

# $J/\Psi D^* D^*$ Form Factor from QCD Sum Rules

M.E. Bracco<sup>1</sup>, M. Chiapparini<sup>1</sup>, F.S. Navarra<sup>2</sup>, and M. Nielsen<sup>2</sup>

<sup>1</sup>Instituto de Física, Universidade do Estado do Rio de Janeiro,

Rua São Francisco Xavier 524, Maracanã, 20559-900, Rio de Janeiro, RJ, Brazil.

<sup>2</sup>Instituto de Física, Universidade de São Paulo, C.P. 66318, 05389-970 São Paulo, SP, Brazil.

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We calculate the  $J/\Psi D^* D^*$  form factor and coupling constant from QCD Sum Rules in the cases where  $J/\Psi$  and  $D^*$  mesons are off-shell. The results show that this method is consistent and allows to extract the same coupling constant for the vertex.

## 1 Introduction

Recent data from  $Pb + Pb$  collisions in the  $NA50$  experiment at CERN-SPS have shown an anomalously large  $J/\Psi$  suppression [1]. This kind of suppression is still considered one of the promising signatures for the quark-gluon plasma (QGP) formation[2], although the conventional mechanism based on  $J/\Psi$  absorption by comoving hadrons has also been shown to contribute significantly to the observed suppression [3]. Since there is no empirical information on the cross sections for charmonium absorption by light hadrons, theoretical models are needed to determine their values. In the meson-exchange model, the cross sections between charmonia and hadrons are evaluated using effective hadronic Lagrangians derived from the  $SU(4)$  flavor symmetry [4, 5]. In these calculations, the cross sections depends sensitively on the form factors and their cut-offs. These form factors are not phenomenologically known.

In this work we report on the use of the QCD Sum Rules (QCDSR) method [6], based on the three-point function, to evaluate the  $J/\Psi D^* D^*$  hadronic form factor and coupling constant, used by meson exchange models in the calculation of the  $J/\Psi - \pi(\rho)$  cross section.

## 2 The QCD Sum Rule Calculation

Following the QCDSR formalism described in our previous works [7, 8, 9], we write the three-point function associated with a  $J/\Psi D^* D^*$  vertex, which is given by

$$\Gamma_{\nu\alpha\mu}^{D^*}(p, p') = \int d^4x d^4y e^{ip'\cdot x} e^{-i(p'-p)\cdot y} \quad (1)$$

$$\times \langle 0 | T \{ j_\nu^{D^*}(x) j_\alpha^{D^*}(y) j_\mu^{\Psi^\dagger}(0) \} | 0 \rangle$$

for the  $D^*$  meson off-shell, and by

$$\Gamma_{\nu\alpha\mu}^{J/\Psi}(p, p') = \int d^4x d^4y e^{ip'\cdot x} e^{-i(p'-p)\cdot y} \quad (2)$$

$$\times \langle 0 | T \{ j_\nu^{D^*}(x) j_\mu^{\Psi^\dagger}(y) j_\alpha^{D^*}(0) \} | 0 \rangle,$$

for the  $J/\Psi$  meson off-shell, where  $j_\nu^{D^*}(x) = \bar{c}(x)\gamma_\nu q(x)$ ,  $j_\alpha^{D^*}(y) = \bar{c}(y)\gamma_\alpha q(y)$  and  $j_\mu^{\Psi^\dagger}(0) = \bar{c}(0)\gamma_\mu c(0)$  are the  $D^*$  and  $J/\Psi$  interpolating fields, written in terms of these mesons. Here  $q$  and  $c$  are the up/down and charm quark field respectively.

The general expressions for the vertex (1) and (2) have fourteen independent Dirac structures. We can write  $\Gamma_{\nu\alpha\mu}$  in terms of the invariant amplitudes associated with each one of these structures in the following form:

$$\begin{aligned} \Gamma_{\mu\nu\alpha}(p, p') = & \Gamma_1(p^2, p'^2, q^2) g_{\mu\nu} p_\alpha \\ & + \Gamma_2(p^2, p'^2, q^2) g_{\mu\alpha} p_\nu + \Gamma_3(p^2, p'^2, q^2) g_{\nu\alpha} p_\mu \\ & + \Gamma_4(p^2, p'^2, q^2) g_{\mu\nu} p'_\alpha + \Gamma_5(p^2, p'^2, q^2) g_{\mu\alpha} p'_\nu \\ & + \Gamma_6(p^2, p'^2, q^2) g_{\nu\alpha} p'_\mu + \Gamma_7(p^2, p'^2, q^2) p_\mu p_\nu p_\alpha \\ & + \Gamma_8(p^2, p'^2, q^2) p'_\mu p'_\nu p_\alpha + \Gamma_9(p^2, p'^2, q^2) p_\mu p'_\nu p_\alpha \\ & + \Gamma_{10}(p^2, p'^2, q^2) p_\mu p_\nu p'_\alpha + \Gamma_{11}(p^2, p'^2, q^2) p'_\mu p'_\nu p_\alpha \\ & + \Gamma_{12}(p^2, p'^2, q^2) p'_\mu p_\nu p'_\alpha + \Gamma_{13}(p^2, p'^2, q^2) p_\mu p'_\nu p'_\alpha \\ & + \Gamma_{14}(p^2, p'^2, q^2) p'_\mu p'_\nu p'_\alpha \end{aligned} \quad (3)$$

The correlation function, Eqs. (1) and (2), can be calculated in two different ways. One using quark degrees of freedom -the theoretical or QCD side- and other using hadronic degrees of freedom -the phenomenological side.

An adequate framework for the calculation of the correlators in the QCD side is the Wilson operator product expansion (OPE). The OPE incorporates the effects of the QCD vacuum through an infinite series of increasing dimension condensate operators. On the other hand, the representation in terms of hadronic degrees of freedom is the place where appears the form factor, decay constants and masses. Both representations are matched using the universality principle, and doing a Borel transformation on both sides, the QCD Sum Rule is complete.

For each one of the structures in Eq. (3) we can write a double dispersion relation of the invariant amplitudes  $\Gamma_i(p^2, p'^2, Q^2)$ ,  $i = 1, \dots, 14$ , over the virtualities  $p^2$  and  $p'^2$ , holding  $Q^2 = -q^2$  fixed:

$$\Gamma_i(p^2, p'^2, Q^2) = -\frac{1}{4\pi^2} \int_{s_{min}}^{\infty} ds \int_{m_c^2}^{\infty} du \times \frac{\rho_i(s, u, Q^2)}{(s-p^2)(u-p'^2)}, \quad (4)$$

where  $\rho_i(s, u, Q^2)$  equals the double discontinuity of the amplitude  $\Gamma_i(p^2, p'^2, Q^2)$ , calculated using the Cutkosky rule, and where  $s_{min} = 4m_c^2$  in the case of  $D^*$  off-shell, and  $s_{min} = m_c^2$  in the case of  $J/\Psi$  off-shell. Note that, *a priori*, the invariant amplitudes receive contributions from all terms in the OPE. The first one of those contributions comes from the perturbative term and it is shown in Figs. 1 and 2 in the case of the correlators (1) and (2) respectively.

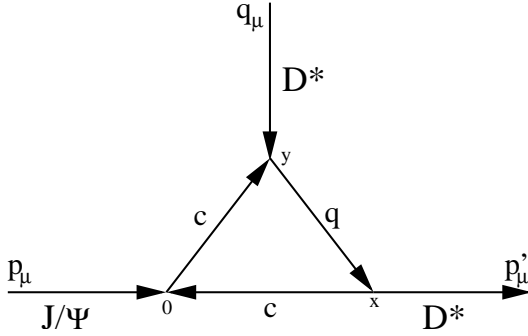


Figure 1. Perturbative diagram for  $g_{J/\Psi D^* D^*}^{(D^*)}(q^2)$ .

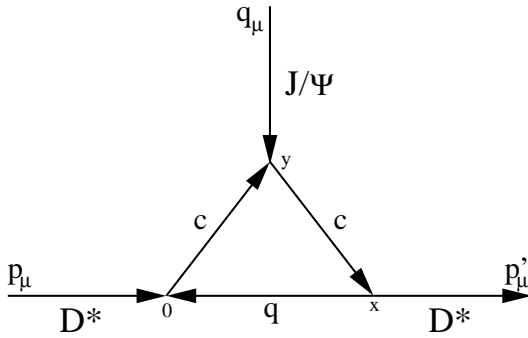


Figure 2. Perturbative diagram for  $g_{J/\Psi D^* D^*}^{(J/\Psi)}(q^2)$ .

In this paper, we work with the structure  $p_\mu p_\nu p_\alpha$  in Eq. (3), namely  $i = 7$ . The corresponding spectral densities, which enter in Eq. (4), are

$$\rho_7^{(D^*)}(s, u, Q^2) = \frac{6}{\pi\sqrt{\lambda}}(A - H) \quad (5)$$

for  $D^*$  off-shell and

$$\rho_7^{(J/\Psi)}(s, u, Q^2) = \frac{6}{\pi\sqrt{\lambda}}(B - J) \quad (6)$$

for  $J/\Psi$  off-shell, where  $\lambda = \lambda(s, u, t) = s^2 + t^2 + u^2 - 2st - 2su - 2tu$ ,  $t = -Q^2$  and  $A, B, H$  and  $J$  are functions of  $(s, t, u)$ . For the structure chosen here, the quark condensate doesn't contribute. We also expect that the perturbative contribution is the dominant one in the OPE, because we are dealing with heavy quarks. For this reason we do not include the gluon and quark-gluon condensates.

The phenomenological side of the vertex function is obtained considering the contribution of the  $J/\Psi$  and one  $D^*$  meson in Eq. (1) and the two  $D^*$  mesons in Eq. (2). Here we introduced the decay constants  $f_{D^*}$  and  $f_{J/\Psi}$ , which are defined by the matrix elements

$$\langle D^* | j_\mu^{D^*} | 0 \rangle = m_{D^*} f_{D^*} \epsilon_\mu^* \quad (7)$$

and

$$\langle J/\Psi | j_\nu^{J/\Psi} | 0 \rangle = m_{J/\Psi} f_{J/\Psi} \epsilon_\nu^* \quad (8)$$

respectively. The resulting phenomenological invariant amplitude associated with the structure  $p_\mu p_\nu p_\alpha$  ( $i = 7$  in Eq. (3)) for the  $D^*$  meson off-shell is:

$$\begin{aligned} \Gamma_7^{(D^*)ph}(p^2, p'^2, Q^2) &= g_{J/\Psi D^* D^*}^{(D^*)}(Q^2) \frac{f_{J/\Psi}}{(p^2 - m_{J/\Psi}^2)} \\ &\times \frac{f_{D^*}}{(Q^2 + m_{D^*}^2)} \frac{f_{D^*}}{(p'^2 - m_{D^*}^2)} \\ &\times (p'^2 - m_{D^*}^2 - m_{J/\Psi}^2) \end{aligned} \quad (9)$$

In the case of the  $J/\Psi$  meson off-shell, the phenomenological invariant amplitude associated with the same structure ( $i = 7$  in Eq. (3)) is:

$$\begin{aligned} \Gamma_7^{(J/\Psi)ph}(p^2, p'^2, Q^2) &= g_{J/\Psi D^* D^*}^{(J/\Psi)}(Q^2) \frac{f_{D^*}}{(p^2 - m_{D^*}^2)} \\ &\times \frac{f_{J/\Psi}}{(Q^2 + m_{J/\Psi}^2)} \frac{f_{D^*}}{(p'^2 - m_{D^*}^2)} \\ &\times (m_{D^*}^2 + m_{J/\Psi}^2 - p^2) \end{aligned} \quad (10)$$

Now we perform a double Borel transformation [10] in both variables  $P^2 = -p^2 \rightarrow M^2$  and  $P'^2 = -p'^2 \rightarrow M'^2$  on both invariant amplitudes  $\Gamma_7$  and  $\Gamma_7^{ph}$ . Equating the results we get the final expressions for the sum rules which allow us to obtain the expressions for the form factors  $g_{J/\Psi D^* D^*}^{(M)}(Q^2)$  appearing in Eqs. (9)–(10). The values for the Borel masses  $M^2$  and  $M'^2$  are

$$\frac{M^2}{M'^2} = \frac{m_{J/\Psi}^2}{m_{D^*}^2} \quad (11)$$

for the  $D^*$  off-shell case and  $M^2 = M'^2$  for the  $J/\Psi$  off-shell case.

For consistency we use in our analysis the QCDSR expressions for the decay constants appearing in Eq. (7) and (8), up to dimension four, coming from the two-point functions in QCDSR. In that calculation the contribution of

the gluon condensate was omitted and the Borel parameters used in the two and three point function are related by  $2M_m^2 = M^2$ , which is a crucial constraint for the incorporation of the HQET symmetries.

The values of the parameters used in this calculation are the following:  $m_q = 7 \text{ MeV}$ ,  $m_c = 1.3 \text{ GeV}$ ,  $m_{D^*} = 2.01 \text{ GeV}$ ,  $m_{J/\Psi} = 3.1 \text{ GeV}$  and  $\langle \bar{q}q \rangle = -(0.23)^3 \text{ GeV}^3$ . The thresholds are given by  $s_0 = (m_M + \Delta_s)^2$ , where  $m_M$  is the mass of the incoming meson, and  $u_0 = (m_{D^*} + \Delta_u)^2$ .

Using  $\Delta_s = \Delta_u \approx 0.7 \text{ GeV}$  for the continuum thresholds and fixing  $Q^2$ , we found a good stability of the sum rule for  $g_{J/\Psi D^* D^*}^{(D^*)}$  as a function of  $M^2$  in the interval  $1 < M^2 < 5 \text{ GeV}^2$ . In the case of  $g_{J/\Psi D^* D^*}^{(J/\Psi)}$  the interval for  $M^2$  is  $4 < M^2 < 10 \text{ GeV}^2$ . Fixing now  $M^2 = 5 \text{ GeV}^2$  we calculated the momentum dependence of the form factor. In Fig. 2, the circles correspond to the  $g_{J/\Psi D^* D^*}^{(D^*)}(Q^2)$  form factor in the interval where the sum rule is valid. The triangles are the result of the sum rule for the  $g_{J/\Psi D^* D^*}^{(J/\Psi)}(Q^2)$  form factor.

In the case of the  $D^*$  meson off-shell, our numerical results can be reproduced by a Gaussian parametrization, the solid curve in Fig. 2, in the following way:

$$g_{J/\Psi D^* D^*}^{(D^*)}(Q^2) = 2.35 e^{-\frac{Q^2}{4.24}} \quad (12)$$

As in Ref.[9], we define the coupling constant as the value of the form factor at  $Q^2 = -m_M^2$ , where  $m_M$  is the mass of the off-shell meson. For the  $D^*$  meson off-shell the coupling constant is:

$$g_{J/\Psi D^* D^*}^{(D^*)} = 6.01 \quad (13)$$

In the case of the  $J/\Psi$  meson off-shell, our sum rule result can be fitted by a monopole parametrization, which corresponds to the dashed line in Fig. 2:

$$g_{J/\Psi D^* D^*}^{(J/\Psi)}(Q^2) = \frac{23.16}{Q^2 + 13.98} \quad (14)$$

giving the following coupling constant, obtained at the  $J/\Psi$  pole:

$$g_{J/\Psi D^* D^*}^{(J/\Psi)} = 5.85 \quad (15)$$

Concluding, the method used to extrapolate the QCDSR results with  $J/\Psi$  and  $D^*$  mesons off-shell permits to extract the same value for the coupling constant (Eqs. (13) and (15)). We can see that the form factor is harder, in the case of the monopole extrapolation, if the off-shell meson is the heavy one, implying that the size of the vertex depends on the exchanged meson, which is consistent with our previous results [9]. In a future work, we intend to analyze the form factor for all the structures appearing in Eq.(3), and the consequences of taking into account increasing dimension OPE

operators for each one. Also, more contributions in the decay constant of  $J/\Psi$  (two point function QCD Sum Rule) will be considered.

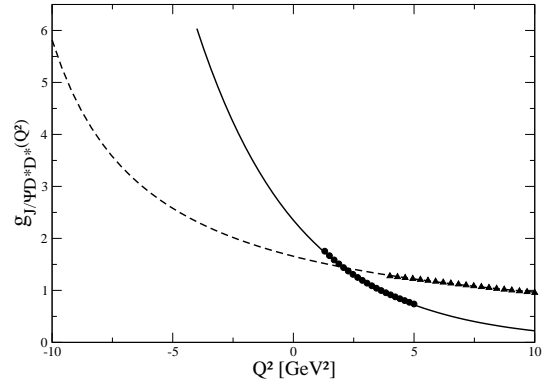


Figure 3. Momentum dependence of the  $J/\Psi D^* D^*$  form factor. The solid line gives the parametrization of the QCDSR results trough Eq.(12), and the dashed line is the parametrization of Eq. (14).

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