## Scientific Paper

Doi: http://dx.doi.org/10.1590/1809-4430-Eng.Agric.v42n2e20210017/2022

# MAXIMIZATION OF PRODUCTION AND NET INCOME IN AGRO-FOREST SYSTEMS 

Vinicius T. do Nascimento ${ }^{1 *}$, Sergio D. Ventura ${ }^{1}$, Angel R. S. Delgado ${ }^{1}$, Washington S. da Silva ${ }^{1}$<br>${ }^{1 *}$ Corresponding author. Universidade Federal Rural do Rio de Janeiro - UFRRJ/ Seropédica - RJ, Brasil.<br>E-mail: vinicius.nascimento@ufrrj.br | ORCID ID: https://orcid.org/0000-0003-0070-4086

## KEYWORDS

radiation, water depth, nitrogen, logarithmic barrier.


#### Abstract

In this study, we present computational procedures to solve problems for the maximization of production and net income from crops or trees in a randomly generated agroforestry system with limited inputs, based on the Logarithmic Barrier Method. The results obtained showed numerical consistency for viability and optimality of both problems in the agroforestry scenarios tested, as well as promoted conditions to solve the problems with real data.


## INTRODUCTION

An agroforestry system (AFS) is a food production practice that aims to conserve and restore nature. This is possible because instead of removing original vegetation and single-crop farming on a large area, this form of production respects and imitates nature, using the relationships among living beings to its advantage and stimulating local biodiversity (Götsch, 1997, Götz et al., 2016). In agroforestry systems, the crops, trees, and animals are managed considering time and space and, to do so, the characteristics of each species used and its relationship with the others must be understood.

Undoubtedly, AFSs are a fusion between food production and environmental conservation, as these systems control soil erosion and recover degraded areas and those used for the production of food and other products. Moreover, AFSs generate economic benefits such as improved family income and reduced external input costs, besides having affordable implementation and maintenance costs.

Solar radiation, water, and nitrogen are important factors for evaluations of crop or tree yield responses in an AFS. Solar radiation is related to photosynthesis and is also responsible for other plant physiological mechanisms. In this sense, studies on interactions between this factor and crop physiology are relevant, especially on photosynthesis and light interception, thus determining the most effective photosynthetic radiation fraction for plant productivity gains.

One strategy to increase radiation-use efficiency by crops is implementing moderate water restrictions. Under such conditions, plants partially close their stomata to reduce water loss to the environment, while photosynthesis remains active but at lower rates (Confalone et al., 1997; Pereira, 2002; Plevich et al., 2019).

Regarding fertilization, AFSs make use of natural resources available and forest nutrient-cycling dynamics, which are supplied by tree pruning and green manuring. Notably, plants growing under dense vegetation have nitrogen concentrations parallel to radiation availability. In this sense, it is known that the greater the uniformity in leaf nitrogen concentrations, the greater the efficiency of its use in photosynthesis.

Considering that in an AFS the competition for sunlight, water, and nutrients (nitrogen fertilization) is high, and seeking to optimize production values, this article presents computational procedures to maximize food production and net income as a function of solar radiation, water, and nutrients (nitrogen) at upper and lower limits.

## MATERIAL AND METHODS

Considering an "ideal SAF", that is, a system that, before its installation, had been considered local physical characteristics such as relief, original vegetation, wind direction and intensity, soil type, solar radiation, water availability, available nutrients, and usage history, as well as the crop and tree species to be grown. In this context, native species should be prioritized to ensure subsistence

[^0]and food security for families, as well as commercial species with greater acceptance in the local markets. We highlight that the main goal of our work is to maximize production and net income generated by each species (crop or tree) in an "ideal SAF", as a function of solar radiation, water depth, and nitrogen dose.

To do so, we supposed that $y(r, w, n)$ analytically represents the production (or response) function of a given plant species $\left(\mathrm{kg} . \mathrm{m}^{-2}\right)$ according to solar radiation ( $r$; in \%), water depth ( $w ;$ in $m m$ ), and nitrogen dose ( $n$; in $\mathrm{kg} \cdot \mathrm{m}^{-2}$ ) in an "ideal AFS". Thus, the production function of this system was given as follows:

$$
y(r, w, n)=a r^{2}+b w^{2}+c n^{2}+d r w+e w n+f r+g w+h n+m
$$

Where: $a, b, c, d, e, f, g, h, i, m \in \mathbb{R} \quad$ (coefficients obtained by regression). Note that $\quad \nabla y(r, w, n)=\binom{\frac{\partial r}{\partial y(r, w, n)}}{\frac{\partial w}{\frac{\partial y(r, w, n)}{\partial n}}}=$
$\left(\begin{array}{c}2 a r+d w+f \\ 2 b w+d r+e n+g \\ 2 c n+e w+h\end{array}\right)$, and the Hessian matrix $H=\left(\begin{array}{ccc}2 a & d & 0 \\ d & 2 b & e \\ 0 & e & 2 c\end{array}\right)$ is asymmetric negative definite matrix if $a, b, c<0, d=$ $e=0$. In this sense, $y(r, w, n)$ is a strictly concave function. We assumed that problems related to the maximization of production and net income for a given plant species can be expressed mathematically as two nonlinear programming problems with the following linear constraints:

$$
\begin{array}{ll}
\operatorname{maximize} & y(r, w, n)=a r^{2}+b w^{2}+c n^{2}+f r+g w+h n+m \\
\text { subject to } & r_{\text {inf }} \leq r \leq r_{\text {sup }} \\
& w_{\text {inf }} \leq w \leq w_{\text {sup }}  \tag{1}\\
& n_{\text {inf }} \leq n \leq n_{\text {sup }}
\end{array}
$$

and
maximize

$$
\begin{align*}
& N I(r, w, n)=p_{c} y(r, w, n)-c_{w} w-c_{n} n-c_{0}  \tag{2}\\
& r_{\text {inf }} \leq r \leq r_{\text {sup }} \\
& w_{\text {inf }} \leq w \leq w_{\text {sup }} \\
& n_{\text {inf }} \leq n \leq n_{\text {sup }},
\end{align*}
$$

subject to

Where:
$0 \leq r_{\text {inf }}, r_{\text {sup }}, w_{\text {inf }}, w_{\text {sup }}, n_{\text {inf }}, n_{\text {sup }}$ - represent the lower and upper limits for the variables $r, w, n$, respectively; $N I(r, w, n)$ is the net income obtained $\left(R \$ . m^{-2}\right)$ as a function of $r, w, n ; p_{c}$ is the price of plant production $\left(R \$ . m^{-2}\right) ; c_{w}$ is the cost of water depth ( $R \$ . \mathrm{mm}^{-1} . h \mathrm{a}^{-1}$ ); $c_{n}$ is the cost of nitrogen dose ( $R \$ . \mathrm{kg}^{-1} \mathrm{ha}^{-1}$ ); and $c_{0}$ is the fixed cost of production $\left(R \$ . m^{-2}\right)$ that may encompass labour and/or machinery costs, etc. Note that problem (2) can be written as:

$$
\begin{array}{ll}
\operatorname{maximize} & N I(r, w, n)=a p_{c} r^{2}+b p_{c} w^{2}+c p_{c} n^{2}+f p_{c} r+\left(g p_{c}-c_{w}\right) w+\left(h p_{c} n-c_{n}\right) n+\left(p_{c} m-c_{0}\right) \\
\text { subject to } & r_{\text {inf }} \leq r \leq r_{\text {sup }} \\
& w_{\text {inf }} \leq w \leq w_{\text {sup }} \\
& n_{\text {inf }} \leq n \leq n_{\text {sup }} .
\end{array}
$$

Therefore, to maximize the production of a given plant species in an "ideal $A F S$ " with limited inputs (1), we developed a computational procedure based on the "logarithmic barrier" method (Bertsekas, 2016; Carvalho et al., 2009; Delgado et al., 2020). Conceptually, this procedure
works as follows: setting a parameter $\mu>0$ and incorporating the constraints that define the objective function using a logarithmic barrier function, then an unconstrained non-linear programming problem is solved as follows:

$$
\begin{equation*}
\operatorname{maximize} \quad \varphi_{\mu}(r, w, n) \tag{3}
\end{equation*}
$$

Where:

$$
\varphi_{\mu}(r, w, n)=y(r, w, n)+\mu B(r, w, n)
$$

and

$$
B(r, w, n)=\operatorname{Ln}\left(r_{\text {sup }}-r\right)+\operatorname{Ln}\left(r-r_{\text {inf }}\right)+\operatorname{Ln}\left(w_{\text {sup }}-w\right)+\operatorname{Ln}\left(w-w_{\text {inf }}\right)+\operatorname{Ln}\left(n_{\text {sup }}-n\right)+\operatorname{Ln}\left(n-n_{\text {inf }}\right) .
$$

Then, the parameter $\mu$ is decreased, and the process is repeated until a stop criterion is met. It is known as a logarithmic barrier because the logarithm function generates interior points away from the limits of a threedimensional constraint box. For each $\mu$, a maximum of $\varphi_{\mu}$ is reached at an interior point in the set of viable solutions to the problem, as $\varphi_{\mu}(r, w, n)$ is a strictly concave function and, when $\mu$ tends to zero, that point moves up to near the
optimal solution of (1). As a function of $\mu$, the set of optimal solutions for the unconstrained problems (3) defines a curve known as the central path (Drumond et al., 2015).

This method is important for maximizing $\varphi_{\mu}(r, w, n)$ for a fixed $\mu$. As $\varphi_{\mu}$ is strictly concave, an optimal solution of (3) is defined by the first-order condition $(r, w, n)=(r(\mu), w(\mu), n(\mu))$ if and only if:

$$
\begin{align*}
& \frac{\partial \varphi_{\mu}(r, w, n)}{\partial r}=2 a r+f-\frac{\mu}{r_{\text {sup }}-r}+\frac{\mu}{r-r_{\text {inf }}}=0  \tag{4}\\
& \frac{\partial \varphi_{\mu}(r, w, n)}{\partial w}=2 b w+g-\frac{\mu}{w_{\text {sup }}-w}+\frac{\mu}{w-w_{\text {inf }}}=0  \tag{5}\\
& \frac{\partial \varphi_{\mu}(r, w, n)}{\partial n}=2 c n+h-\frac{\mu}{n_{\text {sup }}-n}+\frac{\mu}{n-n_{\text {inf }}}=0 \tag{6}
\end{align*}
$$

By defining $\alpha_{\text {sup }}=\frac{\mu}{r_{\text {sup }}-r}, \alpha_{\text {inf }}=\frac{\mu}{r-r_{\text {inf }}}, \beta_{\text {sup }}=\frac{\mu}{w_{\text {sup }}-w}, \beta_{\text {inf }}=\frac{\mu}{w-w_{\text {inf }}}, \gamma_{\text {sup }}=\frac{\mu}{n_{\text {sup }}-n}, \gamma_{\text {inf }}=\frac{\mu}{n-n_{\text {inf }}}$, the system (4)(6) can be written as:

$$
\begin{align*}
& 2 a r+\alpha_{\text {inf }}-\alpha_{\text {sup }}=-f  \tag{7}\\
& 2 b w+\beta_{\text {inf }}-\beta_{\text {sup }}=-g  \tag{8}\\
& 2 c n+\gamma_{\text {inf }}-\gamma_{\text {sup }}=-h  \tag{9}\\
& \alpha_{\text {inf }}\left(r-r_{\text {inf }}\right)=\mu  \tag{10}\\
& \alpha_{\text {sup }}\left(r_{\text {sup }}-r\right)=\mu  \tag{11}\\
& \beta_{\text {inf }}\left(w-w_{\text {inf }}\right)=\mu  \tag{12}\\
& \beta_{\text {sup }}\left(w_{\text {sup }}-w\right)=\mu  \tag{13}\\
& \gamma_{\text {inf }}\left(n-n_{\text {inf }}\right)=\mu  \tag{14}\\
& \gamma_{\text {sup }}\left(n_{\text {sup }}-n\right)=\mu  \tag{15}\\
& r_{, w, n, \alpha_{\text {sup }}, \alpha_{\text {inf }}, \beta_{\text {sup }}, \beta_{\text {inf }}, \gamma_{\text {sup }}, \gamma_{\text {inf }}>0 \text {. }} \begin{array}{l} 
\\
2
\end{array} \\
& \\
& 2
\end{align*}
$$

The points that approximately solve equations (7)(15) are near the central path associated with productivity. Moreover, equations (7)-(8) represent the constraints that define the region of the viability of the corresponding dual problem, while equations (10)-(15) represent the conditions of "approximate complementary slackness".

Among the advantages of dual solutions is the potential provision of economic information about resources such as decision making regarding the acquisition of additional resources or sensitivity analysis. In this case, the variables $\alpha_{\text {sup }}, \alpha_{\text {inf }}, \beta_{\text {sup }}, \beta_{\text {inf }}, \gamma_{\text {sup }}, \gamma_{\text {inf }}$ represent the percentage of changes in production and net income as a function of variations in water volume and nitrogen dose limits.

Conceptually, the numerical procedure implemented to maximize production works as follows: given a parameter $\mu>0$ and a point close to $(r(\mu), w(\mu), n(\mu))$ for each iteration, we approximately solve the nonlinear system (7)-(15) using Newton's method (Fonseca, 2017). Then, the parameter $\mu$ is decreased, and the process is repeated until a predetermined stop condition is met. Likewise, a procedure can be implemented to maximize net income (2).

To test the above procedure computationally, we created an ideal random AFS using the MATLAB 7.4 platform. To do so, we generated four crop or tree production functions ( $i=1,2,3,4$ ), as follows:

$$
y_{i}(r, w, n)=a r^{2}+b w^{2}+c n^{2}+d r w+e w n+f r+g w+h n+m
$$

Where:
$a, b, c<0$, and $d=e=0$ (Table 1). Each one of these functions was maximized over three agroforestry scenarios, considering minimum solar radiation of $2 \%$, water depth range between 50 and 600 mm , and nitrogen dose from 0 to 300 kg . These lower and upper limits for $r, w, n$ were fixed based on the input management recommendations (solar radiation, water, and nitrogen) for agroforestry systems. Thus, the numerical experiments were carried out in 3 three-dimensional boxes: $[0.02,1] \times[50,500] \times$ $[0,100],[0.02,1] \times[150,400] \times[75,300]$ and $[0.02,1] \times[100,600] \times[75,200]$.

TABLE 1. Response or production functions in quadratic forms for the variables $r, w, n$.

| PLANT | PRODUCTION FUNCTION $\left(\boldsymbol{k g} \cdot \boldsymbol{h \boldsymbol { a } ^ { - \mathbf { 1 } } )}\right.$ |
| :---: | :---: |
| 1 | $y_{1}(r, w, n)=-9.47 \cdot 10^{5} r^{2}-4.52 \cdot 10^{-5} w^{2}-4.27 \cdot 10^{-5} n^{2}+4,943.42 r+0.0257349 w+0.0317876 n+0.1225433 r$ |
| 2 | $y_{2}(r, w, n)=-5.07 \cdot 10^{5} r^{2}-8.50 \cdot 10^{-5} w^{2}-5.94 \cdot 10^{-5} n^{2}+73,350.85 r+0.0335390 w+0.0157880 n+4.3457454$ |
| 3 | $y_{3}(r, w, n)=-6.99 \cdot 10^{5} r^{2}-8.15 \cdot 10^{-5} w^{2}-6.92 \cdot 10^{-5} n^{2}+32,280.66 r+0.0000010 w+0.0610159 n+0.1779493$ |
| 4 | $y_{4}(r, w, n)=-7.11 \cdot 10^{5} r^{2}-3.82 \cdot 10^{-5} w^{2}-7.07 \cdot 10^{-5} n^{2}+38,956.53 r+0.0131744 w+0.0432329 n+2.6488897$ |

Afterwards, net incomes associated with each response function in Table 1 were maximized, as follows: $N I_{i}(r, w, n)=p_{i} y_{i}(r, w, n)-c_{w} w-c_{n} n-c_{0}$, for each one of the previously described scenarios. In the numerical experiments, the values fixed for the parameters prices $\left(p_{i}\right)$, input costs $\left(c_{w}, c_{n}\right)$, and fixed costs ( $c_{0}$ ) were randomly determined within the following intervals: $p_{c} \in$ [23.02, 84.33], $c_{w} \in[0.0008,0.16], c_{\mathrm{n}} \in[16.85,34.90]$, and $c_{0} \in[2.800,4.060,10]$. The lower and upper limits of each interval respond approximately to the average values found in the literature for AFSs.

## RESULTS AND DISCUSSION

Tables 2, 3, and 4 show the optimal numerical results of productivity for each plant and scenario considered. Each
table shows the values corresponding to optimal solution $\left(r^{*}, w^{*}, n^{*}\right)$; optimal production $y_{i}\left(r^{*}, w^{*}, n^{*}\right)$, ( $i=1,2,3,4$ ); and iteration number by the implemented procedure. It is worth mentioning that all optimal solutions obtained satisfy each of the constraints imposed on the problem (1).

At first, one can see that all tables (2, 3, and 4) showed that solar radiation values are invariant for the three scenarios. Plants 1, 3, and 4 obtained the lowest solar radiation ( $2 \%$ ), while crop 3 had the highest (7\%). Regarding water depth ( $w$ ) in all scenarios, plant 1 obtained $284,67 \mathrm{~mm}$, just as plant $2(197.29 \mathrm{~mm})$ and 4 $(197.29 \mathrm{~mm})$. Yet for plant 3, optimal water depths were equal to the lower limit imposed in the first ( 50 mm ), second ( 150 mm ), and third ( 100 mm ) scenarios.

TABLE 2. Optimal solution $\left(r^{*}, w^{*}, n^{*}\right)$ and production $y\left(r^{*}, w^{*}, n^{*}\right)$ of $(1)$ in the three-dimensional box $[0.02,1] \times[50,500] \times$ [0,100].

| CROP | $r^{*}$ <br> $(\%)$ | $w^{*}$ <br> $(\mathrm{~mm})$ | $n^{*}$ <br> $(\mathrm{~kg})$ | $y_{i}\left(w^{*}, n^{*}\right)$ <br> $\left(\mathrm{kg} \cdot \mathrm{ha}^{-1}\right)$ | iterations |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.020000 | 284.677897 | 99.997970 | 127.737290 | 22 |
| 2 | 0.072338 | 197.289649 | 99.980918 | $2,661.670057$ | 18 |
| 3 | 0.023091 | 50.002803 | 99.999517 | 378.073711 | 24 |
| 4 | 0.027396 | 172.444253 | 99.997712 | 541.019584 | 21 |

TABLE 3. Optimal solution $\left(r^{*}, w^{*}, n^{*}\right)$ and production $y\left(r^{*}, w^{*}, n^{*}\right)$ of (1) in the three-dimensional box $[0.02,1] \times$ $[150,400] \times[75,300]$.

| CROP | $r^{*}$ <br> $(\%)$ | $w^{*}$ <br> $(\mathrm{~mm})$ | $n^{*}$ <br> $(\mathrm{~kg})$ | $y_{i}\left(w^{*}, n^{*}\right)$ <br> $(\mathrm{kg} \cdot \mathrm{ha})$ | iterations |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.020000 | 284.677198 | 249.993749 | 130.263644 | 22 |
| 2 | 0.072338 | 197.294137 | 132.900164 | $2,661.734410$ | 18 |
| 3 | 0.023091 | 150.001472 | 249.998638 | 381.963170 | 22 |
| 4 | 0.027396 | 172.463383 | 249.995135 | 543.792798 | 22 |

TABLE 4. Optimal solution $\left(r^{*}, w^{*}, n^{*}\right)$ and production $y\left(r^{*}, w^{*}, n^{*}\right)$ of (1) in the three-dimensional box $[0.02,1] \times$ $[100,600] \times[75,200]$.

| CROP | $r^{*}$ <br> $(\%)$ | $w^{*}$ <br> $(\mathrm{~mm})$ | $n^{*}$ <br> $(\mathrm{~kg})$ | $y_{i}\left(w^{*}, n^{*}\right)$ <br> $\left(\mathrm{kg} \cdot \mathrm{ha}^{-1}\right)$ | iterations |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.020000 | 284.679514 | 199.996114 | 129.635030 | 22 |
| 2 | 0.072338 | 197.290397 | 132.896571 | $2,661.734410$ | 18 |
| 3 | 0.023091 | 100.000656 | 199.999679 | 381.488126 | 23 |
| 4 | 0.027396 | 172.442217 | 199.998920 | 543.221925 | 22 |

As for nitrogen dose ( $n$ ) in the first scenario, all plants required a maximum of $100 \mathrm{~kg} . \mathrm{m}^{-2}$, which is the upper limit. In the second scenario, plants 1,3 , and 4 had an optimal nitrogen dose equal to $250 \mathrm{~kg} \cdot \mathrm{~m}^{-2}$, an interior value within the range [of 150,400 ]. In the third scenario, plants 1,3 , and 4 again reached an extreme value of
$200 \mathrm{~kg} \cdot \mathrm{~m}^{-2}$. As for plant 3 in scenarios 2 and 3, an optimal nitrogen dose of $133 \mathrm{~kg} . \mathrm{m}^{-2}$ was obtained, which is also within the intervals $[75,300]$ and $[75,200]$, respectively.

Tables 5, 6, and 7 show the optimal numerical results of net income for each crop and scenario considered (problem [2]).

TABLE 5. Optimal solution $\left(r^{*}, w^{*}, n^{*}\right)$ and net income $N I_{i}\left(r^{*}, w^{*}, n^{*}\right)$ in the three-dimensional box $[0.02,1] \times[50,500] \times$ [0,100], wherein: $\boldsymbol{p}_{\boldsymbol{i}}=77.118775, \boldsymbol{c}_{\boldsymbol{w}}=\mathbf{0 . 1 3 0 1 8 1}, \boldsymbol{c}_{\boldsymbol{n}}=29.924896, \boldsymbol{c}_{\boldsymbol{0}}=3526.003617$.

| PLANT | $r^{*}$ <br> $(\%)$ | $w^{*}$ <br> $(\mathrm{~mm})$ | $n^{*}$ <br> $(\mathrm{~kg})$ | $N I_{i}\left(w^{*}, n^{*}\right)$ <br> $\left(R \$ . h a^{-1}\right)$ | iterations |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.020000 | 266.004924 | 0.000008 | $6,076.890235$ | 82 |
| 2 | 0.072338 | 187.358504 | 0.000007 | $20,1637.752514$ | 79 |
| 3 | 0.023091 | 50.000015 | 0.000008 | $25,206.891131$ | 81 |
| 4 | 0.027396 | 150.344844 | 0.000008 | $37,896.874814$ | 81 |

TABLE 6. Optimal solution $\left(r^{*}, w^{*}, n^{*}\right)$ and net income $N I_{i}\left(r^{*}, w^{*}, n^{*}\right)$ in the three-dimensional box $[0.02,1] \times[150,400] \times$ [75,300], wherein: $\boldsymbol{p}_{\boldsymbol{i}}=73.554399, \boldsymbol{c}_{\boldsymbol{w}}=\mathbf{0 . 0 0 7 1 7 9}, \boldsymbol{c}_{\boldsymbol{n}}=31.521244, \boldsymbol{c}_{\mathbf{0}}=2966.241626$.

| PLANT | $r^{*}$ <br> $(\%)$ | $w^{*}$ <br> $(\mathrm{~mm})$ | $n^{*}$ <br> $(\mathrm{~kg})$ | $N I_{i}\left(w^{*}, n^{*}\right)$ <br> $\left(R \$ . h a^{-1}\right)$ | iterations |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.020000 | 283.598369 | 75.000001 | $4,018.559499$ | 23 |
| 2 | 0.072338 | 196.714125 | 75.000001 | $190,435.879388$ | 19 |
| 3 | 0.023091 | 150.000010 | 75.000001 | $22,267.758534$ | 23 |
| 4 | 0.027396 | 171.162354 | 75.000001 | $34,406.058711$ | 23 |

As can be seen in the above tables, solar radiation values in the three scenarios are again invariant and have the same values as those for productivity maximization. The lowest solar radiation ( $2 \%$ ) was obtained for plants 1,3 , and 4 , while the highest (7\%) was for plant 3 .

TABLE 7. Optimal solution $\left(r^{*}, w^{*}, n^{*}\right)$ and net income $N I_{i}\left(r^{*}, w^{*}, n^{*}\right)$ in the three-dimensional box $[0.02,1] \times[100,600] \times$ [75,200], wherein: $\boldsymbol{p}_{\boldsymbol{i}}=\mathbf{5 5 . 9 2 7 9 7 5}, \boldsymbol{c}_{\boldsymbol{w}}=\mathbf{0 . 1 1 5 6 5 0}, \boldsymbol{c}_{\boldsymbol{n}}=27.399057, \boldsymbol{c}_{\mathbf{0}}=2902.185549$.

| PLANT | $r^{*}$ <br> $(\%)$ | $w^{*}$ <br> $(\mathrm{~mm})$ | $n^{*}$ <br> $(\mathrm{~kg})$ | $N I_{i}\left(w^{*}, n^{*}\right)$ <br> $\left(R \$ . h a^{-1}\right)$ | iterations |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.020000 | 261.803849 | 75.000001 | $2,121.380755$ | 23 |
| 2 | 0.072338 | 185.124526 | 75.000001 | $143,875.052563$ | 19 |
| 3 | 0.023091 | 100.000032 | 75.000001 | $16,073.656423$ | 23 |
| 4 | 0.027396 | 145.374247 | 75.000003 | $25,239.492131$ | 22 |

In the first scenario $([0.02,1] \times[50,500] \times$ $[0,100]$ ), optimal water depths were within the range [of 50,500 ], except for plant 3 which reached a value equal to the lower limit ( 50 mm ). The highest water depth was 283 mm . We can also see in this scenario that $n^{*}=$ 0 for all plants, that is, no nitrogen dose was required. Among all plants, 4 had the highest net income ( $\mathrm{R} \$ 37,896.874814$ ) with 150 mm water depth, while 1 reached the lowest ( $\mathrm{R} \$ 6,076.890235$ ) with 266 mm .

In the second scenario $([0.02,1] \times[150,400] \times$ [75,300]), optimal water depths were within the range [of 150,400 ], except for plant 3 which reached a value equal to the lower limit $(150 \mathrm{~mm})$. All plants in this scenario required at least an application of the lowest nitrogen dose $(75 \mathrm{~kg})$. Among all plants, 2 reached the highest net income ( $\mathrm{R} \$ 190,435.879388$ ) with 197 mm water depth, and again plant 1 had the lowest net income ( $\mathrm{R} \$ 4,018.559499$ ) with 284 mm .

Finally, in the third scenario $([0.02,1] \times$ $[100,600] \times[75,200])$, Table 7 shows that despite the
values of $p_{i}, c_{w}, c_{n}, c_{0}$ being different from those in Table 6 , they had a similar trend in which optimal water depths were within the range $[100,600]$, except for plant 3 that reached a value equal to the lower limit $(100 \mathrm{~mm})$. As in the previous scenario (Table 6), all plants equally required an application of at least the lowest nitrogen dose ( 75 kg ), and plant 2 reached the highest net income ( $\mathrm{R} \$ 143,875.052563$ ) with 185 mm water depth. Likewise, plant 1 also reached the lowest net income ( $\mathrm{R} \$ 2,121.380755$ ) with 262 mm water depth.

## CONCLUSIONS

- This study presents computational procedures to solve problems to maximize production (1) and net income (2) of a certain agroforestry crop under a randomly generated "ideal" agroforestry system with limited inputs, based on the "logarithmic barrier" method.
- The numbers of iterations performed by procedures are low; between 18 and 24 for production maximization, and from 19 to 82 for net income maximization. The first scenario has the largest number of iterations by the procedure implemented and for all plants evaluated.
- The procedures implemented with randomly generated data are consistent and can solve problems (1) and (2) with real data.
- All plants required no nitrogen application ( $n^{*}=$ 0 ). This is feasible because fertilization in AFS can be naturally made using available resources and forest nutrient cycling dynamics, through tree pruning and green manuring.


## REFERENCES

Bertsekas DP (2016) Nonlinear programming. Belmont, Athena Scientific. 773p.

Carvalho D de, Delgado ARS, De Oliveira R, Da Silva WA, Do Forte VL (2009) Maximização da produção e da receita agrícola com limitações de água e nitrogênio utilizando método de pontos interiores. Engenharia Agrícola 29(2):321-327.

Confalone A, Costa LC, Pereira CR (1997) Eficiência do da radiação em distintas fases fenológicas bajo estres hídrico. Revista de la Faculdad de Agronomia de la Universidad del Zulia 17(1): 63-66.

Delgado ARS, Drumond SV, Ferreira PM (2020).
Agricultural optimization with limited resources using duality. Pesquisa e Ensino em Ciências Exatas e da Natureza 4: (e1477): 1-10.

Drumond SV, Delgado ARS, Gonzaga CC (2015) A nonlinear feasibility problem heuristic. Pesquisa Operacional 35(1):107-121. DOI:
https://doi.org/10.1590/0101-7438.2015.035.01.0107.
Fonseca J (2017) Método de Newton generalizado e aplicações. Dissertação, Mestrado, Belém. Universidade Federal do Pará, Instituto de Ciências Exatas e Naturais.

Götsch E (1997) Homem e natureza: cultura na agricultura Recife: Centro Sabiá, 1997. Available: http://www.unigaiabrasil.org/pdfs/SAFS/Homem_e_Natureza.pdf. Accessed Sep 2, 2016.

Götz S, Edenise G, Bronson WG, Wenceslau GT, Lucyana PB (2016) Commodity production as restoration driver in the Brazilian Amazon? Pasture re-agro-forestation with cocoa (Theobroma cacao) in southern Pará. Sustain Sci. 11:277293. DOI: https://doi.org/10.1007/s1 1625-015-0330-8.

Pereira CR (2002) Análise do crescimento e desenvolvimento da cultura de soja sob diferentes condições ambientais. Tese, Doutorado, Viçosa. Universidade Federal de Viçosa.

Plevich JO, Gyenge J, Delgado ARS, Tarico JC, Utello MJ, Fiandino S (2019) Production of fodder in a treeless system and in silvopastoral system in central Argentina. FLORAM 26(1):1-12

## ERRATUM

In the paper "MAXIMIZATION OF PRODUCTION AND NET INCOME IN AGRO-FOREST SYSTEMS", with DOI number: 10.1590/1809-4430-Eng.Agric.v42n2e20210017/2022, published in the journal Agricultural Engineering 42(2):e20210017, on page 1:

Where it reads:

Vinicius T. do Nascimento ${ }^{1 *}$, Sergio D. Ventura ${ }^{1}$, Angel R. S. Delgado ${ }^{1}$, Washigton S. da Silva ${ }^{1}$

It should read:

Vinicius T. do Nascimento ${ }^{1 *}$, Sergio D. Ventura ${ }^{1}$, Angel R. S. Delgado ${ }^{1}$, Washington S. da Silva ${ }^{1}$


[^0]:    ${ }^{1}$ Universidade Federal Rural do Rio de Janeiro - UFRRJ/ Seropédica - RJ, Brasil,

