta - J = 0$, which is the mode used in (12). There are eight bosonic and eight fermionic modes with $\Delta - J = 1$. They arise as follows. First we have the four scalars in the directions not rotated by J, i.e. ϕ^i, $i = 1, 2, 3, 4$. Then we have derivatives of the field Z, $D_i Z = \partial_i Z + [A_i, Z]$, where $i = 1, 2, 3, 4$ are four directions in R^4. Finally there are eight fermionic operators $\chi^a_{J=\frac{1}{2}}$ which are the eight components with $J = \frac{1}{2}$ of the sixteen component gaugino χ (the other eight components have $J = -\frac{1}{2}$). These eight components transform in the positive chirality spinor representation of $SO(4) \times SO(4)$. We will focus first on operators built out of these fields and then we will discuss what happens when we include other fields, with $\Delta - J > 1$, such as $\overline{Z}$.

The state (12) describes a particular mode of ten dimensional supergravity in a particular wavefunction [6]. Let us now discuss how to generate all other massless supergravity modes. On the string theory side we construct all these states by applying the zero momentum oscillators a^i_0, $i = 1, \ldots, 8$ and S^b_0, $b = 1, \ldots 8$ on the light cone vacuum $|0, p_+\rangle_{l.c.}$. Since the modes on the string are massive all these zero momentum oscillators are harmonic oscillators, they all have the same light cone energy. So the total light cone energy is equal to the total number of oscillators that are acting on the light cone ground state. We know that in $AdS_5 \times S^5$ all gravity modes are in the same supermultiplet as the state of the form (12) [12]. The same is clearly true in the limit that we are considering. More precisely, the action of all supersymmetries and bosonic symmetries of the plane wave background (which are intimately related to the $AdS_5 \times S^5$ symmetries) generate all other ten dimensional massless modes with given p_-. For example, by acting by some of the rotations of S^5 that do not commute with the SO(2) symmetry that we singled out we create states of the form

$$\frac{1}{\sqrt{J}}\sum_l \frac{1}{\sqrt{J}N^{J/2+1/2}}Tr[Z^l\phi^r Z^{J-l}] = \frac{1}{N^{J/2+1/2}}Tr[\phi^r Z^J] \tag{13}$$

where ϕ^r, $r = 1, 2, 3, 4$ is one of the scalars neutral under J. In (13) we used the cyclicity of the trace. Note that we have normalized the states appropriately in the planar limit. We can act any number of times by these generators and we get operators roughly of the form $\sum Tr[\cdots z\phi^r z \cdots z\phi^k]$. where the sum is over all the possible orderings of the ϕs. We can repeat this discussion with the other $\Delta - J = 1$ fields. Each time we insert a new operator we sum over all possible locations where we can insert it. Here we are neglecting possible extra terms that we need when two $\Delta - J = 1$ fields are at the same position, these are subleading in a $1/J$ expansion and can be neglected in the large J limit that we are considering. We are also ignoring the fact that J typically decreases when we act with these operators. In other words, when we act with the symmetries that do not leave Z invariant we will change one of the Zs in (12) to a field with $\Delta - J = 1$, when we act again with one of the symmetries we can change one of the Zs that was left unchanged in the first step or we can act on the field that was already changed in the first step. This second possibility is of lower order in a $1/J$ expansion and we neglect it. We will always work in a "dilute gas" approximation where most of the fields in the operator are Zs and there are a few other fields sprinkled in the operator.

For example, a state with two excitations will be of the form

$$\sim \frac{1}{N^{J/2+1}}\frac{1}{\sqrt{J}}\sum_{l=0}^{J} Tr[\phi^r Z^l \psi^b_{J=\frac{1}{2}} Z^{J-l}] \tag{14}$$

where we used the cyclicity of the trace to put the ϕ^r operator at the beginning of the expression. We associate (14) to the string state $a^{\dagger k}_0 S^{\dagger\, b}_0|0, p_+\rangle_{l.c.}$. Note that for planar diagrams it is very important to keep track of the position of the operators. For example, two operators of the form $Tr[\phi^1 Z^l \phi^2 Z^{J-l}]$ with different values of l are orthogonal to each other in the planar limit (in the free theory).

The conclusion is that there is a precise correspondence between the supergravity modes and the operators. This is

[2] In general in the free theory any contraction of a single trace operator with its complex conjugate one will give us a factor of N^n, where n is the number of fields appearing in the operator.

of course well known [5, 6, 7]. Indeed, we see from (4) that their $\Delta - J = -p_+$ is indeed what we compute at weak coupling, as we expect from the BPS argument.

In order to understand non-supergravity modes in the bulk it is clear that what we need to understand the Yang Mills description of the states obtained by the action of the string oscillators which have $n \neq 0$. Let us consider first one of the string oscillators which creates a bosonic mode along one of the four directions that came from the S^5, let's say $a_n^{\dagger\,8}$. We already understood that the action of $a_0^{\dagger\,8}$ corresponds to insertions of an operator ϕ^4 on all possible positions along the "string of Z's". By a "string of Zs" we just mean a sequence of Z fields one next to the other such as we have in (12). We propose that $a_n^{\dagger 8}$ corresponds to the insertion of the same field ϕ^4 but now with a position dependent phase

$$\frac{1}{\sqrt{J}}\sum_{l=1}^{J}\frac{1}{\sqrt{J}N^{J/2+1/2}}Tr[Z^l\phi^4 Z^{J-l}]e^{\frac{2\pi i n l}{J}} \tag{15}$$

In fact the state (15) vanishes by cyclicity of the trace. This corresponds to the fact that we have the constraint that the total momentum along the string should vanish (5), so that we cannot insert only one $a_n^{\dagger\,i}$ oscillator. So we should insert more than one oscillator so that the total momentum is zero. For example we can consider the string state obtained by acting with the $a_n^{\dagger\,8}$ and $a_{-n}^{\dagger\,7}$, which has zero total momentum along the string. We propose that this state should be identified with

$$a_n^{\dagger\,8}a_{-n}^{\dagger\,7}|0,p_+\rangle_{l.c.} \longleftrightarrow \frac{1}{\sqrt{J}}\sum_{l=1}^{J}\frac{1}{N^{J/2+1}} \times Tr[\phi^3 Z^l\phi^4 Z^{J-l}]e^{\frac{2\pi i n l}{J}} \tag{16}$$

where we used the cyclicity of the trace to simplify the expression. The general rule is pretty clear, for each oscillator mode along the string we associate one of the $\Delta - J = 1$ fields of the Yang-Mills theory and we sum over the insertion of this field at all possible positions with a phase proportional to the momentum. States whose total momentum is not zero along the string lead to operators that are automatically zero by cyclicity of the trace. In this way we enforce the $L_0 - \overline{L}_0 = 0$ constraint (5) on the string spectrum.

In summary, each string oscillator corresponds to the insertion of a $\Delta - J = 1$ field, summing over all positions with an n dependent phase, according to the rule

$$\begin{array}{lll} a^{\dagger i} & \longrightarrow & D_i Z \quad \text{for } i = 1,\cdots,4 \\ a^{\dagger j} & \longrightarrow & \phi^{j-4} \quad \text{for } j = 5,\cdots,8 \\ S^a & \longrightarrow & \chi^a_{J=\frac{1}{2}} \end{array} \tag{17}$$

In order to show that this identification makes sense we want to compute the conformal dimension, or more precisely $\Delta - J$, of these operators at large 't Hooft coupling and show that it matches (4). First note that if we set $\frac{gN}{J^2} \sim 0$ in (11) we find that all modes, independently of n have the same energy, namely one. This is what we find at weak 't Hooft coupling where all operators of the form (16) have the same energy, independently of n. Expanding the string theory result (11) we find that the first correction is of the form

$$(\Delta - J)_n = w_n = 1 + \frac{2\pi g N n^2}{J^2} + \cdots \tag{18}$$

This looks like a first order correction in the 't Hooft coupling and we can wonder if we can reproduce it by a a simple perturbative computation.

In order to compute the corrections it is useful to view the $\mathcal{N} = 4$ theory as an $\mathcal{N} = 1$ theory. As an $\mathcal{N} = 1$ theory we have a Yang Mills theory plus three chiral multiplets in the adjoint representation. We denote these multiplets as W^i, where $i = 1, 2, 3$. We will often set $Z = W^3$ and $W = W^1$. The theory also has a superpotential

$$\mathcal{W} \sim g_{YM} Tr(W^i W^j W^k)\epsilon_{ijk} \tag{19}$$

The potential for the Yang Mills theory is the sum of two terms, $V = V_F + V_D$, one coming from F terms and the other from D-terms. The one coming from F terms arises from the superpotential and has the form

$$V_F \sim \sum_{ij} Tr\left([W^i, W^j][\overline{W}^i, \overline{W}^j]\right) \tag{20}$$

On the other hand the one coming from D terms has the form

$$V_D \sim \sum_{ij} Tr([W^i, \overline{W}^i][W^j, \overline{W}^j]) \tag{21}$$

We will concentrate in computing the contribution to the conformal dimension of an operator which contains a W insertion along the string of Zs. There are various types of diagrams. There are diagrams that come from D terms, as well as from photons or self energy corrections. There are also diagrams that come from F terms. The diagrams that come from F terms can exchange the W with the Z. The F term contributions cancel in the case that there are no phases, see Fig. (1) . This means that all other diagrams should also cancel, since in the case that there are no phases we have a BPS object which recieves no corrections. All other one loop diagrams that do not come from F terms do not exchange the position of W, this means that they vanish also in the case that there are phases since they will be insensitive to the presence of phases. In the presence of phases the only diagrams that will not cancel are then the diagrams that come from the F terms. These are the only diagrams that give a momentum, n, dependent contribution.

In the free theory, once a W operator is inserted at one position along the string it will stay there, states with W's at different positions are orthogonal to each other in the planar limit (up to the cyclicity of the trace). We can think of the string of Zs in (12) as defining a lattice, when we insert an operator W at different positions along the string of Zs we are exciting an oscillator $b_l^\dagger$ at the site l on the lattice, $l = 1, \cdots J$. The interaction term (20) can take an excitation

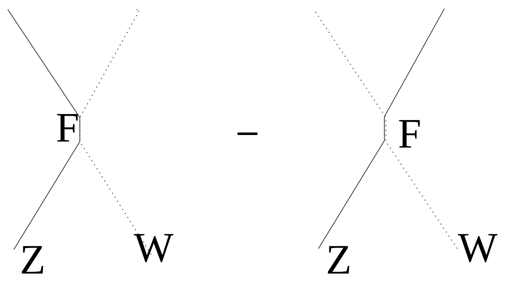

Figure 1. Diagrams that come from F terms. The two diagrams have a relative minus sign. The F terms propagator is a delta function so that we could replace the three point vertex by a four point vertex coming from (20). If there are no phases in the operator these contributions vanish..

from one site in the lattice to the neighboring site. So we see that the effects of (20) will be sensitive to the momentum n. In fact one can precisely reproduce (18) from (20) including the precise numerical coefficient. Below we give some more details on the computation.

We will write the square of the Yang-Mills coupling in terms of what in AdS is the string coupling that transforms as $g \to 1/g$ under S-duality. The trace is just the usual trace of an $N \times N$ matrix.

We define $Z = \frac{1}{\sqrt{2}}(\phi^5 + i\phi^6)$ and similarly for W. Then the propagator is normalized as

$$\langle Z_i^{\ j}(x)\overline{Z}_k^{\ l}(0)\rangle = \delta_i^l \delta_k^j \, \frac{2\pi g}{4\pi^2} \, \frac{1}{|x|^2} \tag{22}$$

In (20) there is an interaction term of the form the form $\frac{1}{\pi g}\int d^4x Tr([Z,W][\overline{Z},\overline{W}])$, where W is one of the (complex) transverse scalars, let's say $W = W^1$. The contribution from the F terms shown in (20) give

$$\begin{aligned} < O(x)O^*(0) > \; &= \; \frac{\mathcal{N}}{|x|^{2\Delta}}\Bigg[1 + N(4\pi g) \\ &\times \; (-2 + 2\cos\frac{2\pi n}{J})I(x)\Bigg] \end{aligned} \tag{23}$$

where $\mathcal{N}$ is a normalization factor and $I(x)$ is the integral

$$I(x) = \frac{|x|^4}{(4\pi^2)^2}\int d^4y \frac{1}{y^4(x-y)^4} \sim \frac{1}{4\pi^2}\log|x|\Lambda + \text{finite} \tag{24}$$

We extracted the log divergent piece of the integral since it is the one that reflects the change in the conformal dimension of the operator.

In conclusion we find that for large J and N the first correction to the correlator is

$$< O(x)O^*(0) >= \frac{\mathcal{N}}{|x|^{2\Delta}}\left[1 - \frac{4\pi g N n^2}{J^2}\log(|x|\Lambda)\right] \tag{25}$$

which implies that the contribution of the operator W inserted in the "string of Zs" with momentum n gives a contribution to the anomalous dimension

$$(\Delta - J)_n = 1 + \frac{2\pi g N n^2}{J^2} \tag{26}$$

which agrees precisely with the first order term computed from (18).

There are similar computations we could do for insertions of D_iZ, $\overline{W}$ or the fermions $\chi^a_{J=1/2}$. In the case of the fermions the important interaction term will be a Yukawa coupling of the form $\overline{\chi}\Gamma_z[Z\chi] + \overline{\chi}\Gamma_{\overline{z}}[\overline{Z},\chi]$. We have not done these computations explicitly since the 16 supersymmetries preserved by the state (12) relate them to the computation we did above for the insertion of a W operator.

The full square root in (11) was recently obtained in the beautifull paper by Santambrogio and Zanon [13]/

In summary, the "string of Zs" becomes the physical string and each Z carries one unit of J which is one unit of $-p_-$. Locality along the worldsheet of the string comes from the fact that planar diagrams allow only contractions of neighboring operators. So the Yang Mills theory gives a string bit model (see [14]) where each bit is a Z operator. Each bit carries one unit of J which is one unit of $-p_-$.

The reader might, correctly, be thinking that all this seems too good to be true. In fact, we have neglected many other diagrams and many other operators which, at weak 't Hooft coupling also have small $\Delta - J$. In particular, we considered operators which arise by inserting the fields with $\Delta - J = 1$ but we did not consider the possibility of inserting fields corresponding to $\Delta - J = 2, 3, \ldots$, such as $\overline{Z}$, $\partial_k\phi^r$, $\partial_{(l}\partial_{k)}Z$, etc.. The diagrams of the type we considered above would give rise to other 1+1 dimensional fields for each of these modes. These are present at weak 't Hooft coupling but they should not be present at strong coupling, since we do not see them in the string spectrum. We believe that what happens is that these fields get a large mass in the $N \to \infty$ limit. In other words, the operators get a large conformal dimension. One can compute the first correction to the energy (the conformal weight) of the of the state that results from inserting $\overline{Z}$ with some "momentum" n. In contrast to our previous computation for $\Delta - J = 1$ fields we find that besides an effective kinetic term as in (18) there is an n independent contribution that goes as gN with no extra powers of $1/J^2$ [1]. This is an indication that these excitations become very massive in the large gN limit. In addition, we can compute the decay amplitude of $\overline{Z}$ into a pair of ϕ insertions. This is also very large, of order gN.

Though we have not done a similar computation for other fields with $\Delta - J > 1$, we believe that the same will be true for the other fields. In general we expect to find many terms in the effective Lagrangian with coefficients that are of order gN with no inverse powers of J to suppress them. In other words, the lagrangian of Yang-Mills on S^3 acting on a state which contains a large number of Zs gives a lagrangian on a discretized spatial circle with an infinite number of KK modes. The coefficients of this effective lagrangian are factors of gN, so all fields will generically get very large masses.

The only fields that will not get a large mass are those whose mass is protected for some reason. The fields with $\Delta - J = 1$ correspond to Goldstone bosons and fermions of the symmetries broken by the state (12). Note that despite the fact that they morally are Goldstone bosons and fermions, their mass is non-zero, due to the fact that the symmetries that are broken do not commute with p_+, the light cone Hamiltonian. The point is that their masses are determined, and hence protected, by the (super)symmetry algebra.

Having described how the single string Hilbert space arises it is natural to ask whether we can incorporate properly the string interactions. Clearly string interactions come when we include non-planar diagrams [2].

Some of the arguments used in this section look very reminiscent of the DLCQ description of matrix strings [15] [16]. It would be interesting to see if one can establish a connection between them. Notice that the DLCQ description of ten dimensional IIB theory is in terms of the M2 brane field theory. Since here we are extracting also a light cone description of IIB string theory we expect that there should be a direct connection.

It would also be nice to see if using any of these ideas we can get a better handle on other large N Yang Mills theories, particularly non-supersymmetric ones. The mechanism by which strings appear in this paper is somewhat reminiscent of [17].

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