TECHNICAL PAPER

NUMERICAL EVALUATION OF THE MODULUS OF LONGITUDINAL ELASTICITY IN STRUCTURAL ROUND TIMBER ELEMENTS OF THE *Eucalyptus GENUS*

ANDRÉ L. CHRISTOFORO[1], TÚLIO H. PANZERA[2], FABIANO B. BATISTA[3],
PAULO H. R. BORGES[4], FRANCISCO A. R. LAHR[5]

ABSTRACT: Currently, the standards that deal with the determination of the properties of rigidity and strength for structural round timber elements do not take in consideration in their calculations and mathematical models the influence of the existing irregularities in the geometry of these elements. This study has as objective to determine the effective value of the modulus of longitudinal elasticity for structural round timber pieces of the *Eucalyptus citriodora* genus by a technique of optimization allied to the Inverse Analysis Method, to the Finite Element Method and the Least Square Method.

KEYWORDS: round timber, Finite Element Method, Least Square Method.

AVALIAÇÃO NUMÉRICA DO MÓDULO DE ELASTICIDADE LONGITUDINAL EM PEÇAS ROLIÇAS ESTRUTURAIS DE MADEIRA DO GÊNERO *Eucalyptus*

RESUMO: Atualmente, os documentos normativos que tratam da determinação das propriedades de rigidez e resistência para elementos estruturais roliços de madeira, não levam em consideração em seus cálculos e modelos matemáticos a influência das irregularidades existentes na geometria dessas peças. Este trabalho tem como objetivo determinar o efetivo valor do módulo de elasticidade longitudinal para peças estruturais roliças de madeira do gênero *Eucalyptus citriodora*, por intermédio de uma técnica de otimização aliada ao Método da Análise Inversa, ao Método dos Elementos Finitos e ao Método dos Mínimos Quadrados.

PALAVRAS-CHAVE: madeira roliça, Método dos Elementos Finitos, Método dos Mínimos Quadrados.

INTRODUCTION

Due to the versatility and availability, for centuries the wood has been used as structural material. Over time, the need for new technologies led to the development of studies on this material, both in its original and processed form, providing increased knowledge about their physical and chemical properties, as well as their uses.

In countries with a tradition in the use of wooden structures, it is common to use mixed systems, both of solid wood and its derivatives. However, the demands associated with processing costs motivate research by finding solutions that combine high efficiency of wood as a structural element to a low production cost. An alternative to this problem is to use this material in its original rounded shape, due to the natural growth of the tree, as shown by PARTEL (1999) in a study that

---

[1] Dr., Departamento de Engenharia Mecânica, Universidade Federal de São João del-Rei (UFSJ), Praça Frei Orlando, 170, Centro, São João del-Rei - MG, alchristoforo@ufsj.edu.br.

[2] Dr., Departamento de Engenharia Mecânica, Universidade Federal de São João del-Rei (UFSJ), Praça Frei Orlando, 170, Centro, São João del-Rei - MG, tuliopanzera@ufsj.edu.br.

[3] Dr., Departamento de Engenharia Mecânica, Universidade Federal de São João del-Rei (UFSJ), Praça Frei Orlando, 170, Centro, São João del-Rei - MG, fabianchini@ufsj.edu.br.

[4] Dr., Departamento de Engenharia de Produção Civil, Centro Federal Tecnológico de Minas Gerais (CEFET-MG), Av. Amazonas, 7675, Nova Gameleira, Belo Horizonte - MG, pborges@civil.cefetmg.br.

[5] Dr., Departamento de Engenharia de Estruturas, Laboratório de Estruturas de Madeira, Escola de Engenharia de São Carlos, Universidade de São Paulo, Av. Trabalhador Sancarlense, 400, São Carlos - SP, frocco@sc.usp.br.

Recebido pelo Conselho Editorial em: 10-2-2011
Aprovado pelo Conselho Editorial em: 9-6-2011

consisted in an assessment of the main structural systems of housing, buildings, electrification towers, and bridges using round timber in Brazil and abroad.

The Brazilian Standard for the Design of Wood Structures, NBR 7190:1997, of the Brazilian Association of Technical Standards (ABNT), specifies the rounded elements from the base and top diameter, regardless of the used genus. This document also determines that the strength and stiffness properties should be obtained through bodies-of-proof tests with small dimensions and free of defects, even though the wood is not a homogeneous and isotropic material.

The verification of the differences in mechanical properties between bodies-of-proof and parts of structural dimensions is a topic to further investigation.

BATISTA et al. (2000) developed an experimental research comparing the modulus values obtained from bodies-of-proof free of defects and sawn pieces of structural dimensions. Of the three genera studied, two of them, Eucalyptus and Cambará, showed faithful results, which are not the same from to the Cupiúba wood, which presented values for the reduced models about 30% lower than the structural models.

MINÁ et al. (2004) evaluated the strength and stiffness between wood rounded poles from the *Eucalyptus citriodora* genus with bodies-of-proof. The results found for the elasticity modulus of the bending tests were higher for structural parts, being lower in compression tests.

CORSINI et al. (2004) used the classification of visual and mechanical bodies-of-proof free of defects and structural parts of the *Eucalyptus citriodora* genus. The wooden parts were visually classified based on the text of the review of NBR 7190:1997, and tested for compression, traction, shearing and bending. As the final part of the conclusion of the research, the authors draw attention to the need of standardization of testing of structural pieces of wood.

Countries with a tradition of using wood for various purposes have a wider range of normative documents when compared to countries that are still trying to settle in this scenario. We can say that the experience gained over years of use of the material amounted to the large number of works extensively developed by research laboratories corroborate for the vast amount of information found in technical standards.

Concerning the determination of the mechanical properties, specifications and standardization of experimental methods for wood structural dimensions some international normative documents can be cited, such as ANSI O5.1 (2002), ASTM D198 (1976), ASTM D1036 (1997), DIN EN 14251 (2004) and AS 2209 (1994). The modulus of elasticity according to these standards can be calculated from bending tests at four points or at the condition of cantilever embedded. It is worth mentioning that the equations for determining the modulus of elasticity for rounded elements consider the hypothesis that the piece is truncated cone, using one or two values of circumference to determine the moment of inertia.

In Brazil, the documents dealing with existing rounded structural elements are intended primarily to serve the market of poles, which are valid for at least 20 years without technical review.

The standard NBR 6231:1980 (Wood Poles: Bending Strength) prescribes the manner in which the test should be made of the bending strength of wood poles. One end of the element is embedded and in the other end a concentrated force is applied, producing a displacement. Using an equation where factors such as the applied force, the geometric characteristics, and the displacement measured are considered, we determine the modulus of elasticity of the piece.

The standard NBR 8456:1984 (Poles of Preserved Eucalyptus for Electricity Distribution Networks) establishes the conditions for the preparation and receipt of eucalyptus poles preserved under pressure, which would be deployed in aerial electricity distribution networks.

The standard NBR 8456:1984 (Poles of Preserved Eucalyptus for Electricity Distribution Networks) also standardize the poles of preserved eucalyptus to use them in aerial electricity distribution networks. These are specified: length of the pole, type, nominal resistance, maximum deflection, length and diameter of embedding, diameter at 20 cm of the top (minimum and maximum) and top and bottom perimeter.

The standard NBR 6122:1996 (Design and Implementation of Foundations) recommends the use of NBR 7190:1997 for the resistance calculation of wooden piles, the latter being limited to tests of resistance in bodies-of-proof with small dimensions and free of defects, even though it is convenient to use the structural part to determine its mechanical properties.

As the international normative documents, the national ones which deal with round timber are also based on the taper shape assumption, even though the existence of possible irregularities in the part geometry is known.

Regarding the application and study of properties of strength and stiffness for wood structural rounded pieces, it is possible to highlight the studies of RANTA-MAUNUS (2000), WOLF & MOSELEY (2000), ROSS et al. (2001), CALIL et al. (2004), PINTO NETO et al. (2004), SALES et al. (2004), LARSON et al. (2004), MINÁ (2005), MINÁ & DIAS (2008), ZANGIÁCOMO & LAHR (2008), CARREIRA & DIAS (2009), and SALES et al. (2010).

Thus, it is necessary to develop of research projects to determined, reliably, the properties of strength and stiffness of round structural pieces as fundamental subsidies for producers and engineers.

This study aims to present an alternative methodology of calculation for determining the modulus of longitudinal elasticity of round structural pieces of timber from *Eucalyptus grandis* genus, taking into account the possible influence of irregularities of form. The modulus of effective elasticity is here calculated by the use of one computer program, developed in the fundamentals of Finite Element Method coupled with Inverse Analysis Method and Method of Least Squares.

MATERIALS AND METHODS

The experiments were developed at the Laboratory of Wood and Wood Structures, of the Department of Structural Engineering of the Engineering School of São Carlos, São Paulo University.

To determine the modulus of longitudinal elasticity 24 pieces of round structural pieces of timber from *Eucalyptus citriodora* genus were used, with average length of 750 cm and diameter at breast height around 30 cm, both donated by IRPA company from São Carlos.

The round structural pieces of timber used in the determination of the modulus of longitudinal elasticity respect the relation $L/D_{eq} \geq 21$, validating the theory of Bernoulli beams.

The modulus of longitudinal elasticity in this work is evaluated according to two different mathematical models of calculation, both by making use of the structural scheme of at three-point bending.

For the first case (Figure 1), the elasticity modulus is calculated with the aid of Equation 1, ΔF being the load increase, E the modulus of longitudinal elasticity, L the effective length of the element, D_{eq} the equivalent diameter, and δ the linear displacement, measured below the point of force application.

$$E = \frac{3 \cdot \Delta F \cdot L^3}{4 \cdot \pi \cdot \delta \cdot D_{eq}{}^4} \tag{1}$$

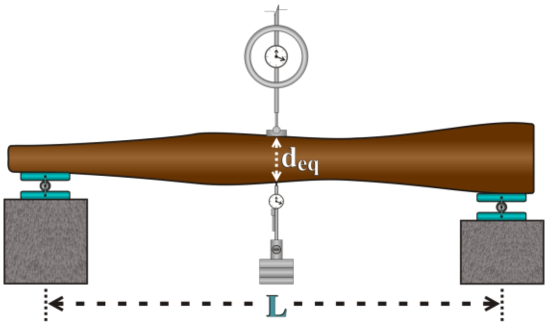

FIGURE 1. Bending test of three-point considering the equivalent diameter.

In Equation 1, it is worth emphasizing that the equivalent diameter (D_{eq}) is measured at the point of force application, obtained by measuring the circumference at the midpoint of the piece, assuming that the sections of the structural elements are perfectly circular, the diameters vary linearly in the length, and the maximum displacement occurs at the point of force application (short taper).

In the second case, as an alternative form of calculation, it is proposed that the value of the effective elasticity modulus (E_o) is calculated according to the structural scheme of test illustrated in Figure 2. Here it is titled of modulus of effective elasticity the values obtained from numerical simulation, due to more reliable results since the consideration of the irregularities of the shape of the associated parts with an optimization technique.

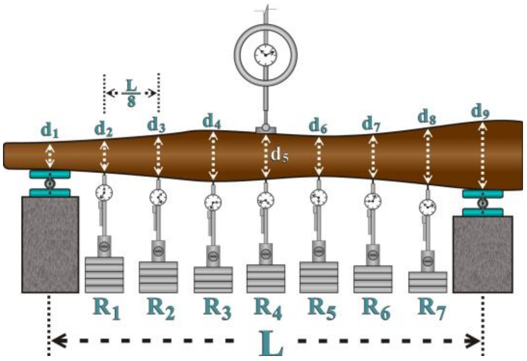

FIGURE 2. Alternative test for determination of the modulus of longitudinal elasticity.

In the determination of the modulus of effective elasticity, for each structural test performed were leased seven dial gauges along the parts, distant L/8 from each other, and nine values of circumference were also measured along its length, for the determination of their respective diameters.

Readings of the displacements on the clocks 1; 2; 3; 5; 6 and 7 (Figure 2) are performed when the displacement in the mid-span is approximately L/200, where L is the effective length of the piece, expressed in centimeters. This value ensures the linear-elastic behavior of the material and geometric linearity for the piece, since it is a measure for small displacements.

To calculate the modulus of effective elasticity, one computer program has been developed (Eotm) in the basis of Finite Element Method (MEF), according to the use of the second kinematic model of deformation of beams in the Bernoulli Principle of Virtual Work (PTV), disregarding in these calculations the forces per unit of volume (own weight).

The bar finite element has two degrees of freedom per node, two translations and two rotations, formulated with the use of an interpolated polynomial of third degree.

The approximate geometry for the part is considered piecewise linear. For each successive two circles (Figure 2) we use a finite element variation with truncated cone in its domain (Figure 3), and the moment of inertia is calculated by Equation 2.

$$I(x)=\frac{\pi}{4}\cdot\left(\frac{r_f-r_i}{h_e}\cdot x+r_i\right)^4 \qquad (2)$$

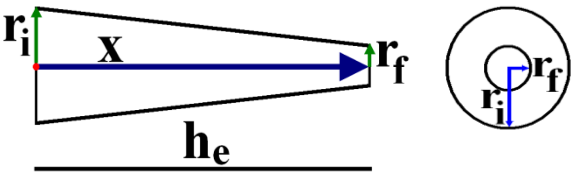

FIGURE 3. Variation on moment of inertia in the finite element.

From Equation 2, r_i and r_f are the calculated radii from successive measurements of their respective circles, h_e is the finite element length, and x is one real number between zero and h_e.

Using the Inverse Analysis Method, the values of displacements, as well as the values of the diameters measured along the element (depending on the shape of approximation of its geometry), are provided to the Eotm program, in order to calculate the optimal value of the modulus of elasticity of the piece. It is noteworthy that the inverse problem in this work consists in determining the modulus of longitudinal elasticity of the piece from one set of displacements measured experimentally, since the direct problem consists in using this variable in determining the allowable displacements for a structure or one structural element.

The Eotm, based on the basis of the MEF, determines one (numeric) displacement vector, accounting to the influence of irregularities on the piece geometry, considering the modulus of elasticity of the structural element the dependent variable.

With possession of the displacement vector determined by the program, and the experimental displacement vector, a function is built, based on the Least Square Method (Equation 3), whose objective is to determine the value of the modulus of elasticity so that the waste generated by both solutions, numerical and experimental, is minimal.

The modulus of elasticity is obtained by employing the Newton Raphson Method in Equation 3, where $U^{(e)}$ is the experimental displacement vector, and $U^{(n)}$ the vector of numeric displacement.

$$f(E)=\frac{1}{2}\sum_{i=1}^{n}\left(U_i^{(e)}-U_i^{(n)}\right)^2 \qquad (3)$$

In order to verify the differences between the values of moduli of elasticity calculated using the simplified model presented by Equation 1 (E_{Deq}) and the proposed alternative method of calculation (E_o - effective or excellent), the statistical test of the analysis of the confidence interval from the difference between two means was used, expressed by Equation 4, where μ is the population mean of differences, $\bar{x}_m$ the sample arithmetic mean of the differences, n the sample size, S_m the sample standard deviation of the differences, and $t_{\alpha/2,n-1}$ the tabulated value by the distribution "t" of Student, with n-1 degrees of freedom and significance level α.

$$\bar{x}_m-t_{\frac{\alpha}{2},n-1}\frac{S_m}{\sqrt{n}}\le\mu\le\bar{x}_m+t_{\frac{\alpha}{2},n-1}\frac{S_m}{\sqrt{n}} \qquad (4)$$

For the assumed conditions, the table t parameter is equal $t_{2,5\%;47} = 2,02$, with 95% reliability, and a degree of freedom equals 23.

RESULTS AND DISCUSSION

The values of E_{Deq} and E_o obtained for round structural pieces of timber from *Eucalyptus citriodora* genus are presented in Table 1.

TABLE 1. Values of the modulus of longitudinal elasticity for round timbers.

Pieces	E_{Deq} (kN cm^{-2})	E_o (kN cm^{-2})	Pieces	E_{Deq} (kN cm^{-2})	E_o (kN cm^{-2})
1	2,043.23	2,137.72	13	2,192.81	2,278.77
2	2,198.63	2,116.41	14	1,939.94	2,043
3	1,979.92	2,103.67	15	1,925.08	1,855.84
4	2,001.44	2,076.37	16	1,682.23	1,732.76
5	1,785.78	1,924.44	17	1,620.92	1,553.49
6	1,945.39	1,827.27	18	1,912.17	2,013.03
7	1,832.07	1,926.81	19	1,934.49	2,089.35
8	2,112.61	2,065.89	20	1,559.86	1,528.54
9	2,120.71	2,037.22	21	1,659.53	1,781.28
10	1,685.53	1,467.47	22	1,889.68	1,674.39
11	1,550.62	1,597.58	23	1,651.23	1,482.96
12	2,022.73	2,165.91	24	1,646.81	1,821.34

The mean, standard deviation and coefficient of variation for E_{Deq} values are respectively equal 1871; 195 and 0.1.

The mean, standard deviation and coefficient of variation for the values of E_o are respectively equal 1889; 236 and 0.22.

The confidence interval between the values for E_{Deq} e E_o is $-32 \leq \mu \leq 68.99$ and, as the zero belongs to the interval, one can say these are statistically equivalent.

CONCLUSIONS

For the statistical analysis performed on the values of modulus of longitudinal elasticity for both methods of calculation used, simplified and numerical, it appears that the wood of the *Eucalyptus citriodora* genus tested, on average, have slightly conical geometry, allowing the modulus of longitudinal elasticity be determined by the direct application of simplified methodology.

The combined use of both forms of calculation presented here shows itself as an alternative methodology for the classification for the round element. If the results of the modulus of longitudinal elasticity for both calculation methods are statistically equivalent, this implies that the piece has little taper and regular geometry, otherwise the taper to be taken into consideration and the value of the modulus of elasticity of the piece must be obtained with the use of the numerical calculation strategy presented.

REFERENCES

AMERICAN NATIONAL STANDARDS INSTITUTE. *ANSI O5.1 – American national standard for wood poles.* Washington, 2002.

AMERICAN SOCIETY FOR TESTING AND MATERIALS. *ASTM D1036 – Standard test methods of static tests of wood poles.* Philadelphia, 1997.

AMERICAN SOCIETY FOR TESTING AND MATERIALS. *ASTM D198 – Static tests of timbers in structural sizes.* Philadelphia, 1976.

ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. *NBR 6122 – Projeto e execução de fundações*. Rio de Janeiro, 1996.

ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. *NBR 6231 – Postes de madeira – resistência à flexão*. Rio de Janeiro, 1980.

ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. *NBR 7190 – Projeto de estruturas de madeira*. Rio de Janeiro. 1997.

ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. *NBR 8456 – Postes de eucalipto preservado para redes de distribuição de energia elétrica*. Rio de Janeiro, 1984.

AUSTRALIAN STANDARD. AS 2209 – Timber – poles for over heads lines. Australia. 1994.

BATISTA, A.M.; ROSSI, N.; MASCIA, N.T.; FURLANI, J.E. Estudo da flexão estática: relação entre peças de madeira com dimensões estruturais e dimensões reduzidas. In: ENCONTRO BRASILEIRO EM MADEIRAS E EM ESTRUTURAS DE MADEIRA, 7., 2000. São Carlos.

CALIL JR., C.; ESPINOSA, M.M.; MINÁ, A.J.S.; DIAS, A A. Confiabilidade em elementos estruturais de madeira roliça. *Revista Madeira*, São Carlos, v.14, 2004. 1 CD-ROM.

CARREIRA, M.R.; DIAS, A.A. Avaliação da rigidez à flexão de toras de madeira por meio de vibração transversal. *Cadernos de Engenharia de Estruturas*, São Carlos, v.11, p.75-79, 2009. Disponível em: <http://caderno.set.usp.br/index.php/cee>. Acesso em: 8 ago. 2011.

COMITÉ EUROPÉEN DE NORMALISATION. DIN EN 14251 – *Structural round timbers – test methods*. Bruxelas, 2004.

CORSINI, T.A.; FONTE, T.F.; CALIL JR., C. Propriedades mecânicas de peças estruturais versus corpos-de-prova isentos de defeitos. In: ENCONTRO BRASILEIRO EM MADEIRAS E EM ESTRUTURAS DE MADEIRA, 9., 2004. Cuiabá. *Anais*...

LARSON, D.; MIRTH, R.; WOLFE, R. Evaluation of small-diameter ponderosa pine logs in bending. *Forest Products Journal*, Madison, v.54, p.52-58, dez. 2004.

MINÁ, A.J.S. Estudo de estacas de madeira para fundações de pontes de madeira. 2005. 117 f. Tese (Doutorado em Engenharia de Estruturas) - Escola de Engenharia de São Carlos, Universidade de São Paulo, São Carlos, 2005.

MINÁ, A.J.S.; DIAS, A.A. Estacas de madeira para fundações de pontes de madeira. *Cadernos de Engenharia de Estruturas*, v.10, p.129-155, 2008. Disponível em: <http://cadernos.set.eesc.usp.br/index.php/cee>. Acesso em: 5 set. 2011.

MINÁ, A.J.S.; DIAS,A.A.; CALIL JR., C. Avaliação da rigidez e da resistência de postes de madeira para uso como estacas para fundações. In: ENCONTRO BRASILEIRO EM MADEIRAS E EM ESTRUTURAS DE MADEIRA, 9., 2004. Cuiabá. *Anais*...

PARTEL, P.M.P. Sistemas estruturais e construtivos utilizando madeira roliça de reflorestamento. 1999. 134 f. Dissertação (Mestrado em Arquitetura e Urbanismo) - Escola de Engenharia de São Carlos, Universidade de São Paulo, São Carlos, 1999.

PINTO NETO, J.; CALIL JR., C.; ESPINOZA, M.M. Propriedades de resistência e rigidez de peças estruturais roliças de pequeno diâmetro. In: ENCONTRO BRASILEIRO EM MADEIRAS E EM ESTRUTURAS DE MADEIRA, 9., 2004. Cuiabá. *Anais*...

RANTA-MAUNUS, A. Bending and compression properties of small diameter round timber. In: WORLD CONFERENCE ON TIMBER ENGINEERING, 2000, Whistler.

ROSS, R.J.; WANG, X.; MATTSON, J.A.; ERICKSON, J.R.; FORSMAN, J.W.; GESKE, E.A.; WEHR, M.A. *Comparison of several nondestructive evaluation techniques for assessing stiffness and MOE of small-diameter logs*. Madison: U. S. Department of Agriculture, Forest Service, Forest Products Laboratory, 2001. (Research Paper).

SALES, A.; CANDIAN, M.; SALLES, V.C. Nondestructive evaluation of timber: the new Brazilian code for the design of timber structures. *Materials and Structures*, London, v.43, p.213-221, 2010.

SALES, A.; PELIZAN, T.R.; OLIVEIRA, F.G.R.; CANDIAN, M.; LUCCHETTE, F.F.; SALGON, J.L.; MILLER, K.P. Avaliação de propriedades mecânicas de peças roliças de Eucalipto por meio de ultra-som. In: ENCONTRO BRASILEIRO EM MADEIRAS E EM ESTRUTURAS DE MADEIRA, 9., 2004. Cuiabá. *Anais...*

WOLFE, R.; MOSELEY, C. Small-diameter log evaluation for value-added structural applications. *Forest Products Journal*, Madison, v.50. p.48-58, 2000.

ZANGIÁCOMO, A.L.; ROCCO LAHR, F.A. Avaliação do efeito de cisalhamento na flexão de elementos roliços da espécie Eucalyptus citriodora. In: ENCONTRO BRASILEIRO EM MADEIRAS E EM ESTRUTURAS DE MADEIRA, 11., 2008, Londrina.