ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

ARTICLE

A Systematic Literature Review on Proportionality

Uma Revisão Sistemática da Literatura sobre Proporcionalidade

Henrique Rizek **Elias***
ORCID iD 0000-0002-9660-7303
Arthur Belford **Powell****
ORCID iD 0000-0002-6086-3698
Hanna Nadim **Haydar*****
ORCID iD 0000-0003-3113-0054
Alessandro Jacques **Ribeiro******
ORCID iD 0000-0001-9647-0274

Abstract

In this article, we carry out a Systematic Literature Review (SLR) regarding the mathematical topic of proportionality. With this SLR, we seek to answer the following research questions: How are the studies on proportionality in Mathematics Education found in the literature review? What interactions emerge from the analyses carried out according to the tetrahedral model of the didactic situation? The survey was carried out considering 41 journals, and based on the search procedures and established filters, we found 58 articles. The analysis was carried out in two moments. At moment 1, to answer the first investigative question, we noticed that there is a predominance of some theoretical references that support the work. Still, there is, on the other hand, diversity in the contexts in which the research took place. At moment 2, to answer the second investigative question, we used the tetrahedral model of the didactic situation to analyze 22 of the 58 articles, as they carried out research involving elementary, middle, and high school students. We identified four types of interactions based on the tetrahedral model: student-content, student-teacher-content, student-content-artifact, and student-teacher-content-artifact. More than half of the articles (13 of 22) explore student-content interaction. Four articles explore the student-teacher-content-artifact interaction, with only one explaining the importance of this interaction to promote the compression of the notion of proportionality. We finalize the SLR by indicating possible paths for new research, especially those investigating the potential of student-teacher-content-artifact interaction for teaching proportionality and developing students' proportional reasoning.

* Doctor in Mathematics Education from Universidade Estadual de Londrina (UEL). Professor at Universidade Tecnológica Federal do Paraná (UTFPR), Londrina, PR, Brazil. E-mail: henriqueelias@utfpr.edu.br.
** Doctor in Mathematics Education from Rutgers University-New Brunswick, Full Professor of Mathematics Education at Rutgers University-Newark, Newark, New Jersey 07102, USA. Email: powellab@newark.rutgers.edu
*** Ph.D. in Mathematics Education from Columbia University. Professor and Researcher at Brooklyn College, City University of New York (CUNY), Brooklyn, NY, USA. Email: Haydar@brooklyn.cuny.edu.
**** Doctor in Mathematics Education from Pontifícia Universidade Católica de São Paulo. Associate professor at Universidade Federal do ABC, Santo André, SP, Brazil. E-mail: alessandro.ribeiro@ufabc.edu.br.

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

Keywords: Proportionality. Proportional Reasoning. Proportion. Tetrahedron model of the didactical situation. Systematic Literature Review.

Resumo

Neste artigo, realizamos uma Revisão Sistemática da Literatura (RSL) a respeito do tema matemático proporcionalidade. Com essa RSL, buscamos responder às questões de investigação: como são as pesquisas em Educação Matemática sobre proporcionalidade encontradas no levantamento? Que interações emergem das análises feitas segundo o modelo tetraédrico da situação didática? O levantamento foi realizado considerando 41 periódicos e, a partir dos procedimentos de busca e dos filtros estabelecidos, foram encontrados 58 artigos. As análises foram realizadas em dois momentos. No momento 1, para responder à primeira questão investigativa, percebemos que há um predomínio de alguns referenciais teóricos que fundamentam os trabalhos, mas há, por outro lado, diversidade nos contextos em que as pesquisas ocorreram. No momento 2, para responder à segunda questão investigativa, utilizamos o modelo tetraédrico da situação didática para analisar 22 dos 58 artigos, pois são aqueles que realizaram a pesquisa envolvendo estudantes da Educação Básica. Identificamos quatro tipos de interações a partir do modelo tetraédrico: estudante-conteúdo, estudante-professor-conteúdo, estudante-conteúdo-artefato, estudante-professor-conteúdo-artefato. Mais da metade dos artigos (13 dos 22) exploram a interação estudante-conteúdo. Quatro artigos exploram a interação estudante-professor-conteúdo-artefato, sendo que apenas um dos artigos explicita a importância dessa interação para promover a compressão da noção de proporcionalidade. Finalizamos a RSL, indicando caminhos possíveis para novas pesquisas, principalmente aquelas que investiguem potencialidades da interação estudante-professor-conteúdo-artefato para o ensino de proporcionalidade e para o desenvolvimento do raciocínio proporcional dos estudantes.

Palavras-chave: Proporcionalidade. Raciocínio Proporcional. Proporção. Modelo tetraédrico da situação didática. Revisão Sistemática da Literatura.

1 Introduction

The concept of proportionality officially appears, according to the National Common Curricular Base (BNCC), in the fourth year of Elementary School, in the Thematic Unit Numbers, in the context of the different meanings of multiplication and division, and continues to be a central notion in the later school years of Elementary School, in different Thematic Units: Algebra, Geometry, Quantities, and Measurements. In High School, the concept is also mentioned in the BNCC, in the Skills linked to *Specific Competence 4*[1], linked to the study of polynomial functions (Brazil, 2018).

The expression "proportional thinking" also appears in the BNCC, only once[2], when it mentions that "[...] an aspect emphasized in Elementary School is the development of proportional thinking" (Brazil, 2018, p. 528). In the United States of America, the National Council of Teachers of Mathematics (NCTM), in the 1989 Curriculum and Evaluation Standards document, already emphasized that proportional reasoning: "[...] it is of such great

[1] According to the BNCC (Brazil, 2018, p. 538), in the context of Mathematics for Secondary Education, Specific Competence 4 aims to "Understand and use, with flexibility and precision, different mathematical representation records (algebraic, geometric, statistical, computational, etc.), in the search for solutions and communication of problem results".
[2] The expression *proportional thinking* does not appear in the BNCC for High School.

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

importance that it merits whatever time and effort that must be expended to assure its careful development. Students need to see many problem situations that can be modeled and then solved through proportional reasoning” (NCTM, 1989, p. 82).

Scientific research (e.g., Langrall; Swafford, 2000; Burgos; Godino, 2022) also highlights the relevance of the proportionality concept and proportional reasoning for students in elementary and secondary schools. According to Langrall and Swafford (2000), proportional reasoning is one of the most important skills to be developed during the final years of Elementary School, as it allows students to consolidate their mathematical knowledge from the early years of Elementary School and build a solid foundation for Mathematics and algebraic reasoning in High School. According to the authors, students who do not develop proportional reasoning will probably encounter obstacles in understanding higher-level Mathematics, especially Algebra.

Given the importance of the topic for students education, in this article, we conducted a Systematic Literature Review (SLR) aiming to identify relevant aspects of how researchers have dedicated themselves to investigating the topic of proportionality, proportion, and proportional reasoning or thinking throughout elementary and secondary school. Kaiser and Schukajlow (2024), when presenting the recent proposal of the journal ZDM - *Mathematics Education* to introduce, in the journal, a new category of special issues, dedicated specifically to *Reviews of Important Topics in Mathematics Education*, defend the importance of review research (narrative, scoping or systematic) in Mathematics Education. According to the authors,

> Mathematics educators are obliged to negotiate the ever-increasing number of scientific studies in mathematics education that generate new knowledge for researchers, teachers, and policy makers. This is no easy task, and finding and systematizing relevant information are major challenges of our time (Kaiser; Schukajlow, 2024, p. 1).

Thus, we recognize the importance of SLR research, such as the one we are proposing. There are already other literature reviews on the topic of proportionality or proportional reasoning in Brazilian academic literature (for example, Menduni-Bortoloti; Barbosa, 2017; Vieira; Santos, 2019; Melara; Silva, 2021). In the next section, we discuss these works in more detail, but it is worth mentioning that our research differs from those reviews, since our purpose in carrying out SLR is to identify, among other aspects, interactions between Student, Teacher, Content, and Artifact present in the research studies. The interaction between Student, Teacher, Content, and Artifact, called the tetrahedron model of the didactical situation (Rezat; Sträßer, 2012), consists of an extension of the triangular model[3], since, in addition to considering the interactions between teachers, students and content in the classroom, it highlights the relevant role of artifacts in the teaching of Mathematics.

Therefore, in this SLR, we seek to answer the following research questions: *what research about proportionality is found in mathematics education? What interactions emerge from the analyses made according to the tetrahedron model of the didactic situation?*

2 Theoretical basis

According to Petticrew and Roberts (2006, p. 2),

> [...] Systematic literature reviews are a method of making sense of large bodies of information, and a means of contributing to the answers to questions about what works and what does not – and many other types of questions too. They are a method of mapping out areas of uncertainty, and identifying where little or no relevant research has been done, but where new studies are needed. Systematic reviews also flag up areas where spurious certainty abounds. These are areas where we think we know more than we do, but where in reality there is little convincing evidence to support our beliefs.

In the case of the chosen mathematical theme - proportionality - there is a large number of publications, indicating the need to organize, through an SLR, all this information. The existing bibliographic survey research on the topic is relevant and points to some pertinent directions regarding how scientific productions have treated the teaching and learning of proportionality.

Menduni-Bortoloti and Barbosa (2017) carried out a systematic review of the literature based on seven relevant journals (Brazilian and non-Brazilian) in Mathematics Education, namely: *Boletim do Grupo de Estudos e Pesquisas em Educação Matemática* (Bulletin of GEPEM); *Zetetiké; Boletim de Educação Matemática (Bolema); Educação Matemática Pesquisa; Acta Scientiae, Journal of Mathematics Teacher Education* and *ZDM - International Journal on Mathematics Education*. According to the authors, the analysis of seventeen articles showed a diversity of realizations of this concept, distributed in three scenarios: (a) the proportionality concept realized as a ratio; (b) the concept of proportionality realized as equality between ratios; and (c) the concept of proportionality realized as a function. In scenario (a), the concept of proportionality as a ratio was realized as a comparison between parts, the

[3] Triadic conceptions of teaching and learning a discipline are not new. According to Herbst and Chazan (2012), research from the 1960s already provided triadic conceptions (triangular model) of teaching that allow us to distinguish the roles played in the action of teaching and investigate the relationships between teacher and students and between students and content in the classroom..

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

equivalence of ratios, rate, scale, division, vector, and musical intervals. In scenario (b), the proportionality concept was described by the equality between ratios, supported by Thales' theorem, whose realizations were the rule of three and percentage. In scenario (c), the concept of proportionality was presented as a function, through multiplicative relations, rate of variation, scale, and percentage.

Vieira and Santos (2019) conducted a literature review and analyzed nine academic articles. To select the texts, the authors searched the *CAPES* Journal Portal (Coordination for the Improvement of Higher Education Personnel) for works that contained the terms proportionality, proportion, or proportional reasoning in the title. According to the authors, the results indicated that the reasons for this knowledge lie in the development of proportional reasoning, mainly in conjunction with the teaching of the concepts of ratio, proportion, and linear function. The teaching approach most often mentioned in the survey was problem-solving in different contexts. Finally, the authors state that proportionality, in the elementary and secondary school Mathematics ecosystem, mainly inhabits the subareas of Algebra, Geometry, and Numbers and has as its ecological niche: ratio, proportion, similarity, and linear function.

Melara and Silva (2021) conducted a bibliographical research aiming at analyzing three doctoral theses, produced in Brazil between 2000 and 2019, which investigated issues related to proportional reasoning in training processes for Mathematics teachers. Among the main conclusions of the article, the authors mention that the training processes investigated favored the development of knowledge and the professional outlook of teachers. Based on the theses analyzed, they also concluded that these highlighted the importance of training processes that provide teachers with spaces that favor reflection and allow them to become aware of their pedagogical work (Melara and Silva, 2021).

The three literature reviews mentioned (Menduni-Bortoloti; Barbosa, 2017; Vieira; Santos, 2019; Melara; Silva, 2021) offer us complementary contributions in terms of results. While Menduni-Bortoloti and Barbosa (2017) focus their review on the *concept* of proportionality, in order to present a Mathematics for teaching this concept, Vieira and Santos (2019), when investigating *the reasons for being and the didactic ecology of proportionality*, point to proportional reasoning and problem solving as a teaching approach as central aspects of the research analyzed in the review. Melara and Silva (2021), on the other hand, present, through the doctoral theses analyzed, contributions within the scope of the *training processes* of Mathematics teachers.

The SLR proposed in this research differs from those mentioned above both in scope and purpose. In terms of focus, our interest is in looking at the interaction between Student,

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

Teacher, Content, and Artifact present in the investigations carried out. In terms of scope, as explained in the next section, a greater number of scientific articles will be considered than the number of works considered in the three surveys mentioned.

Our focus is on the interactions because, as Cohen, Raudenbush, and Ball (2000) point out, teaching is not created only by teachers, by students, or by content, but by their interactions. For the authors, *interaction* does not refer to a particular form of discourse, but to the connected work of teachers and students on content, in environments such as the classroom.

As Rezat and Sträßer (2012) point out, the relationship between teacher and students in the mathematics classroom has never really been dual, because it has always relied on the mediation of artifacts (i.e., books, rulers, compasses, tasks or, more recently, digital technologies) incorporated into the teaching activity[4]. Due to the ability of artifacts to restructure the didactic situation as a whole, Rezat and Sträßer (2012) argue that these should be considered as a fourth vertex, extending the model of the didactic triangle[5] to the tetrahedral model of the didactic situation[6], as shown in Figure 1.

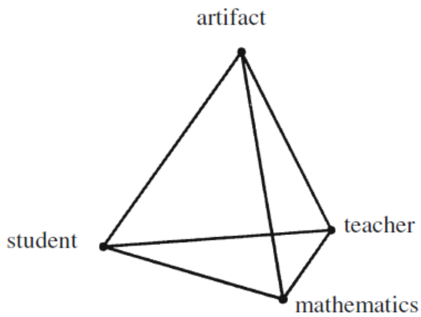

Figure 1- Tetrahedral model of the teaching situation
Source: REZAT; STRÄßER (2012, p. 645)

From this perspective, it is important to consider that not only do students manipulate artifacts while learning Mathematics, but teachers also interact as they select, produce, and adapt artifacts while planning or teaching their Mathematics classes.

Thus, for Rezat and Sträßer (2012), each of the triangular faces of the tetrahedron represents a particular perspective on the role of artifacts: the teacher-student-content triangle (the didactic triangle) is the basis of the model; the student-artifact-content triangle represents the activity mediated by the mathematics learning instrument; the teacher-artifact-student

[4] In the methodological aspects section, we explain what type of artifacts we are considering in this research.
[5] Volume 44, issue 5 of the journal *ZDM - Mathematics Education* is dedicated exclusively to articles that discuss new perspectives for the didactic triangle: teacher, student and content.. Link: https://link.springer.com/journal/11858/volumes-and-issues/44-5. Accessed on June 7, 2024.
[6] Rezat and Sträßer (2012) emphasize that the extension of the didactic triangle to the tetrahedron is not new and that they were not the first to do so. There are other precursors in Mathematics Education, such as David Tall.

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

triangle describes the role of the teacher as an orchestrator and a mediator; the teacher-artifact-content triangle represents the teacher's activities of doing mathematics and planning mathematics teaching mediated by the artifact.

Rezat and Sträßer (2012) go further in their proposed model since they include the social and cultural dimensions, proposing a socio-didactic tetrahedral model. We agree with the socio-didactic tetrahedral model, which considers that social, political, and cultural issues (e.g., family, community, the public image that one has about Mathematics and its role in society, the appreciation and role of the teacher in society, etc.) influence the teaching and learning of Mathematics. We believe that the tetrahedral model shapes and is shaped by the broader social, economic, and political context. However, in our research, we will not focus on these aspects (socio-political-economic-cultural) that influence the teaching and learning of Mathematics[7]. We will focus *only* on the four vertices presented in Figure 1: Student (S), Teacher (T), Content (C), and Artifact (A).

In a classroom environment, each of the four elements of the tetrahedron performs specific roles and responsibilities that influence and manifest themselves in the way they interact with each other. These roles, responsibilities, and interactions produce different pedagogical models.

From this, it is necessary to understand and explain the roles, responsibilities, and interactions of the elements that make up the didactic tetrahedron. Thus, in this article, we seek to understand how such interactions emerge from research in Mathematics Education.

3 Methodological aspects

We begin this section by presenting the seven steps for implementing an SLR, according to Petticrew and Roberts (2006). In presenting them, we describe and justify our inclusion and exclusion criteria for both journals and articles to be considered in the survey. For the inclusion and exclusion criteria of articles, we carried out the process in three phases. Finally, to answer our research question, we divided the data analysis into two stages. The seven steps for implementing an SLR, the three phases for selecting articles, and the two stages of data analysis are explained throughout this section.

Petticrew and Roberts (2006) present seven steps for conducting an SLR: 1) Clearly define the question that the review is setting out to answer; 2) Determine the types of studies

[7] Considering these aspects with regard to the content of proportionality will be a later stage of our studies, in subsequent articles.

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

that need to be located to answer your question; 3) Carry out a comprehensive literature search to locate those studies; 4) Screen the results of that search (that is, sift through the retrieved studies, deciding which ones look as if they fully meet the inclusion criteria, and thus need more detailed examination, and which do not; 5) Critically appraise the included studies; 6) Synthesize the studies and assess the heterogeneity among the study findings; 7) Disseminate the findings of the review.

The questions that the research aims to answer, stage 1 of the RSL, are: *what research about proportionality is found in mathematics education? What interactions emerge from the analyses made according to the tetrahedral model of the didactic situation?*

For stage 2 of the SLR, we defined scientific articles published in journals as the type of studies to be considered to answer the research question. For stage 3 of the SLR, on the Sucupira Platform, in Qualis Periódicos[8], we performed an initial search considering the following inclusion criteria for journals: (i) *classification event*: Classification of Journals Quadrennial 2017-2020; (ii) *evaluation area*: Teaching; (iii) *title*: Mathematics; and (iv) *classification*: A1, A2, A3 and A4. With these filters, we sought to find journals in the Teaching area, with Qualis A1, A2, A3, or A4, and that have the word Mathematics in the journal name. These inclusion criteria are justified by the fact that they include journals in the Teaching area, with a focus on Mathematics, well evaluated in the latest Qualis Periódicos classification. From this search, we included 19 journals.

However, we noticed that relevant journals that do not contain the word *Mathematics* in their names were left out. For this reason, we returned to the Sucupira Platform and Qualis Periódicos to conduct a complementary survey, including Qualis A1 and A2 journals (Classification of Journals for the 2017-2020 Quadrennial Period) with a *primary area* of Education. This time, we considered, in the inclusion criteria, journals whose scope indicated being Mathematics Education, but that do not have Mathematics in their name (for example, Zétetiké), and journals specific to Science Education that publish works on Mathematics Education (for example, Areté - Revista Amazônica de Ensino de Ciências). In this complementary survey to the first one, we chose to include only Qualis A1 and A2 journals to limit the number of journals and, consequently, prevent the high number of articles included in the SLR from making more in-depth analyses unfeasible.

As exclusion criteria for this complementary survey, we considered non-Brazilian

[8] Link:
https://sucupira.capes.gov.br/sucupira/public/consultas/coleta/veiculoPublicacaoQualis/listaConsultaGeralPeriodicos.jsf

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

journals[9] and those that are explicitly from another area. For example, the following journals were excluded: *Revista Brasileira de Ensino de Física*; *Contemporâneos: Revista de Artes e Humanidades*; *Química Nova na Escola*; *Revista de Ensino de Biologia from the Brazilian Brasileira de Ensino de Biologia*; *English Language Teaching*; *Chemistry Education; Revista de investigación en didáctica de las matemáticas.*

After exclusions, we selected 21 other journals (discounting the Qualis A1 and A2 journals already considered in the first search). Table 1 presents the journals included in the SLR.

Qualis (Quantity included)	**JOURNALS**
A1 (9)	Areté (Manaus) - Revista Amazônica de Ensino de Ciências
	Bolema: Boletim De Educação Matemática
	Ciência & Educação
	Educação Matemática Pesquisa
	Ensaio: Pesquisa Em Educação Em Ciências
	Investigações Em Ensino De Ciências
	Pesquisa Em Foco (UEMA)
	Revista Brasileira De Pesquisa Em Educação Em Ciências
	Revista Internacional De Pesquisa Em Educação Matemática (RIPEM)
A2 (18)	Acta Scientiae (ULBRA)
	Alexandria (UFSC)
	Amazônia - Revista De Educação Em Ciências E Matemáticas (Online)
	Caderno Pedagógico
	Conexões: Ciência E Tecnologia
	Contexto & Educação
	EAD Em Foco - Revista De Educação A Distância
	Educação Matemática Em Revista
	Perspectivas Da Educação Matemática
	RBPG - Revista Brasileira Da Pós-Graduação
	RENCIMA
	Revista Brasileira Da Educação Profissional E Tecnológica
	Revista Brasileira De Ensino De Ciência E Tecnologia
	Revista Do Instituto Geogebra Internacional De São Paulo
	Revista Interagir
	Revista Thema
	Vidya

[9] To include non-Brazilian journals, we believe that the criteria should be different from those adopted in this research. Qualis Periódicos is not an appropriate criterion for international journals, since Qualis is built based on the scientific production of Brazilian stricto sensu Graduate Programs (PPG) for the purpose of evaluating them. This means that relevant international journals may not even appear in a Qualis Periódicos stratum if, in the period evaluated, they did not have a publication by a member of a stricto sensu PPG. For example, the relevant journal *Mathematical Thinking and Learning* (Impact Factor (2022): 1.6; *CiteScore (Scopus*) (2022): 3.0; *SNIP* (2022) 1.552; *SJR* (2022) 0.929) is not included in Qualis Periódico (2017-2020).

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

	Zetetiké
A3 (7)	Jornal Internacional de Estudos em Educação Matemática
	Revemat: Revista eletrônica de Educação Matemática
	Revista Brasileira de História da Matemática
	Revista de Educação Matemática
	Revista de História da Educação matemática
	Revista Paranaense de Educação Matemática
	Tangram: Revista de Educação Matemática
A4 (6)	Caminhos da Educação Matemática em Revista
	Ensino da Matemática em Debate
	Rematec: Revista de Matemática, Ensino e Cultura
	Revista de Educação, Ciências e Matemática
	Revista de Produção discente em Educação Matemática
	Revista Sergipana de Matemática e Educação Matemática
40	**Total number of journals considered in SLR**

Table 1 – Journals considered in the Systematic Literature Review
Source: prepared by the authors, 2025

As shown in Table 1, we considered a total of 40 journals, namely: 9 Qualis A1 journals, 18 Qualis A2 journals, 7 Qualis A3 journals, and 6 Qualis A4 journals. In addition to these journals, we included the journal *Quadrante*, from Lisbon, Portugal, a Qualis B3 journal that could contain additional information for our survey. We included *Quadrante*, even though it deviates from the pre-established criteria, because it contains publications in Portuguese (which also facilitates the publication of research conducted by Brazilians in the journal) and because Portuguese research often has repercussions in Brazil[10]. Thus, *Quadrante* is the only non-Brazilian journal included in our review.

Once the journals were selected, we began searching for articles that addressed the topic of proportionality. This is stage 4 of the SLR, which is dedicated to deciding which studies appear to fully meet the inclusion criteria and therefore require more detailed examination and which do not. We divided the procedure involving this stage into three phases: Phase 1, Phase 2, and Phase 3.

In Phase 1, we used the search tool, available on the journals' platforms. In the first journals searched, we used the word *proportionality* as a keyword. We noticed that words such as *proportion* and *proportional thinking* are also used in research on the topic and, for this reason, we included these words as keyword. Therefore, we used the words: *proportionality,*

[10] Quadrante is not the not the only journal with these characteristics. For example, there is the journal *Educação e Matemática: Revista da Associação de Professores de Matemática*, also from Portugal and with publications in Portuguese. The inclusion of Quadrante as the only non-Brazilian journal with publications in Portuguese was an exception adopted by the authors, simply to have a parameter of research published outside Brazil. As mentioned in footnote 9, including non-Brazilian journals would require other criteria.

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

proportion, proportional thinking, and proportional reasoning as search terms.

The search in the journals was carried out throughout the month of February 2024 and considered all articles that appeared when the aforementioned search keyword were considered, without limiting by year of publication. Thus, in Phase 1, we collected 197 articles (194 articles found in national journals and 3 articles found in *Quadrante*).

Then, in Phase 2, after reading the abstracts and searching for our chosen keywords (*proportionality, proportion, proportional reasoning,* and *proportional thinking*) throughout the text to verify in which contexts they were being considered, 117 articles were discarded, as they clearly did not focus on the theme of proportionality in the context we intended. We illustrate this with four examples: (a) the article by Cintra, Marques Junior, and Sousa (2016), which is dedicated to making a correlation between the reference matrix and the items involving Chemistry concepts present in the National High School Exam (ENEM) from 2009 to 2013; (b) the article by Cavadas and Sá-Pinto (2020), which aims to understand the conceptions of Portuguese students in initial teacher training about evolution and the origin of life; (c) the article by Correia and Nardi (2019), in which the word proportion appears in the context of indicating the proportion between concepts associated with classical astronomy and modern astronomy presented in the students' responses; and (d) the article by Oliveira and Souza (2018, p. 240), in which the word proportionality appears twice throughout the text to say that a student investigated "used a notion of proportionality to argue about the existence of a relationship between a person's height and the size of their feet". In other words, these examples indicate that several articles were disregarded because they only mention some of the search terms, but do not focus on the topic of proportionality.

After these studies were discarded, 80 articles remained. Then, in Phase 3, the 80 articles were read in full, seeking to understand whether they were in fact focused on the topic of proportionality. During this reading, the following information was recorded in an Excel spreadsheet: *the context of the research* (i.e., whether it was with students in elementary and secondary schools, in initial teacher training, in continuing teacher training, with textbooks, with curricular documents, or whether it was a theoretical work), *main theoretical references* (emphasizing references on teaching and learning proportionality, proportion or proportional reasoning and thinking) and the way in which the *topic of proportionality was addressed* (whether it was within another mathematical topic, such as Thales' Theorem or function, or in the context of multiplicative structures, or whether it was directly the topic of proportionality via some teaching approach, for example, problem-solving, mathematical modeling, mathematical tasks, etc.).

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

From this reading of the 80 articles, we noticed that there were still articles that did not focus on the proportionality theme. For example, in the article by Pontes and Guimarães (2021, p. 48), which discusses the construction of bar graphs, the theme of proportionality appears when students needed to represent "a simple bar graph, with a non-unitary scale, taking into account the proportionality of the bars". Thus, some articles with this characteristic were excluded, resulting in 58 articles that effectively constituted the *corpus* of our research.

From these 58 articles, we proceeded to stages 5 and 6 of data processing, following Petticrew and Roberts (2006). We carried out the analyses in two stages. In stage 1, we extracted two relevant pieces of information from the articles: the context of the research and the main theoretical frameworks used. With this, we sought to understand, in a broad way, how the theme of proportionality has been addressed in Mathematics Education research (first researchquestion). In stage 2, using information obtained in stage 1, we selected only articles whose research was conducted with students in elementary or secondary schools. With this, we sought to identify, using the tetrahedral model, which themes and interactions emerge between Students, Teachers, Content, and Artifact (second research question).

A summary of the procedures for the SLR is presented in Figure 2.

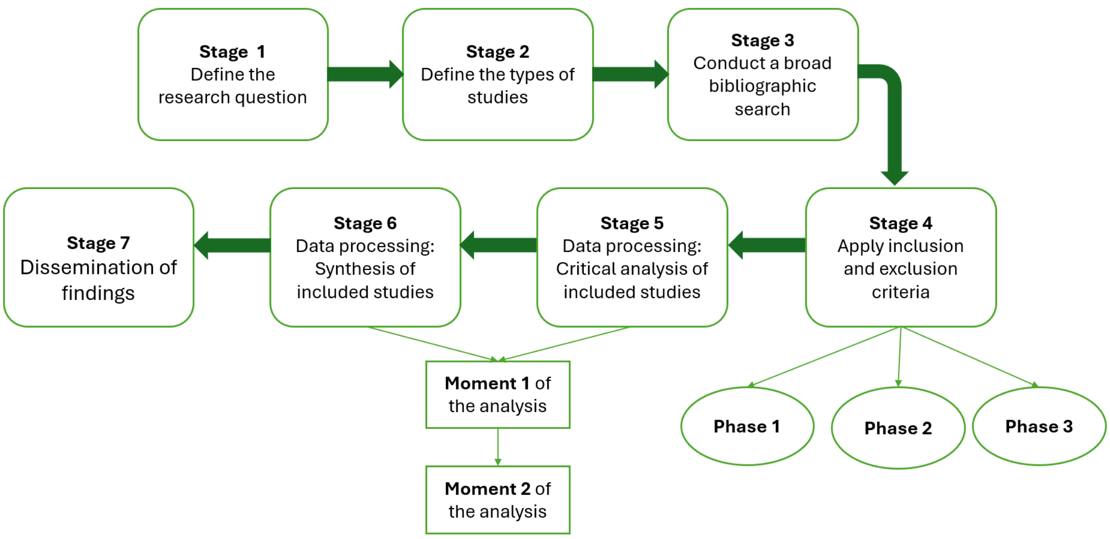

Figure 2 - Summary of the stages, phases and moments carried out in the RSL
Source: prepared by the authors, 2025

Finally, it is worth noting that, at vertex A of the tetrahedron, we are interested in

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

specific artifacts, such as manipulable materials[11] and digital technologies[12]. Our interest is in identifying, in the research reviewed, how interactions (Student, Teacher, Content, and Artifact) occur involving artifacts other than those routinely used in the classroom. Therefore, in our analyses, we are not considering the use of language, writing, textbooks, or mathematical tasks (problems) at vertex A of the tetrahedral model.

4 Data analysis

4.1 Stage 1 of analysis

In Table 2, we seek to organize the main theoretical frameworks on proportionality (proportion, proportional reasoning, proportional thinking) adopted in the research surveyed. We present only the three theoretical frameworks most mentioned in the research. Others were also mentioned, but on fewer occasions, if compared to the three presented in Table 2. For example, the perspective of Silvestre and Ponte (2013) and Ponte *et al*. (2010) appeared in four studies, and the perspective of Maranhão and Machado (2011) was also mentioned in four studies. Considering this discrepancy, we chose to organize and discuss only the three most mentioned theoretical frameworks.

Theoretical Frameworks	Survey articles
Proportional reasoning, according to Susan Lamon (e.g., Lamon (2012)[13])	Faria and Maltempi (2019); Cyrino (2016); Fernandes (2021); Cebola and Brocardo (2021); Garcia Silva, Cândido and Souza (2018); Fernandes (2022); Bezerra, Martins, Borssoi, and Silva (2022); Scheller and Bonotto (2020); Coutinho and Tortola (2020); Souza, Galvão, and Poggio (2016); Melara and Silva (2021); Cândido and Garcia Silva (2018); Costa, Martins, Pagani, and Allevato (2023); Garcia and Cyrino (2019); Neres and Almeida (2021); Barros Nunes and Costa (2016); Garcia Silva, Cândido, and Pietropaolo (2018)
Proportional reasoning, according to Lesh, Post and Behr (1988) or other works by the *Rational Number Project* group (RNP)[14].	Cyrino (2016); Garcia Silva, Lopes, and Galvão (2020); Garcia Silva, Cândido, and Souza (2018); Bezerra, Martins, Borssoi, and Silva (2022); Scheller and Bonotto (2020); Silva and Palanch (2021); Souza, Galvão, and Poggio (2016); Melara and Silva (2021); Cândido and Garcia Silva (2018); Viana and Miranda (2016); Garcia and Cyrino (2019); Barros Nunes and Costa (2016); Garcia Silva, Cândido and Pietropaolo (2018)
Theory of Conceptual Fields	Ramos, Silva, Luz, Firme, and Saraiva (2021); Alpha and Almouloud

[11] "Manipulative materials are those which the pupil is able to feel, touch, handle, and move. They may be real objects which have social applications in our everyday affairs, or they may be objects which are used to represent an idea or a characteristic of number or of the number system. Such objects as a measuring cup, ruler, scales, thermometer, and milk bottles of different size are used in our daily affairs" (Grossnickle; Junge; Metzner, 1951, p. 162).

[12] Digital technologies are understood here as digital artifacts "both physical (computers, cell phones, tablets, etc.) and virtual (internet, social networks and data clouds, among others)" (Brasil, 2018, p. 474).

[13] We mentioned the third edition of the book (Lamon, 2012), but several studies use previous editions.

[14] The RNP began in 1979 with researchers Merlyn Behr, Thomas Post and Richard Lesh as members. Other researchers joined the project, such as Kathleen Cramer. This group of researchers presented several empirical and theoretical research results on the teaching and learning of rational numbers.

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

(Multiplicative Structure) according to the works of Gérard Vergnaud (e.g. Vergnaud (1990)).	(2021); Tieppo, Cappelin, Zanatta, Nogueira, and Rezende (2023); Fioreze, Barone, Basso, and Isaia (2013); Cebola and Brocardo (2021); Aragão, Lauterta, and Schliemann (2022); Milagre and Santana (2018); Silva, Pessoa, and Carvalho (2020); Scheller and Bonotto (2020); Rodrigues and Rezende (2021); Miranda, Rezende, and Nogueira (2021); Viana and Miranda (2018); Espindola and Moura (2018); Peron, Nogueira, and Rezende (2019); Vergnaud (2018); Rodrigues and Rezende (2023); Viana and Miranda (2016)

Table 2 – Main theoretical references considered in the articles
Source: prepared by the authors, 2025

From Table 2, we can see which theoretical references have supported the research published in Brazil (and three in Portugal) when it comes to the topic of proportionality. Gérard Vergnaud's Theory of Conceptual Fields and the notion of Proportional Reasoning according to Susan Lamon's work were predominant[15].

Regarding the strong presence of Vergnaud's work, mainly involving the Conceptual Field of Multiplicative Structures, the insertion and influence of this important researcher (and of Mathematics Didactics in general) in Mathematics Education in Brazil is recognized, whether through his own productions (also) in Portuguese (as is the case of the article Vergnaud, 2018), mentioned in Table 2), or through consolidated research groups in the country that carry out investigations from Vergnaud's perspective. For example, the Study and Research Group on Mathematics Didactics (GEPeDiMa)[16] one of the leaders of which is researcher Veridiana Rezende, appeared in several articles in Table 2.

Regarding the presence of Lamon's works, it is clear that her book "*Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers*" has a great influence on the research surveyed that deals with Proportional Reasoning. We also noted that most of the articles that mention Susan Lamon's works also consider proportional reasoning from the perspective of Lesh, Post and Behr (1988) or the RNP.

Table 2 indicates that, at least with regard to the theoretical perspective, much of the research is strongly centered on the same theoretical references, suggesting little foundational plurality on the topic.

In Table 3, we organize the articles based on the contexts in which the research was developed.

Research contexts	Articles	Quantity
A literature review or article analysis	Menduni-Bortoloti and Barbosa (2017); Vieira and Santos (2019); Melara and Silva (2021)	3

[15] In this text, we are not interested in presenting a number that indicates exactly how many times a given theoretical framework has appeared. That was not the purpose of Table 2. The purpose was to provide an overview, a broad idea of what has predominated in the research analyzed.
[16] Link to access the Group in the CNPq Directory: https://dgp.cnpq.br/dgp/espelhogrupo/521078 . Acessed on April 27, 2024.

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

Textbook analysis	Ramos, da Silva, da Luz, Firme, and Saraiva (2021); Alpha and Almouloud (2021); Rodrigues and Rezende (2021); Lazzaretti and Mathias (2023); Santos and Santos-Wagner (2016); Peron, Nogueira, and Rezende (2019);	6
Document analysis	Alpha and Almouloud (2021); Vieira, Barros and Santos (2020); Tieppo, Cappelin, Zanatta, Nogueira, and Rezende (2023); Silva, Pessoa, and Carvalho (2020); Silva and Palanch (2021); Miranda, Rezende, and Nogueira (2021); Lima (2013).	7
Early Years or Early Childhood Education students	Bezerra, Martins, Borssoi, and Silva (2022); Castro and Castro Filho (2018); Coutinho and Tortola (2020); Rodrigues and Rezende (2023).	4
Early Secondary or Middle School Students	Oliveira and Clareto (2020); Fioreze, Barone, Basso, and Isaia (2013); Cebola and Brocardo (2021); Aragão, Lautert, and Schliemann (2022); Siqueira and Gaertner (2015); Savioli and Silva (2015); Viana and Miranda (2016); Quitembo and Domingas (2020); Viseu (2015); Viana and Miranda (2018); Espindola and Moura (2018); Teixeira (2020b); Neres and Almeida (2021); Cabral, Dias, and Lobato Junior (2019); Oliveira, Fernandes, and Fermé (2007); Andreatta; Allevato, and Pinto (2018).	16
Secondary School Students	Parameswari, Purwanto, Sudirman, and Susiswo (2023); Souza, Galvão, and Poggio (2016); Teixeira (2020a); Costa, Martins, Pagani, and Allevato (2023).	4
Initial Teacher Training (Bachelor's Degree in Mathematics, Bachelor's Degree in Pedagogy)	Fernandes (2021); Burgos, Godino, and Rivas (2019); Garcia Silva, Cândido, and Souza (2018); Fernandes (2022); Scheller and Bonotto (2020); Bisognin, Fiorez, and e Cury (2005); Cândido and Garcia Silva (2018); Nunes and Costa (2016); Garcia Silva, Cândido, and Pietropaolo (2018).	9
Continuing training of teachers who teach mathematics	Faria and Maltempi (2019); Cyrino (2016); Menduni-Bortoloti and Barbosa (2018); Garcia Silva, Lopes, and Galvão (2020); Milagre and Santana (2018); Garcia and Cyrino (2019).	6
Construction workers	Silva (2011).	1
Theoretical work	Silva (2012); Fossa (2020); Vergnaud (2018).	3

Table 3 – Contexts in which the research was developed
Source: prepared by the authors, 2025

From Table 3, we noticed[17] that the context in which research is most often conducted is with students in the Early Secondary or Middle School. Regrouping the studies into four larger categories, we obtain: (a) *document analysis* (including literature review, analysis of textbooks and documents); (b) *elementary and secondary school students* (including Early Childhood Education, Elementary Education, and Upper Secondary or High School); (c*) teacher training* (including initial and continuing teacher training); and (d) *other contexts* (including construction workers and theoretical studies).

We noted that the *elementary and secondary school students*' category is the one with the most studies conducted, totaling 24, followed by the *teacher training category*, with 16 studies.

[17] The sum of the number of articles presented in Table 3 is 59 and not 58, as the article Alpha and Almouloud (2021) appears twice, as it investigated textbooks and also curricula in the Republic of Mali.

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

Identifying the context of the research was also relevant for moment 2 of data analysis, which considered the category of elementary and secondary school students to analyze the interactions between Student (S), Teacher (T), Content (C), and Artifacts (A), according to the tetrahedral model.

4.2 Stage 2 of analysis

At this stage of the analysis, we focused on 22 of the 24 articles in the category of *elementary and secondary school students*. The article Teixeira (2020a) was excluded because, despite involving 7th-grade or Early Secondary School students, it presents characteristics of a report of classroom experience, with the absence of a theoretically based analysis. The same occurred with Andreatta, Allevato and Pinto (2018), a work also with characteristics of a report of a classroom experience and focusing on the problem situations presented, not on the discussion involving proportionality.

From the 22 remaining papers, we conducted the analyses by categorizing each of the articles depending on the focus of the interaction considered. Such interaction may involve an edge of the tetrahedron, for example, SC (student and content) and SA (student and artifact); a face of the tetrahedron, for example, STC (student, teacher, and content); or the tetrahedron as a whole, that is, STCA (student, teacher, content, and artifact).

We based our coding process on Stylianides, Stylianides, and Moutsios-Rentzos (2024) based on the idea of *focus*. For example, consider an article investigating how students deal with proportionality in the context of classroom discussions. If the teacher, although present in these discussions and sometimes influential in the issues discussed by the students, did not appear prominently[18] in the analysis of the article's data, the article was coded as SC. If the role of the teacher and the role of the students are explicit in the article (not only theoretically, but also during the data analysis), then the article was coded as STC. The same happens with regard to the use of the artifact; if the use of the material in the interaction between student, teacher, and content is explicit, the article was coded as STCA. In the subsections below, we present the four categories that emerged from the data.

[18] By *prominence*, we refer to whether or not the teacher appears during classroom dialogues with students. If, during the analysis of the data in the articles, no speech by the teacher appears or if only one speech by the teacher appears to present a task, we consider that this teacher did not appear prominently. However, if during the analysis of the data, different speeches by the teacher are presented, in the sense of interacting with the students, we consider that this teacher appeared prominently..

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

4.2.1 Student and Content (SC)

The articles that fall into this category are: Castro and Castro Filho (2018), Rodrigues and Rezende (2023), Aragão, Lautert and Schliemann (2022), Viana and Miranda (2016, 2018), Quitembo and Domingas (2020), Espindola and Moura (2018), Teixeira (2020b), Neres and Almeida (2021), Cabral, Dias and Lobato Junior (2019), Parameswari *et al.* (2023), Souza, Galvão and Poggio (2016) and Costa *et al.* (2023).

It is possible to perceive that most of the articles, 13 of the 22, focus on SC interaction. The central point of the articles in this category is almost always the written production of students when solving a problem (or test). Occasionally, there is mention of a didactic intervention (for example, in Viana and Miranda (2018) and Castro and Castro Filho (2018)), but the analyses are aimed at understanding the strategies used by students when solving the requested problems. Castro and Castro Filho (2018) mention that one of the objectives was to verify how an intervention, based on the use of digital technologies, can significantly contribute to changing the performance of these children. However, in the analyses, the authors only deal with the pre-test and post-test, without indicating how the mentioned intervention actually took place.

To illustrate the main characteristic of this category, that is, the focus on SC interaction, we present the objectives of some research. Aragão, Lautert, and Schliemann (2022) sought to analyze how students solve problems of double and multiple proportions. Rodrigues and Rezende (2023) aimed to analyze operational invariants associated with the concept of function, mobilized by 5th-grade elementary school students, while solving mixed situations of the simple proportion and measurement transformation type. Espindola and Moura (2018) aimed to identify the difficulties of 6th-grade elementary school students in relational and numerical calculation when solving problem situations in the multiplicative conceptual field.

With regard to the results presented by the research contained in this category, in general, SC interaction is often analyzed based on what the problem (or test) provided (or not) or would have the potential to provide to students. For example, Aragão, Lautert, and Schliemann (2022, p. 202, our translation) conclude that:

> The totality of correct answers associated with the mixed strategy [...] deserves to be highlighted because it suggests that students when using this strategy, consider the multiple relationships present in the problem statement. [...] The lack of use of the rule of three, despite this algorithm being part of the 7th-grade textbook, may be related to the fact that the numerical relationships in the problems are easy to determine mentally.

Viana and Miranda (2016, p. 194) conclude that "The results indicate that there was

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

no parallel between the strategies for the two types of problem and the work suggests that different situations should be offered to broaden the conceptual field that refers to proportionality".

Teixeira (2020b, p. 19) states that most students "[...] sought to solve the problems with the knowledge they had already acquired in previous years" and that "[...] the concept of proportionality through the model of a linear function was not used by any of the study subjects in solving the problems proposed in the evaluation sheet".

Here, we present the objectives of some research (Aragão; Lautert; Schliemann, 2022; Rodrigues; Rezende, 2023; Espindola; Moura, 2018) and the results of others (Aragão; Lautert; Schliemann, 2022; Viana; Miranda, 2016; Teixeira, 2020b) to illustrate the main characteristic of all the works in this category: the focus is on the way students deal with mathematical content, regardless of the teacher's actions or the use of some artifact. The research in this category may differ in the way they approach students (using problems, tasks, or tests) or in the mathematical theme to be considered (direct proportionality, inverse proportionality, double and multiple proportions, function, aspects of proportional reasoning), but they all investigate SC interaction.

4.2.2 Student, Teacher, and Content (STC)

The articles that fall into this category are: Oliveira and Clareto (2020), Cebola and Brocardo (2021), and Savioli and Silva (2015).

To illustrate the main characteristic of this category, that is, the focus on STC interaction, we present the objectives of the three studies. Oliveira and Clareto (2020) aimed to present a writing essay from a mathematics classroom through fables. Cebola and Brocardo (2021) aimed to analyze the articulation between concepts, resolution strategies and representations used by students, supported by numerical relations and properties of operations to characterize a model of the conceptual evolution of multiplicative comparison. Savioli and Silva (2015) sought to identify, analyze, and discuss aspects of algebraic thinking and language manifested by 6th-grade students of a public school in Palotina, Paraná when solving problems involving algebraic content.

Regarding the results presented by the research contained in this category, we noticed that the STC interactions that emerged point in different directions: the diversity of ways of doing mathematics present in a classroom, the strategies for the teacher to provide a conceptual understanding of multiplicative comparison by students, and the aspects of algebraic thinking

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

manifested by students.

Oliveira and Clareto (2020) underscore the different ways of doing (mathematics) in a classroom. Student, teacher, textbook, History of Mathematics, mathematicians, each one has a way, a way of doing mathematics (in this case, the topic of proportionality) and, in the end, “This is how the classroom is made: with lives and inventions and productions and code shuffling: *this is how I do it*!” (Oliveira and Clareto, 2020, p. 950, emphasis in the original).

Cebola and Brocardo (2021, p. 147) present results regarding the model of the conceptual evolution of multiplicative comparison, stating, for example, that “the construction of the concept of multiplicative comparison prevails in the work in two measurement spaces and in the exploration of the multiplicative relationship between the corresponding quantities within each one”. Thus, in their conclusions, the authors suggest to teachers to organize teaching aimed at students' conceptual understanding of multiplicative comparison.

Savioli and Silva (2015) point out results that concern the algebraic thinking expressed by sixth-grade students, indicating that they used, for example, syncopated language to express themselves mathematically and direct proportionality to solve the proposed problem.

In all three cases, although the focus of the data analysis was not on the teacher, their role is well defined in the work. In all of them, during the data analysis, the teachers clearly appeared as mediators in the student-content interaction and, for this reason, they were considered in this category (STC).

4.2.3 Student, Content, and Artifact (SCA)

The articles that fall into this category are: Fioreze et al. (2013), and Siqueira and Gaertner (2015).

To illustrate the main characteristic of this category, that is, the focus on SCA interaction, we present the objectives of the two studies. Fioreze *et al.* (2013) aimed to analyze the construction of proportionality concepts using digital resources, especially the Virtual Geoplane. The objective of Siqueira and Gaertner (2015) was to analyze the contributions of the teaching methodology called Interdisciplinary Islands of Rationality[19], in the learning of the concept of proportionality.

[19] According to Siqueira and Gaertner (2015), the Interdisciplinary Islands of Rationality are a teaching methodology focused on Scientific and Technological Literacy, proposed by Gérard Fourez, which aims to develop critical and autonomous students. This methodology is a means of directing the work that will be done in the classroom, and this work is not determined by the various disciplines linked to the theme, but by the project, its purpose and the context.

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

Regarding the results presented by the studies contained in this category, we perceive that the SCA interactions that emerged go in the direction of recognizing the importance of artifacts in the student-content relationship.

Fioreze *et al*. (2013, p. 275-276) conclude that comparing the results obtained in the previous analysis[20], after experimenting using the Virtual Geoplane software as an artifact, the student "expanded the concept of similarity of flat figures explicitly, that is, in addition to making this concept explicit, it is closer to the concept accepted by the scientific community".

Siqueira and Gaertner (2015, p. 160) used the Interdisciplinary Islands of Rationality teaching methodology to address the concept of proportionality by exploring food labels as an artifact. In conclusion, the authors state that "teaching, when presented in a contextualized and interdisciplinary manner, provokes in students the desire to construct their own knowledge in a more sophisticated way, promoting Scientific Literacy".

In both cases, the artifacts (Virtual Geoplane software and food labels) seem, as indicated by the authors of the research, to allow students to carry out experiments that lead them to the formalization of the concept of proportionality.

Neither article focuses on the teacher. Siqueira and Gaertner (2015) mention, in the Research Methodology, the interactions between students and students and teacher, but, during the data analysis, the teacher did not appear. Fioreze et al. (2013) also emphasize the role of the teacher (in this case, the teacher and researcher are the same person), but they do so only in the theoretical discussion and in the conclusions and do not provide data (transcribed excerpts of speech, for example) related to the teacher's interventions during the analysis.

Because they did not include teachers during the data analysis, both articles were included in this category (SCA).

4.2.4 Student, Teacher, Content and Artifact (STCA)

The articles that fall into this category are: Bezerra et al. (2022), Coutinho and Tortola (2020), Viseu (2015), Oliveira, Fernandes, and Fermé (2007).

To illustrate the main characteristic of this category, that is, the focus on STCA interaction, we present the objectives of the four studies. Bezerra *et al*. (2022) aimed to investigate the mobilization of Proportional Reasoning in a mathematical modeling activity developed in the context of remote teaching. Coutinho and Tortola (2020) aimed to investigate

[20] According to the authors, the preliminary analysis refers to a stage prior to the use of the Virtual Geoplano software, in which the aim was to verify what the students knew about proportionality.

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

how Early Childhood Education students (nursery school III class, 3 and 4 years old) mobilize proportional reasoning in a mathematical modeling activity. Viseu (2015) sought to characterize the activity that 9th-grade students carry out with modeling tasks, using a graphing calculator and sensors, in the study of functions and the difficulties they reveal in these activities. Finally, Oliveira, Fernandes, and Fermé (2007) aimed to describe, analyze, and understand how students learn Mathematics using robots as mediating elements of learning.

Regarding the results presented by the research contained in this category, we noticed that the STCA interactions that emerged are closely linked to the teaching approach adopted by the teacher in the context of the research. Three out of the four articles (Bezerra et al., 2022; Coutinho; Tortola, 2020; Viseu, 2015) used mathematical modeling[21] as a teaching methodology. Oliveira, Fernandes, and Fermé (2007) focused on the use of technologies in Mathematics classes, investigating the contribution of robots to the learning of Mathematics.

Bezerra *et al*. (2022) carried out a mathematical modeling activity called *Cooking with Mathematics* and had as its theme the production of a sweet. The ingredients and utensils to make the recipe[22] were used as artifacts. Regarding the research results, the authors consider that some aspects of proportional reasoning (Lamon, 2012), such as measurement, were mobilized more spontaneously because the activity is part of the daily lives of some students, involving informal measurement strategies. According to the authors, considering situations that may be present in the daily lives of students is inherent to mathematical modeling, a fact that contributed to the evidence of aspects of proportional reasoning.

The authors also state that not all aspects of proportional reasoning (Lamon, 2012) were present in the development of the mathematical modeling activity, and emphasize that their mobilization could have been required by the teachers through questions. Thus, the research indicates possibilities and the importance of the teacher in mediating the student-content-artifacts interaction.

Coutinho and Tortola (2020) carried out a mathematical modeling activity called *Swinging or balancing on the seesaw*?, and the students investigated the factors that determine their movement and balance. The artifacts used were seesaws and the students' and teachers' own bodies or packages containing 1 kg of beans and 5 kg of rice. Regarding the results of the research, the authors indicate that five of the seven aspects of proportional reasoning (Lamon,

[21] Viseu (2015) uses modeling tasks, but here we are including them in the same category of mathematical modeling, used by Bezerra *et al.* (2022), Coutinho and Tortola (2020).
[22] It is worth mentioning that the research was developed in a remote context, that is, the recipe was produced together with the students' families, in their homes.

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

2012) were mobilized by the students during the activity: quantities and covariation, relative reasoning, sharing and comparison, unitization, and measurement.

Just like Bezerra *et al.* (2022), Coutinho and Tortola (2020) indicate that aspects of proportional reasoning that were not highlighted can be stimulated by the teacher so that students can mobilize them.

Viseu (2015) proposed seven modeling tasks to students, two of which were modeled by the direct proportionality function, three by the inverse proportionality function, one by the quadratic function, and one by the cubic function. The author only analyzes three of the tasks. A graphing calculator and sensors were used as artifacts. Regarding the results of the research, the author suggests that the use of artifacts was essential for the students to solve the proposed tasks, not only to perform calculations but also to collect data, organize them in statistical lists, consult these lists, create other lists, and construct the graph that modeled the situation under study.

Despite importance of the graphing calculator and sensors, the author noticed some difficulties that the students had when dealing with the tasks. The students did not always distinguish between direct and inverse proportionality, they expressed confusion in the designation of the type of proportionality and were only able to distinguish through the graphical representation of the function to decide which of the two hypotheses was the most correct.

Oliveira, Fernandes, and Fermé (2007) engaged students in five tasks, focusing on direct proportionality as a function. Tape measures, cardboard, boards, and robots were used as artifacts. Regarding the results of the research, the authors indicate that the robots helped students to attribute a new meaning to the concept of direct proportionality, previously seen as depending solely and exclusively on the quotients between the quantities. Furthermore, they conclude that the robots allowed students to associate the constant of proportionality with a real and observable characteristic, in this case, the robots speed.

The four articles in this category evidenced STCA interactions during the data analysis. The teacher appears explicitly in the dialogues analyzed during the research, interacting with the students and leading the discussions in the classroom (a fact that did not occur in the articles classified as CS and SCA). As we stated previously, perhaps due to the teaching approaches (mathematical modeling and use of technologies in Mathematics classes) adopted in the four studies, the role of the teacher was evident throughout the texts. Such evidence, combined with the fact that the research also involved the use of the mentioned artifacts, allowed these four articles to be included in the STCA category.

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

Finally, it is worth highlighting that Oliveira, Fernandes, and Fermé (2007), in their general considerations of the results obtained, weave reflections that are close to the tetrahedral model (STCA) considered in this research. As shown in Figure 3, the authors highlight interactions between Instruments (robots), Object (proportionality), and Subject (students). The teacher is present both in the Instruments, through the way he questioned the students, and in the Community (classroom). Other interactions are established, such as the Rules (of school Mathematics) and the Division of Labor (horizontal division of tasks among the different members of the community, and vertical division of power and statutes).

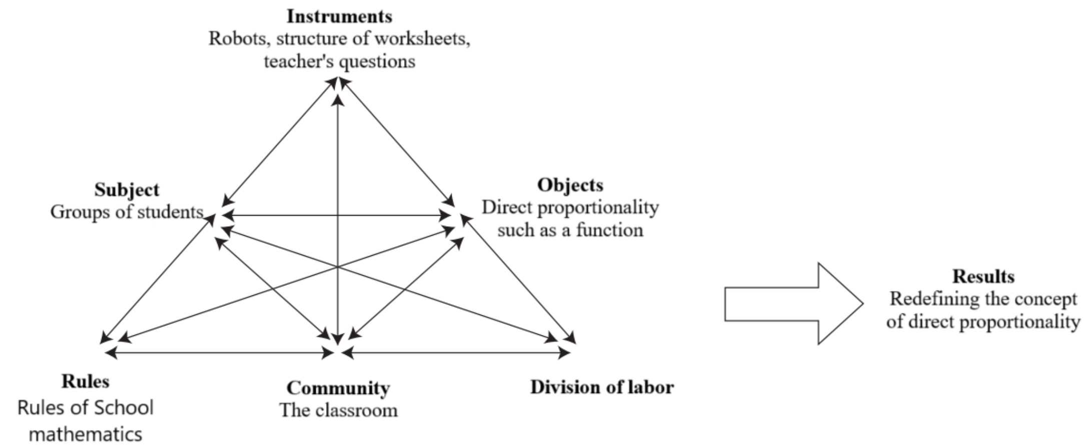

Figure 3 - Structure of the mathematical activity of the class analyzed by Oliveira, Fernandes, and Fermé (2007)
Source: Oliveira, Fernandes and Fermé (2007, p. 102)

As a result of these interactions, Figure 3 indicates the redefinition of the concept of direct proportionality by students.

5 Conclusions and Final Considerations

We return to our research questions to carry out stage 7 of the SLR, that is, to disseminate the conclusions of the review: *what research about proportionality is found in mathematics education? What interactions emerge from the analyses carried out according to the tetrahedral model of the didactic situation?*

Regarding what the research in Mathematics Education on proportionality is like, in stage 1 of the analysis, we note that, in the theoretical foundations of the articles, there is a predominance of the works of Lamon (2012) and Lesh, Post, and Behr (1988) or works of Vergnaud, especially when the authors of the researches are Brazilian. If we consider, for example, Burgos, Godino, and Rivas (2019), Parameswari *et al*. (2023), and Oliveira,

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

Fernandes, and Fermé (2007), articles whose authors are not Brazilian, we perceive a variation in relation to the perspectives adopted. These articles do not make use of Lamon (2012) and Lesh, Post, and Behr (1988) or the work of Vergnaud, which suggests that research carried out outside Brazil can offer us new perspectives on the teaching and learning of proportionality.

In addition to the theoretical references, we also noted that there is research involving students from Early Childhood Education, through Elementary School, to High School, and most of it is in the context of the final years of Elementary School. There are studies involving the initial and continuing education of teachers who teach Mathematics, as well as theoretical research or analysis of documents and textbooks. Therefore, we found that there is a diversity of contexts in which research on proportionality is being conducted.

Based on the diversity of contexts and theoretical frameworks used, in stage 2 of the analyses, we concluded that the interactions that emerged based on the tetrahedral model were: SC, STC, SCA, and STCA. The SC interaction was the one that appeared in the greatest quantity and the focus of these interactions concerns the way in which students solve a problem, a task, or a test. In these cases, the research often seeks to understand which strategy was used to solve the problem or identify the difficulties expressed by the students. In the works identified in this category, the interest was centered on the direct interaction between SC, without the intermediation of the teacher, and artifacts different from those traditional in the classroom (tasks, pencils, notebooks).

In the STC interaction, we did not notice a specific focus. When considering the teacher in the interactions, the studies diversify their perspectives. They pay attention to the different ways of doing mathematics in a classroom, to the teacher's strategies, and to aspects of the students' thinking.

In the SCA interaction, the focus is on the potential that the artifacts (Virtual Geoplano software and food labels) have to provide students with opportunities for experimentation that lead them to formalize the mathematical concept.

We noticed in the STCA interaction that the attention is not only focused on the artifact used but on the classroom environment in a broader way. The teaching approaches (mathematical modeling and use of technologies) enhanced the STCA interaction, since they allowed students to carry out mathematical experiments using artifacts and, evidently, mediated by the teacher.

We highlight that the work of Oliveira, Fernandes, and Fermé (2007) was the only one that explicitly considered the importance of STCA interaction in a classroom environment as a way of promoting proportionality understanding (Figure 3). It is worth noting that this article

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

was one of two articles found in the only non-Brazilian journal (*Quadrante*) included in our survey. In other words, the only study that more explicitly addresses STCA interaction is not published in Brazil. Therefore, we suggest that new research on the potential of this STCA interaction in the teaching of proportionality and the development of proportional reasoning be carried out in the Brazilian context.

Thus, based on the SLR presented in this article, a questions for a research agenda arises: what theoretical approaches underlie international research on proportionality? How does international research address STCA interaction in the teaching of proportionality? What contributions can this international research bring to Brazilian research and, mainly, to the teaching of proportionality (and the development of proportional reasoning) in Brazilian schools? What mathematical tasks can be planned for teaching proportionality to explore STCA interaction in the classroom? What formative tasks can be planned to work on in the (initial and continuing) training of teachers who teach Mathematics, in the Brazilian context, with a view to promoting STCA interaction in the classroom?

Finally, we highlight that a limitation of this research concerns the choices made to restrict what we are considering as Artifacts. Other research can complement this one, in the sense of including, for example, mathematical tasks as Artifacts and investigating STCA interactions in the classroom.

Acknowledgments

We would like to thank the National Council for Scientific and Technological Development – CNPq for the financial support granted (postdoctoral scholarship abroad (PDE) to the first author) through Public Call MCTI/CNPq No. 14/2023 - Support for International Scientific, Technological and Innovation Research Projects, process 441920/2023-3.

Authorship contributions

All authors contributed substantially to the conception and planning of the study; to the collection, analysis, and interpretation of data; to the writing and critical review; and approval of the final version to be published.

Referências

ALPHA, O.; ALMOULOUD S. A. Das proporções à proporcionalidade: o impacto crucial ou

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

hegemonia da regra de três. **Educação Matemática Pesquisa**, São Paulo, v. 23, n. 1, p. 769-809, 2021.

ANDREATTA, C.; ALLEVATO, N. S. G.; PINTO, A. H. Ensino e aprendizagem de matemática através de situações-problema em uma escola comunitária rural. **Revista de Educação Matemática**, São Paulo, v. 15, n. 19, p. 373-384, 2018.

ARAGÃO, A. B. B. L.; LAUTERT, S. L.; SCHLIEMANN, A. D. Solving Double and Multiple Proportion Problems in the Final Years of Elementary School. **Acta Scientiae**, Canoas, v. 24 n. 4, p. 183-206, 2022.

BARROS NUNES, C.; SANTOS COSTA, M. O Raciocínio Proporcional e a Resolução de Problemas na Formação Inicial de (futuros) Professores de Matemática. **REMATEC**, Belém, v. 11, n. 21, p. 47-63, 2016.

BEZERRA, M. DO C. C.; MARTINS, N.; BORSSOI, A. H.; SILVA, K. A. P. DA. Modelagem Matemática nos Anos Iniciais no Contexto Remoto: uma análise do Raciocínio Proporcional. **Perspectivas da Educação Matemática**, Campo Grande, v. 15, n. 37, p. 1-22, 27 abr. 2022.

BRASIL. **Base Nacional Comum Curricular**. Brasília: Ministério da Educação, 2018.

BURGOS, M.; GODINO, J. D. Prospective Primary School Teachers' Competence for the Cognitive Analysis of Students' Solutions to Proportionality Tasks. **Journal für Mathematik-Didaktik**, Hamburg, v. 43, [s.n.], p. 347-376, 2022.

BURGOS, M.; GODINO, J. D.; RIVAS, G. M. Epistemic and Cognitive Analysis of Proportionality Tasks from the Algebraization Levels Perspective, **Acta Scientiae**, Canoas, v. 21, n. 4, p. 63-81, 2019.

CABRAL, N. F.; DIAS, G. N.; LOBATO JUNIOR, J. M. D. S. O ensino de razão e proporção por meio de atividades. **Ensino Da Matemática Em Debate**, São Paulo, v. 6, n. 3, p. 174-206, 2019.

CÂNDIDO, A. S.; GARCIA SILVA, A. F. Relação Entre Conhecimento e Olhar Profissional para o Ensino de Situações Envolvendo o Raciocínio Proporcional na Formação Inicial. **Jornal Internacional de Estudos Em Educação Matemática**, São Paulo, v. 10, n. 3, p. 208-219, 2018.

CASTRO, J. B.; CASTRO FILHO, J. A. Desempenho de estudantes do 5° ano na construção de gráficos de setores: dificuldades e possibilidades pedagógicas. **Revista de Ensino de Ciências e Matemática**, São Paulo, v. 9, n. 2, p. 12-31, 2018.

CAVADAS, B.; SÁ-PINTO, X. Conceções de Estudantes Portugueses em Formação Inicial de Professores sobre a Evolução e a Origem da Vida. **Revista Brasileira De Pesquisa Em Educação Em Ciências**, v. 20, p. 1339–1362, 2021.

CEBOLA, G. M.; BROCARDO, J. Modelo de flexibilidade na evolução conceptual da comparação multiplicativa. **Acta Scientiae**, Canoas, v. 23, n. 6, p. 147-178, 2021.

CINTRA, E. P.; MARQUES JUNIOR, A. C.; SOUSA, E. C. Correlação entre a matriz de referência e os itens envolvendo conceitos de Química presentes no ENEM de 2009 a 2013. **Ciênc. Educ**., Bauru, v. 22, n. 3, p. 707-725, 2016.

COHEN, D. K.; RAUDENBUSH, S. W.; BALL, D. L. Resources, instruction, and research. **Educational Evaluation and Policy Analysis**, London, v. 25, n. 2, p. 119-142, 2000.

CORREIA, P. R. M.; NARDI, A. O que revelam os mapas conceituais dos meus alunos? Avaliando o

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

conhecimento declarativo sobre a evolução do universo. **Ciênc. Educ**., Bauru, v. 25, n. 3, p. 685-704, 2019.

COSTA, M. S.; MARTINS, A. C. DE J.; PAGANI, E. M. L.; ALLEVATO, N. S. G. Resolução de problemas: estratégia para ensinar, aprender e avaliar o conteúdo proporcionalidade na educação de jovens e adultos. **Revista Paranaense de Educação Matemática**, Campo Mourão, v. 12, n. 27, p. 76-99, 2023.

COUTINHO, L.; TORTOLA, E. Raciocínio proporcional em uma atividade de modelagem matemática por alunos da educação infantil. **Vidya**, Santa Maria, v. 40, n. 2, p. 65-85, 2020.

CYRINO, M. C. C. T. Mathematics Teachers' Professional Identity Development in Communities of Practice: Reifications of Proportional Reasoning Teaching. **Bolema**, Rio Claro, v. 30, n. 54, p. 165-187, abr. 2016.

ESPINDOLA, E. B. de M.; MOURA, I. F. M. Resolução de situações-problemas do campo conceitual multiplicativo: o cálculo relacional e o numérico. **Revemat**, Florianópolis, v.12, n. 2, p. 82-100, 2018.

FARIA, R. W. S. C.; MALTEMPI, M. V. Intradisciplinaridade Matemática com GeoGebra na Matemática Escolar. **Bolema**, Rio Claro, v. 33, n. 63, p. 348-367, abr. 2019.

FERNANDES, J. A. Aplicação da proporcionalidade direta à resolução de uma situação do quotidiano por futuros professores dos primeiros. **Educação Matemática Pesquisa**, São Paulo, v. 23, n. 2, p.76-98, 2021.

FERNANDES, J. A. Desempenho de Futuros Professores numa tarefa de Proporcionalidade Envolvendo Quantidades de uma Grandeza. **Perspectivas da Educação Matemática**, Campo Grande, v. 15, n. 37, p. 1-20, abr. 2022.

FIOREZE, L. A.; BARONE, D.; BASSO, M.; ISAIA, S. Análise da construção dos conceitos de proporcionalidade com a utilização do software geoplano virtual. **Ciênc. educ**., Bauru, v. 19, n. 2, p. 267-278, 2013.

GARCIA, T. M. R.; CYRINO, M. C. C. T. Identidade profissional de professores que ensinam matemática em uma comunidade de prática. **Revista Paranaense de Educação Matemática**, Campo Mourão, v. 8, n. 15, p. 33-61, 2020.

GARCIA SILVA, A. D. F.; CÂNDIDO, A. S.; PIETROPAOLO, R. C. Formação Inicial em Pedagogia: um estudo sobre conhecimentos relativos à proporcionalidade. **Revista Sergipana de Matemática e Educação Matemática**, Aracaju, v. 3, n. 2, p. 24-51, 2018.

GARCIA SILVA, A. F.; CÂNDIDO, A. S.; SOUZA, V. H. G. Raciocínio proporcional: um estudo sobre as estratégias de estudantes de pedagogia ao resolverem diferentes situações. **Acta Scientiae**, Canoas, v. 20, n. 1, p. 20-35, 2018.

GARCIA SILVA, A. F.; LOPES, S. C.; GALVÃO, M. E. E. L. Conhecimentos Profissionais e Reflexões de uma Professora ao Ensinar uma situação envolvendo a ideia de Proporcionalidade. **Acta Scientiae**, Canoas, v. 22, n. 5, p.143-164, 2020.

GROSSNICKLE, F. E.; JUNGE, C.; METZNER, W. Instructional Materials for Teaching Arithmetic. *In:* HENRY, N. B. (ed.). **The Teaching of Arithmetic Fiftieth Yearbook of the National Society for the Study of Education, pt. 2**. Chicago: University of Chicago Press, 1951. p. 155-185.

HERBST, P.; CHAZAN, D. On the instructional triangle and sources of justification for actions in mathematics teaching. **ZDM Mathematics Education**, Berlim, v. 44, n. 5, p. 601-612, 2012.

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

KAISER, G.; SCHUKAJLOW, S. Literature reviews in mathematics education and their significance to the field. **ZDM – Mathematics Education**, Berlim, v. 56, n. 1, p. 1-3, 2024.

LAMON, S. **Teaching fractions and ratios for understanding**: essential content knowledge and instructional strategies for teachers. 3thed. New York: Routledge, 2012.

LANGRALL, C. W.; SWAFFORD, J. Three balloons for two dollars: Developing proportional reasoning. **Mathematics Teaching in the Middle School**, Reston, v. 6, n. 4, p. 254-261, 2000.

LESH, R.; POST, T. R.; BEHR, M. Proportional Reasoning. *In:* HIEBERT, J.; BEHR, M. (eds.). **Number Concepts and Operations in the Middle Grades**. Reston: Lawrence Erlbaum & NCTM. 1988. p. 93-118.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS -NCTM. **Curriculum and Evaluation Standards for School Mathematics**. Reston: NCTM, 1989.

MARANHÃO, C.; MACHADO, S. Uma meta-análise de pesquisas sobre o pensamento proporcional. **Educar em Revista**, Curitiba, v. 27, n. especial, p. 141-156, 2011.

MELARA, J. F. T.; SILVA, A. F. G. Pensamento Proporcional e a Formação de professores: um Estudo Sobre Teses Produzidas de 2000 a 2019. **Jornal Internacional de Estudos em Educação Matemática**, São Paulo, v. 14, n. 3, p. 320-328, 2021.

MENDUNI-BORTOLOTI, R. D'A., BARBOSA, J. C. A construção de uma Matemática para o ensino do conceito de proporcionalidade direta a partir de uma revisão sistemática de literatura. **Bolema**, Rio Claro, v. 31, n. 59, p. 947-967, 2017.

MILAGRE, P. H.; SANTANA, E. R. DOS S. Situações de proporção simples: uma análise dos enunciados elaborados por professoras em um processo formativo. **Perspectivas da Educação Matemática**, Campo Grande, v. 11, n. 27, p. 933-954, 2018.

MIRANDA, C. DE A.; REZENDE, V.; NOGUEIRA, C. M. I. Uma Análise de Problemas de Função Afim Fundamentada na Teoria dos Campos Conceituais. **Jornal Internacional De Estudos Em Educação Matemática**, São Paulo, v. 14, n. 4, p. 485-495, 2021.

NERES, R. L.; ALMEIDA, J. M. P. A importância do ensino e aprendizagem de proporcionalidade sob a perspectiva das representações semióticas. **Ensino da Matemática em Debate**, São Paulo, v. 8, n. 1, p. 1-18, 2021.

OLIVEIRA, M. E.; CLARETO, S. M. É assim que eu faço! Proporcionalidade e Invenção em uma Aula de Matemática. **Bolema**, Rio Claro, v. 34, n. 68, p. 934-951, 2020.

OLIVEIRA, R.; FERNANDES, E.; FERMÉ, E. Proporcionalidade directa como função: da perfeição à realidade a bordo de um *robot*. **Quadrante**, Lisboa, v. 16, n. 1, p. 81-110, 2007.

OLIVEIRA, P. P. S.; SOUZA, A. C. Um estudo sobre a participação de alunos do 6º ano do Ensino Fundamental em uma pesquisa estatística. **Educ. Matem. Pesq.**, São Paulo, v. 20, n.1, p.224-247, 2018.

PARAMESWARI, P.; PURWANTO; SUDIRMAN; SUSISWO. Correct-Incorrect Proportional Reasoning Strategies on the Proportional Problems and SOLO Taxonomy. **Acta Sci.**, Canoas, v. 25, n. 5, p. 86-117, 2023.

PERON, L. D. C.; NOGUEIRA, C. M. I.; REZENDE, V. Análise de problemas do campo conceitual

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

multiplicativo presentes em livros didáticos de 5º ano ofertados pelo PNLD. **Ensino da Matemática em Debate**, São Paulo, v. 6, n. 3, p. 89-119, 2019.

PETTICREW, M.; ROBERTS, H. **Systematic reviews in the social sciences**: A practical guide. Oxford: Blackwell Publishing, 2006.

PONTES, M., GUIMARÃES, G. L. Construção de gráficos de barras a partir de diferentes recursos: lápis e papel e software excel. **Revista Sergipana de Matemática e Educação Matemática**, Aracaju, v. 6, n. 1, p. 43-64, 2021.

PONTE, J. P.; SILVESTRE, A. I.; GARCIA, C.; COSTA, S. **O desenvolvimento do conceito de proporcionalidade directa pela exploração de regularidades** Tarefas para o 1° e 2 ° ciclos do Ensino Básico: Materiais de apoio ao professor. Lisboa, 2010.

QUITEMBO, A. D. J.; DOMINGAS, A. A proporcionalidade na 9ª classe do 1º ciclo do ensino secundário. Uma análise sobre as competências desenvolvidas. **Revemat**, Florianópolis, v. 15, n. 2, p. 01-21, 2020.

RAMOS, R. DE C. DE S. S.; SILVA, J. A.; LUZ, V. S.; FIRME, S. M.; SARAIVA, D. R. Situações de Expressões Numéricas em Livros Didáticos de 6° ano: uma análise segundo a Teoria dos Campos Conceituais. **Bolema**, Rio Claro, v. 35, n. 71, p. 1294-1315, set. 2021.

REZAT, S.; STRÄßER, R. From the didactical triangle to the socio-didactical tetrahedron: artifacts as fundamental constituents of the didactical situation. **ZDM**, Berlim, v. 44, n. 5, p. 641-651, 2012.

RODRIGUES, C. L. B. H.; REZENDE, V. Problemas mistos em livros didáticos: uma classificação com base na teoria dos campos conceituais. **Amazônia:** Revista de Educação em Ciências e Matemáticas, Belém, v. 17, n. 39, p. 271-287, 2021.

RODRIGUES, C. L. B.; REZENDE, V. Conhecimentos associados ao conceito de função manifestados por estudantes dos anos iniciais ao resolverem situações mistas. **Quadrante**, Lisboa, v. 32, n. 1, p. 26-49, 2023.

SAVIOLI, A. M. P. das D.; SILVA, E. P. Aspectos da linguagem e do pensamento algébrico manifestados por estudantes do 6º ano em um episódio de ensino. **Vidya**, Santa Maria, v. 35, n. 1, 2015.

SCHELLER, M.; BONOTTO, D. de L. Estratégias de resolução de situação problema de modelagem matemática e o pensamento proporcional: um estudo com estudantes de pedagogia. **Amazônia:** Revista de Educação em Ciências e Matemáticas, Belém, v. 16, n. 36, p. 197-223, 2020.

SILVA, N. R.; PESSOA, C.; CARVALHO, A. B. G. Quem é Esse Pokémon? Uma Análise do Jogo de Cartas Pokémon à Luz da Teoria dos Campos Conceituais. **Perspectivas da Educação Matemática**, Campo Grande, v. 13, n. 32, p. 1-21, 2020.

SILVA, T. M.; PALANCH, W. B. de L. O raciocínio proporcional nas atividades de proporcionalidade do sétimo ano no Caderno da Cidade Saberes e Aprendizagens. **Revista de Ensino de Ciências e Matemática**, São Paulo, v. 12, n. 1, p. 1-22, 2021.

SILVESTRE, A. I., PONTE, J. P. Raciocínio proporcional: uma perspetiva atual. **Educação e Matemática**: Revista da APM, Lisboa, [s.v.], n. 123. p. 17-20, 2013.

SIQUEIRA, J. B.; GAERTNER, R. Ilhas Interdisciplinares de Racionalidade: conceito de proporcionalidade na compreensão de informações contidas em rótulos alimentícios. **R. Bras. de Ensino de C&T**, Curitiba, v. 8, n. 2, p. 160-175, 2015.

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v39a2402011

SOUZA, V. H. G.; GALVÃO, M. E. E. L.; POGGIO, A. M. P. P. O Conceito de Proporcionalidade Direta de Alunos Brasileiros de 14-16 Anos na Perspectiva dos Três Mundos da Matemática. **Jornal Internacional de Estudos em Educação Matemática**, São Paulo, v. 9, n. 1, p. 30-64, 2016.

STYLIANIDES, G. J.; STYLIANIDES, A. J.; MOUTSIOS-RENTZOS, A. Proof and proving in school and university mathematics education research: a systematic review. **ZDM – Mathematics Education**, Berlim, v. 56, n. 1, p. 47-59, 2024.

TEIXEIRA, P. J. M. Análise da produção de alunos do 7º ano do Ensino Fundamental, veiculada em cartazes, acerca da presença ou não de proporcionalidade em situações cotidianas de preços de produtos. **Tangram,** Dourados, v. 3, n. 3, p. 110-137, 2020a.

TEIXEIRA, P. J. M. Mobilizando o Raciocínio Proporcional na resolução de problemas de percentual. **Revemat**, Florianópolis, v. 15, n. 2, p. 01-21, jan./dez., 2020b.

TIEPPO, S. M.; CAPPELIN, A.; ZANATTA, L. F.; NOGUEIRA, C. M. I.; REZENDE, V. Um panorama de situações do tipo misto em provas do Exame Nacional do Ensino Médio. **Ciência & Educação**, Bauru, v. 29, p. e23009, 2023. Disponível em: https://doi.org/10.1590/1516-731320230009. Acesso em: fev. 2025.

VERGNAUD, G. La théorie de champs conceptuels. **Recherches en Didactique de Mathématiques**, Grenoble, v. 10, n. 23, p. 133-170, 1990.

VERGNAUD, G. Quais questões a teoria dos campos conceituais busca responder? **Caminhos da Educação Matemática em Revista**, Aracaju, v. 9, n. 1, p. 1-28, 2019.

VIANA, O. A.; MIRANDA, J. A. O raciocínio proporcional e as estratégias de resolução de problemas de valor omisso e de comparação. **Revemat**, Florianópolis, v. 11, n. 1, p. 194-213, 2016.

VIANA, O. A.; MIRANDA, J. A. Problemas de comparação de razões: uma avaliação do raciocínio proporcional de alunos do sexto ano. **Revemat**, Florianópolis, v. 13 n. 1, p. 163182, 2018.

VIEIRA, M. S. L. M.; SANTOS, M. C. Proporcionalidade: um olhar a partir da TAD, **Educação Matemática Pesquisa**, São Paulo, v. 21, n. 5, p. 514-528, 2019.

VISEU, F. A atividade de alunos do 9.º ano com tarefas de modelação no estudo de funções. **Revemat**, Florianópolis, v.10, n. 1, p. 24-51, 2015.

Submetido em: 03 de Julho de 2024.
Aprovado em: 05 de Novembro de 2024.

Editor-chefe responsável: Prof. Dr. Marcus Vinicius Maltempi
Editor associado responsável: Profa. Dra. Celi Espasandin Lopes